Collisional model with correlated environment in the matrix product form

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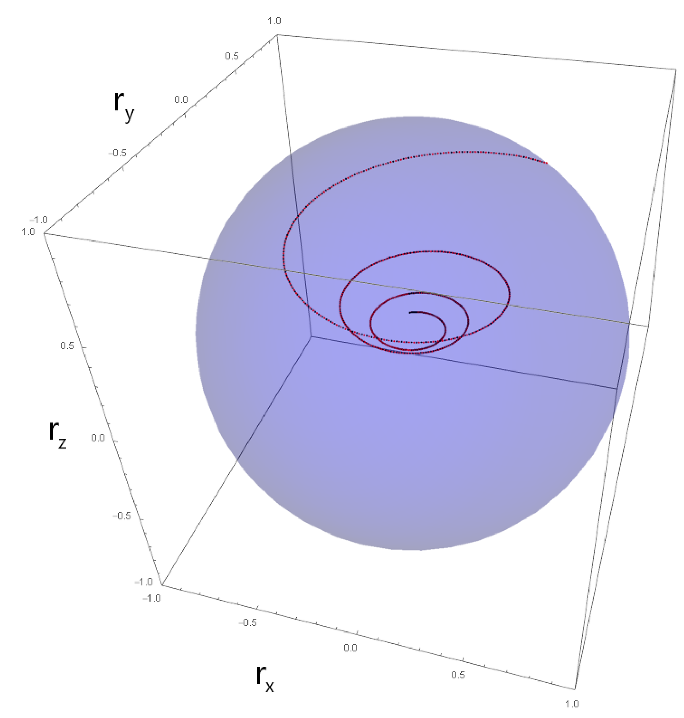

Abstract

Collisional model describes an open quantum system interacting sequentially with subenvironments. If all the subenvironments are initially identical and there are no correlations among them, then the system dynamics is Markovian in the stroboscopic limit [1]. However, if the environment is in the correlated state, then such correlations naturally affect system dynamics and can make it strictly non-Markovian [2]. Divisibility property of collisional models is analyzed in [3]. In present report, we consider a particular form of correlations in the environment, namely, the environment is in the matrix product state (MPS). We develop Nakajima-Zwanzig projective techniques for such a case and derive the corresponding master equation. The memory kernel is expressed through matrices defining MPS. The considered model is able to describe spin transport in carbon chains [4].

Collisional model

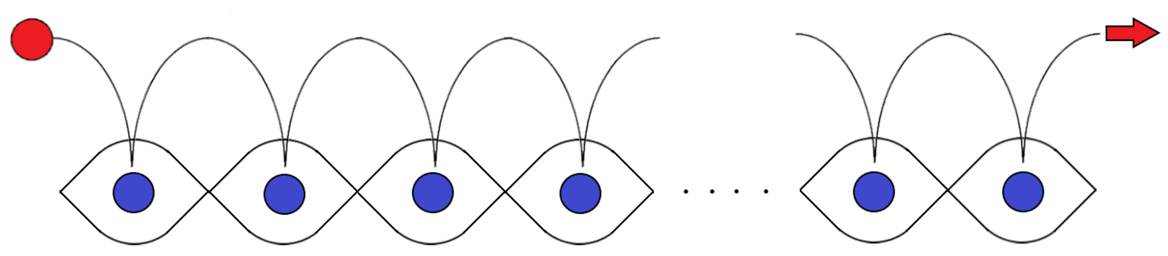

Figure 1: Collisional model for a system (red) interacting sequentially with particles of correlated environment (blue).

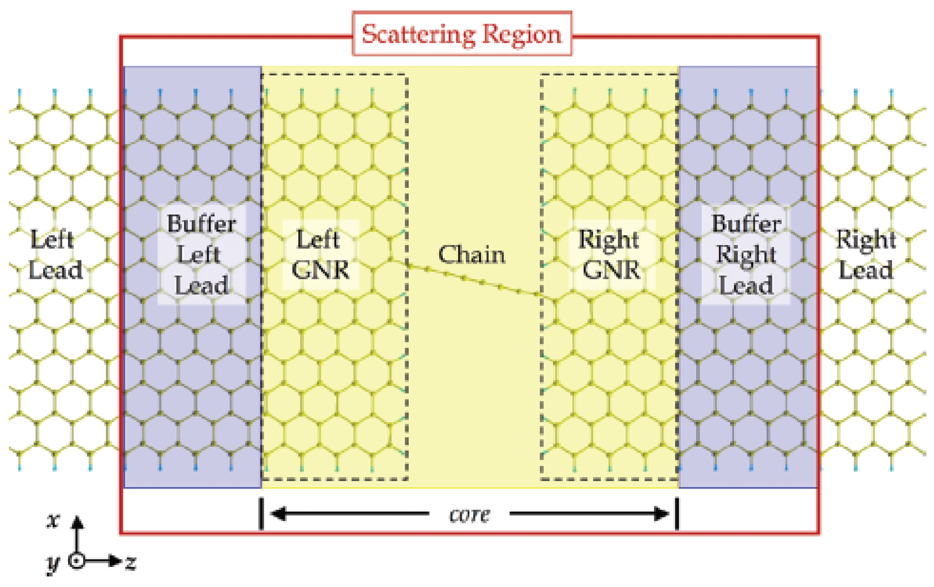

Figure 2: Relevance to calculation of spin current through one-dimensional structures. Image from the paper [4].

