Surface states in p-wave superconductors and odd-frequency pairing

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ABS
F/S junction
Odd-frequency pairing
Vortex
Proximity

MESA+
INSTITUTE FOR NANOTECHNOLOGY

МФТИ
Collaboration

- Y. Tanaka and K. Yada, Nagoya University, Japan
- Y. Asano, Hokkaido University, Japan
- S. Kashiwaya, AIST, Tsukuba, Japan
- S.V. Bakurskiy and M. Yu. Kupriyanov, Moscow State University, Russia
Contents

- What is odd-frequency pairing
- Relation to Andreev bound states
- Impurity scattering effect
- Anomalous surface impedance
- Effect of surface roughness
Chiral p-wave superconductors

Sr$_2$RuO$_4$

Maeno (1994)

Spin-triplet p-wave

Time-reversal symmetry broken

Topological index for chirality

\[ N = \frac{1}{4\pi} \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y \hat{m} \cdot \left( \frac{\partial \hat{m}}{\partial k_x} \times \frac{\partial \hat{m}}{\partial k_y} \right) \]

\[ \hat{m} = \frac{m}{|m|}, \quad m = (\text{Re}d_z, \text{Im}d_z, \epsilon_k) \]

Volovik
Conventional Classification of Symmetry of Cooper pair

Spin-singlet Cooper pair  $\rightarrow$  Even Parity

Spin-triplet Cooper pair  $\rightarrow$  Odd Parity

BCS

s-wave

d-wave

p-wave

$^3\text{He}$  Sr$_2$RuO$_4$
Odd-frequency pairing

**Fermi-Dirac statistics**

Symmetry of pair wave functions:

\[ \mathbf{k} \otimes \sigma \otimes \omega = \text{odd} \]

*Momentum x Spin x Frequency*

**Berezinskii**  
(1974):  
*Spin-triplet s-wave*

**Balatsky & Abrahams**  
(1992):  
*Spin-singlet p-wave*

\[ \Delta(t-t') \]

\[ \Delta(\omega), f(\omega) \]

\( \omega \), Matsubara frequency
Pair amplitude

Exchange of time

Even-frequency pairing (conventional pairing)

\[ F_{\alpha,\beta}(r_1t_1, r_2t_2) = F_{\alpha,\beta}(r_1t_2, r_2t_1) \]

Odd-frequency pairing

\[ F_{\alpha,\beta}(r_1t_1, r_2t_2) = -F_{\alpha,\beta}(r_1t_2, r_2t_1) \]
Symmetry of the pair amplitude
+ symmetric, − anti-symmetric

<table>
<thead>
<tr>
<th></th>
<th>Frequency (time)</th>
<th>Spin</th>
<th>Orbital</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>ESE</td>
<td>+(even)</td>
<td>− (singlet)</td>
<td>+(even)</td>
<td>−</td>
</tr>
<tr>
<td>ETO</td>
<td>+(even)</td>
<td>+ (triplet)</td>
<td>−(odd)</td>
<td>−</td>
</tr>
<tr>
<td>OTE</td>
<td>−(odd)</td>
<td>+ (triplet)</td>
<td>+(even)</td>
<td>−</td>
</tr>
<tr>
<td>OSO</td>
<td>−(odd)</td>
<td>− (singlet)</td>
<td>−(odd)</td>
<td>−</td>
</tr>
</tbody>
</table>

ESE (Even-frequency spin-singlet even-parity)
ETO (Even-frequency spin-triplet odd-parity)
OTE (Odd-frequency spin-triplet even-parity) Berezinskii
OSO (Odd-frequency spin-singlet odd-parity) Balatsky, Abrahams

**BCS Cuprate**
**$^3$He Sr$_2$RuO$_4$**
• **Odd-frequency pairing state** is possible in inhomogeneous superconductors even for conventional even-frequency pairing in the bulk

**Broken spin rotation symmetry or spatial invariance symmetry can induce odd-frequency pairing state:**

- **ferromagnet/superconductor junctions:**
  Bergeret, Volkov & Efetov, 2001

- **non-uniform systems:**
  
  *Junctions:* Tanaka & Golubov, 2007; Eschrig & Lofwander, 2007
  
  *Vortices:* Yokoyama *et al.*, 2008; Tanuma *et al.*, 2009)
How to produce long range odd-frequency triplet from singlet s-wave pairing in S/F hybrid system?