Matrix product states (MPS)

General state

$$|\psi\rangle = \sum_{i_1,\dots,i_N} c_{i_1\dots i_N}|i_1\dots i_N\rangle. \tag{1}$$

Matrix product approximation

$$|\psi\rangle = \sum \langle l|A^{(1)}_{i_1}\dots A^{(N)}_{i_N}|r\rangle|i_1\dots i_N\rangle. \tag{2}$$

Figure 3: Tensor network representation of MPS.

Examples:
GHZ state $\frac{1}{\sqrt{2}}(|00\dots0\rangle + |11\dots1\rangle)$ corresponds to

$$A_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad A_1 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \tag{3}$$

AKLT state of qutrit-chain [5] with finite correlation length:

$$A_+ = \begin{pmatrix} 0 & \sqrt{2/3} \\ 0 & 0 \end{pmatrix}, \quad A_0 = \begin{pmatrix} -1/\sqrt{3} & 0 \\ 0 & 1/\sqrt{3} \end{pmatrix}, \quad A_- = \begin{pmatrix} 0 & 0 \\ -\sqrt{2/3} & 0 \end{pmatrix}. \tag{4}$$

Density matrix of environment:

$$\rho_{\rm env} = \sum_{\substack{i_1,\dots,i_N \\ j_1,\dots,j_N}} \langle\xi| \left(A^{(1)}_{i_1} \otimes A^{(1)*}_{j_1}\right) \dots \left(A^{(N)}_{i_N} \otimes A^{(N)*}_{j_N}\right) |\xi\rangle \; |i_1\dots i_N\rangle\langle j_1\dots j_N|. \tag{5}$$

Change of notation:

$$\sum_{i,j} A_i \otimes A_j^* = \sum_k \mathcal{A}_k. \tag{6}$$

Then

$$\rho_{\rm env} = \sum_{k_1,\dots k_N} \langle\xi|\mathcal{A}_{k_1}\dots\mathcal{A}_{k_N}|\xi\rangle \; |i_1\dots i_N\rangle\langle j_1\dots j_N|. \tag{7}$$

Derivation of master equation

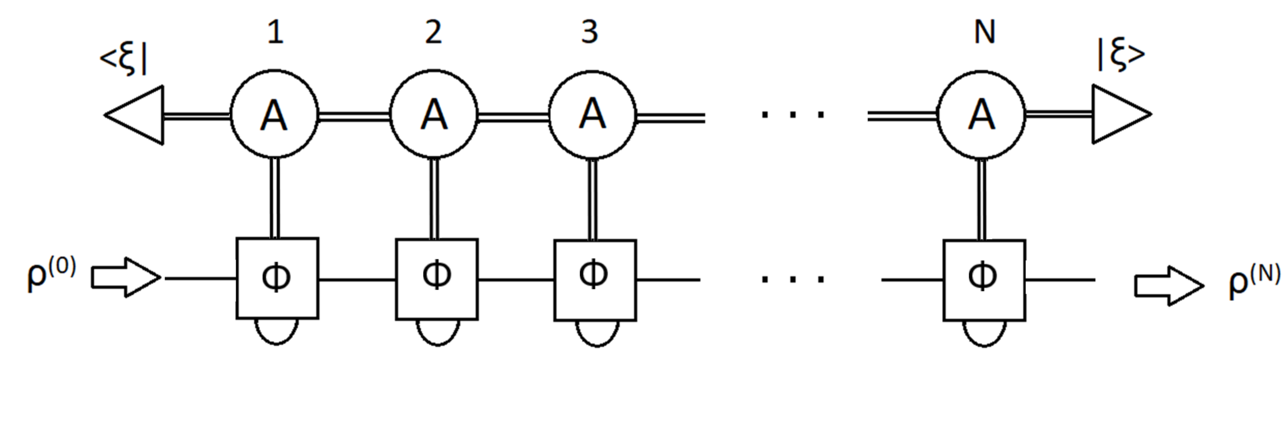

Figure 4: Collisional model in terms of tensor networks.