F (Ferromagnet) : Diffusive

We consider only s-wave pairing here.

One needs two ferromagnets to generate long range odd-frequency pairing

OTE $S_z = 0$, $d \parallel z$-axis, $S \perp z$-axis

OTE $S_z = \pm 1$, $d \perp z$-axis, $S \parallel z$-axis


ESE (Even-frequency spin-singlet even-parity)
OTE (Odd-frequency spin-triplet even-parity)
Contents

- What is odd-frequency pairing
- Relation of odd-$\omega$ to Andreev bound states
- Impurity scattering effect
- Anomalous surface impedance
- Effect of surface roughness
Andreev reflection

Normal metal

Superconductor

- Electron
- Hole

Cooper pair

- Fermi energy
- Energy gap

Andreev 1964
Andreev reflection and Andreev bound state

Inner gap state (Andreev bound state) is realized by Andreev reflection. (McMillan Rowell, 1966)

Josephson current can be expressed by the probability of Andreev reflection. (Furusaki, Tsukada, 1990)
Are Andreev bound states related to odd-frequency state?

Yes
Andreev bound state
(non-topological and topological)

Andreev bound state with non zero energy (de Gennes, Saint James)

Not edge state

Non topological

Mid gap (zero energy) Andreev bound state

Surface Andreev bound state

Edge state  Topological

L. Buchholtz & G. Zwicknagl (81); J. Hara & K. Nagai: Prog. Theor. Phys. 74 (86)
C.R. Hu: (94)
Odd-frequency pairing in the presence of nonzero energy ABS

OSO component is generated from superconductor with ESE symmetry.

Y. Tanaka, Y. Tanuma A.A. Golubov, PRB 76 054522 (2007)
Andreev bound state (non-zero) generated by quasiparticle interference in $N$

**Andreev Bound states condition** *(non zero energy)*
(*de Gennes, Saint-James, McMillan Thomas Rowell)*

$$\epsilon_n = \pm \frac{\pi v_F x}{2L} (n + 1/2), \quad n = 0, 1, 2, \ldots \quad \Delta_0 \gg |\epsilon_n|$$

Fully transparent interface

$$\frac{|f_{1+}^{(N)}(\epsilon, \theta)|}{|f_{2+}^{(N)}(\epsilon, \theta)|} = |\tan (\pi/2 + \pi n)| = \infty.$$

$\Delta_0$ odd-frequency pairing

$\Delta_0$ even-frequency pairing

Bound state in the junctions can be expressed by the generation of the odd-frequency Cooper pair (pair amplitude).

Y. Tanaka, Y. Tanuma and A.A. Golubov, PRB 76 054522 (2007)
Superconducting Materials where MARS is observed

YBa$_2$CuO$_{7-\delta}$ (Geerk, Kashiwaya, Iguchi, Greene, Yeh, Wei..)
Bi$_2$Sr$_2$CaCu$_2$O$_y$ (Ng, Suzuki, Greene....)
La$_{2-x}$Sr$_x$CuO$_4$ (Iguchi)
La$_{2-x}$Ce$_x$CuO$_4$ (Cheska)
Pr$_{2-x}$Ce$_x$CuO$_4$ (R.L. Greene)
Sr$_2$RuO$_4$ (Mao, Meno, Kawamura, Laube)
κ-(BEDT-TTF)$_2$X, X=Cu[N(CN)$_2$]Br (Ichimura)
UBe$_{13}$ (Ott)
CeCoIn$_5$ (Wei Greene)
PrOs$_4$Sb$_{12}$ (Wei)
Superfluid $^3$He (Okuda, Nomura, Higashitani, Nagai)
Andreev bound state in unconventional pairing

Mid gap Andreev bound state (MARS)

$\Delta_+ \Delta_- < 0$

Local density of state has a zero energy peak.
(Sign change of the pair potential at the interface)

Mid gap Andreev resonant (bound) state

$$\Delta_+ \Delta_- < 0$$

Electron-like quasiparticle

Hole-like quasiparticle

Odd-frequency Cooper pair (Odd-frequency pair amplitude)