Goal: $\frac{d\rho}{dt} = \int_0^t \mathcal{K}(t-t')[\rho(t')]dt'$.
Environment + system:

$$R^{(0)} = |\xi\rangle \otimes \rho^{(0)}, \tag{8}$$

$$R^{(N+1)} = \sum_k \hat{\mathcal{A}}_k \otimes \Phi_k[R^{(N)}], \tag{9}$$

$$R^{(N+1)} = \sum_{k,k_0,\dots,k_{N-1}} \langle\xi|\hat{\mathcal{A}}_k\hat{\mathcal{A}}_{k_0}\dots\hat{\mathcal{A}}_{k_{N-1}}|\xi\rangle\Phi_k\Phi_{k_0}\dots\Phi_{k_{N-1}}[\rho^{(0)}], \tag{10}$$

$$\Phi_k[\rho] = tr_{env}[U(|i\rangle\langle j| \otimes \rho)U^\dagger]. \tag{11}$$

Nakajima-Zwanzig projectors

$$\hat{P} = |\xi\rangle\langle\xi|, \tag{12}$$

$$\hat{Q} = \hat{I} - |\xi\rangle\langle\xi|. \tag{13}$$

satisfy relations $\hat{P}\hat{P} = \hat{P}$, $\hat{Q}\hat{Q} = \hat{Q}$, $\hat{P}\hat{Q} = \hat{q}\hat{P} = 0$, $\hat{P} + \hat{Q} = \hat{I}$ and allow to express the system density operator:

$$\hat{P}[R^{(n)}] = |\xi\rangle \otimes \rho^{(n)}. \tag{14}$$

Applying projection technique to (9), we obtain

$$\begin{cases} \hat{P}R^{(N+1)} = \sum_k \hat{P}\hat{\mathcal{A}}_k \otimes \Phi_k[\hat{P}R^{(N)}] + \sum_k \hat{P}\hat{\mathcal{A}}_k \otimes \Phi_k[\hat{Q}R^{(N)}], \\ \hat{Q}R^{(N)} = \sum_{k_0} \hat{Q}\hat{\mathcal{A}}_{k_0} \otimes \Phi_{k_0}[\hat{Q}R^{(N-1)}] + \sum_{k_0} \hat{Q}\hat{\mathcal{A}}_{k_0} \otimes \Phi_{k_0}[\hat{P}R^{(N-1)}]. \end{cases} \tag{15}$$

Taking into account that $\hat{Q}R^{(0)} = 0$, we get

$$\hat{Q}R^{(N)} = \sum_{n=0}^{N-1} \sum_{k_0,\dots,k_n} \hat{Q}\hat{\mathcal{A}}_{k_0}\dots\hat{Q}\hat{\mathcal{A}}_{k_n} \otimes \Phi_{k_0}\dots\Phi_{k_n}[\hat{P}R^{(N-n-1)}]. \tag{16}$$

Substituting (16) in (15) yields

$$\hat{P}R^{(N+1)} = \sum_k \hat{P}\hat{\mathcal{A}}_k \otimes \Phi_k[\hat{P}R^{(N)}] + \sum_{n=0}^{N-1} \sum_{k,k_0,\dots,k_n} \hat{P}\hat{\mathcal{A}}_k\hat{Q}\hat{\mathcal{A}}_{k_0}\dots\hat{Q}\hat{\mathcal{A}}_{k_n} \otimes \Phi_k\Phi_{k_0}\dots\Phi_{k_n}[\hat{P}R^{(N-n-1)}]. \tag{17}$$

Finally, we get the master equation in the form of recurrence relation

$$\rho^{(N+1)} = \sum_k \langle\xi|\hat{\mathcal{A}}_k|\xi\rangle\Phi_k[\rho^{(N)}] + \sum_{n=0}^{N-1} \sum_{k,k_0,\dots,k_n} \langle\xi|\hat{\mathcal{A}}_k\hat{Q}\hat{\mathcal{A}}_{k_0}\dots\hat{Q}\hat{\mathcal{A}}_{k_n}|\xi\rangle\Phi_k\Phi_{k_0}\dots\Phi_{k_n}[\rho^{(N-n-1)}]. \tag{18}$$

Let τ be the duration of collision and γ be the interaction strength. Assume $\gamma\tau \ll 1$, then

$$\rho^{(N+1)} = \rho^{(N)} + \gamma\tau L_1[\rho^{(N)}] + \gamma^2\tau^2 L_2[\rho^{(N)}] + \gamma^2\tau^2 \sum_{n=0}^{N-1} K(n)[\rho^{(N-1-n)}]. \tag{19}$$

Since $\frac{\rho(t+\tau)-\rho(t-\tau)}{2\tau} = \frac{d\rho(t)}{dt} + O(\tau^2)$, in the limit $\gamma\tau \to 0$ and $\gamma^2\tau \to {\rm const}$ we get:

$$\begin{aligned} \frac{d\rho(t)}{dt} &= \frac{1}{2}\gamma L[\rho(t-\tau)] + \frac{1}{2}\gamma L[\rho(t)] \\ &+ \frac{1}{2}\gamma^2\tau \sum_{n=0}^{t/\tau-1} K(n\tau)[\rho(t-n\tau-\tau)] + \frac{1}{2}\gamma^2\tau \sum_{n=0}^{t/\tau-2} K(n\tau)[\rho(t-n\tau-2\tau)]. \end{aligned} \tag{20}$$

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