Tanaka, Golubov, Asano:

**Sr$_2$RuO$_4$(1.5K phase)/Au junction fabrication**

Sr$_2$RuO$_4$ vacuum cleavage

↓

Au deposition in-situ

SIM image of junction

Zero-bias conductance peak observation between Sr$_2$RuO$_4$ (1.5K phase) and Au

Consistent with theoretical calculation based on chiral p-wave pairing


Feature of the Andreev bound states

$\mathbf{d}_{xy}$ wave

$p_x + ip_y$

$E = 0$

$E = 0$

$\text{Hu}(94)$

Tanaka Kashiwaya (95)

Tanaka Kashiwaya (97)

Sigrist Honerkamp (98)
spin-triplet superconductor junctions

Ballistic Normal metal | Superconductor
---|---

Impurity scattering (isotropic)

Diffusive Normal metal (DN) | Superconductor

Proximity effect in aerogel, Higashitani, Nagato, and Nagai (2009)
Eilenberger Equation

\[-i \mathbf{v}_F \cdot \nabla \hat{g}^R = \left[ \hat{H}_{qc} + \sum^R, \hat{g}^R \right],\]

\[\hat{H}_{qc} = \begin{pmatrix} \varepsilon & \Delta(\hat{p},r) \\ -\Delta^*(\hat{p},r) & -\varepsilon \end{pmatrix}, \quad \sum^R = \frac{i}{2 \tau_{imp}} \langle \hat{g}^R \rangle\]

\[\hat{g}^R \hat{g}^R = \hat{1} \quad \sum^R = 0 \text{ (ballistic)}\]

Quasiclassical approximation

( 1 ) Cooper pair is formed by two electrons on the Fermi surface

( 2 ) The effective pair potential is determined by the direction of the motion of electrons.

Eilenberger equation

Quasiclassical Green’s function (Matsubara representation)

\[\mp i v_F x \partial_x f_{1\pm} = 2\omega_n f_{2\pm} - 2\bar{\Delta}_{\pm}(x) g_{\pm}\]
\[\mp i v_F x \partial_x g_{\pm} = 2\bar{\Delta}_{\pm}(x) f_{1\pm},\]
\[\mp i v_F x \partial_x f_{2\pm} = -2\omega_n f_{1\pm},\]

Pair potential \(\bar{\Delta}_{\pm}(x)\)

Quasiparticle \(g_{\pm}\)

Cooper pair which does not exist in bulk \(f_{1\pm}\)

Cooper pair in bulk \(f_{2\pm}\)

Direction of motion \(+\)

Normal metal \(\bar{\Delta}_{+}(x)\)

Superconductor \(\bar{\Delta}_{-}(x)\)

Odd-frequency

Even-frequency
Condition of the generation of odd-frequency pairing

\[
\mp iv_{Fx} \partial_x f_{1\pm} = 2\omega_n f_{2\pm} - 2\bar{\Delta}_{\pm}(x) g_{\pm} \\
\mp iv_{Fx} \partial_x g_{\pm} = 2\bar{\Delta}_{\pm}(x) f_{1\pm}, \\
\mp iv_{Fx} \partial_x f_{2\pm} = -2\omega_n f_{1\pm},
\]

\[
v_{F_x}^2 \partial_x^2 f_{1\pm} - 4(\omega_n^2 + \bar{\Delta}_{\pm}^2(x)) f_{1\pm} \pm 2i |v_{Fx}| [\partial_x \bar{\Delta}_{\pm}(x)] g_{\pm} = 0
\]

Spatial change of pair potential
+ nonzero quasiparticle state

Odd-frequency pairing

General theory, Higashitani 2011
Underlying physics

Near the interface, even and odd-parity pairing states (pair amplitude) can mix due to the breakdown of the translational symmetry.

Fermi-Dirac statistics

The interface-induced state (pair amplitude) should be odd in frequency where the bulk pair potential has an even-frequency component since there is no spin flip at the interface.
Contents

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Impurity scattering effect

Diffusive Normal metal (DN)  Superconductor

Impurity scattering (isotropic)

Only s-wave pair amplitude exists in DN

(1) ESE
(2) OTE

ESE (Even-frequency spin-singlet even-parity)
Usadel equation

Available for diffusive limit $\tau_{imp}T \ll 1$

$$D \nabla (\hat{g}_0^R \nabla \hat{g}_0^R) + i[\hat{H}_0, \hat{g}_0^R] = 0$$

Angular average

$\hat{g}_0^R \rightarrow \hat{g}_R$  

Diffusive limit

Diffusive normal metal region attached to superconductor

$$\hat{H}_0 = \varepsilon \hat{\tau}_3$$

Boundary condition available for unconventional superconductors

Even frequency spin singlet even parity (ESE) pair potential

\[ \text{Real}[f(\varepsilon)] = \text{Real}[f(-\varepsilon)] \]
\[ \text{Imag}[f(\varepsilon)] = -\text{Imag}[f(-\varepsilon)] \]

Even frequency spin singlet \text{s-wave (ESE)} pair is induced in DN.
Odd frequency spin triplet s-wave (OTE) pair is induced in DN

New type of proximity effect
Unconventional proximity effect

Odd-frequency pairing at the interface includes s-wave component

No proximity effect

Odd-frequency pairing at the interface: Odd-parity
**STS experiments**

Proximity effect via odd-frequency pairing

- **LDOS in DN has a zero energy peak**
- **Diffusive normal Metal (DN)**
- **Spin-Triplet superconductor**

Conventional proximity effect

- **LDOS in DN has a gap**
- **Diffusive normal Metal (DN)**
- **Spin-Singlet superconductor**

DoS peak is robust against impurity scattering
Anomalous proximity effect in nano-wire

Topological

\[ V_Z > \sqrt{\Delta^2 + \mu^2} \]

(Effective triplet \( p_x \)-wave superconductor)

Non Topological

\[ V_Z < \sqrt{\Delta^2 + \mu^2} \]

LDOS in Nano wire (Disordered segment)

\[ \mu = t \]

robust zero energy peak of LDOS in topological phase!!

\[ V_{ex} = V_Z > \mu \]

Similar anomalous proximity effect has been predicted in spin-triplet \( p \)-wave superconductor. Tanaka and Kashiwaya, PRB 2004
Odd-frequency pairing and LDOS

LDOS and pair amplitude at $\varepsilon = 0$

Re($g_{\uparrow\uparrow}$)  LDOS
Im($f_{\uparrow\uparrow}$)  Odd-frequency pair amplitude

Even-frequency pairing
$f_{\sigma\sigma'}(\varepsilon) = f_{\sigma\sigma'}^*(-\varepsilon)$

Odd-frequency pairing
$f_{\sigma\sigma'}(\varepsilon) = -f_{\sigma\sigma'}^*(-\varepsilon)$

Odd-frequency pairing always exists in topological phase where Majorana mode (fermion) exists

Y. Asano and Y. Tanaka  PRB87 104513 (2013)
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Surface impedance of a superconductor

Response to electromagnetic field

\[ Z = \frac{4\pi E}{c H} = R + iX \]

\[ R + iX = \left[ \frac{i\omega \mu_0}{\sigma_1 - i\sigma_2} \right]^{1/2} \]

- **R**: surface resistance
  - loss of microwave power due to normal carrier

- **X**: reactance
  - response of superconducting carriers
Figure (a) shows the ratio $Z_s/Z_n$ as a function of $T/T_c$ for an s-wave superconductor, with $\omega / \Delta_0 = 0.1$. The graph displays two curves: one in red labeled X and another in blue labeled R.

Figure (b) illustrates a cube representing the transition region between a superconductor and a normal metal, with dimensions labeled W and W+2L.

Figure (c) depicts the overlap between a superconductor and a normal metal, with normal modes labeled s-wave, d-wave, and p-wave, and an angle $\alpha$ indicated.
Unusual electromagnetic response

\[
J = -\frac{c}{4\pi\lambda^2} A
\]

\[
\lambda(x)^{-2} = \frac{K}{\xi_N^2(T_c)} \frac{T}{T_c} \sum_{\omega_n > 0} F_{\omega_n}^2(x)
\]

Tanaka, Asano, Golubov, Kashiwaya, PRB 72, 140503R (05); PRB 73, 059901 (06):

even-freq. proximity \( \lambda^2 > 0 \)
odd-freq. proximity \( \lambda^2 < 0 \)

\[
R + iX = \left[ \frac{4\pi i\omega}{c^2(\sigma_1 - i\sigma_2)} \right]^{1/2}
\]

\[
\sigma_1 = \frac{n_n e^2 \tau}{m}
\]
\[
\sigma_2 = \frac{n_n e^2 \tau}{m} \omega \tau + \frac{c^2}{4\pi\lambda_L^2 \omega}
\]

even freq. pairs \( R < X \)
odd freq. pairs \( R > X \): unusual relation
The result: local impedance of the n-layer

Contents

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- Effect of surface roughness in $p$-wave superconductors
Model for surface roughness

Eilenberger equations

\[ 2\omega f(x, \theta) + v \cos(\theta) \frac{d}{dx} f(x, \theta) = 2\Delta g(x, \theta) + \frac{1}{\tau} (g \langle f \rangle - f \langle g \rangle), \]
\[ 2\omega g(x, \theta) - v \cos(\theta) \frac{d}{dx} f^+(x, \theta) = 2\Delta g(x, \theta) + \frac{1}{\tau} (g \langle f^+ \rangle - f^+ \langle g \rangle), \]
\[ 2v \cos(\theta) \frac{d}{dx} g(x, \theta) = 2\Delta (f - f^+) + \frac{1}{\tau} (f \langle f \rangle - f^+ \langle g \rangle), \]

Self-Consistent equation (for p_x-pairing)

\[ \delta \ln \frac{T}{T_c} + 2\pi T \sum_\omega \langle 2 \cos(\theta' - \alpha) f(x, \theta') \rangle = 0. \]

Ovchinnikov model: roughness parameter \( d/l_e \)

Clean p-wave superconductor

Dirty spacer
Pair potential $\Delta$

$p_x$ state:
$\Delta = \Delta_0 \cos(\theta)$

Chiral $p_x + i p_y$ state:
$\Delta = \Delta_x \cos(\theta) + i \Delta_y \sin(\theta)$

Pair amplitude $f$

($p_x$-pairing)

$$f = f(\theta)$$

Show average value

$$\langle f \rangle = \int f(\theta) \, d\theta$$

Interface splits $f$ into two components

$$f = f_1 + f_2$$

Odd-frequency component $f_1$

Even-frequency component $f_2$

In a bulk only even-frequency component $f_2$ exists
Chiral p-wave state: pair amplitudes

Angle-resolved pairing amplitudes at a rough surface ($k_y = k_F \sin \theta$)

Odd-$\omega$

Even-$\omega$

$x=0$  \hspace{1cm} $d/l=1$

Clean p-wave SC \hspace{1cm} Dirty spacer
Angle-resolved surface density of states ($k_y = k_F \sin \theta$)

- subgap spectrum broadened by surface roughness
- additional subgap bound states appear:
  enhancement of low-energy DoS by disorder
Spatial distribution of angle-resolved DOS at $k_y = 0$

Angle-averaged surface DOS: topologically protected midgap bound states

Clean $p_x$-wave SC

Dirty spacer
Spatial distribution of angle-resolved DOS at $k_y = 0$

Angle-averaged surface DOS: enhancement of subgap DOS by disorder
Angle-averaged surface DOS

Enhancement of subgap DOS by disordered in chiral $p$=wave

See also S. Murakawa, et al., PRL (2009), JPSJ (2011)
3He-B with a wall coated by 4He
Emergence of sharp edge in DoS at $\Delta_* < \Delta$

Y. Nagato, et al., JLTP (1998)

Origin: suppression of $\Delta_y$ by surface disorder
Summary

- Odd-frequency pairing state is realized in superconductors with broken translational or spin rotation symmetry

- Manifestations of odd-frequency pairing:
  Andreev bound states, anomalous surface impedance, long-range proximity effects in S/F hybrids

- Odd-frequency pairing is generated at rough surfaces in p-wave superconductors. Subgap surface bound states are robust with respect to disorder, as a consequence of odd-frequency pairing