

## Report on the content of the dissertation

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(name of the member of the committee)

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(name of the candidate for the degree)

*Families of Sets with Forbidden Configurations and Applications to Discrete Geometry*  
(Title of the dissertation, degree, specialization)

*doctor of physical and mathematical sciences,*

*05.13.17 – theoretical foundations of computer science*

Date of the defense: 18.05.2019

The evaluation of the dissertation in accordance with the Regulations on the award of scientific degrees of candidates and doctors of sciences at MIPT (in what follows, referred to as Regulations):

1. Relevance of the topic of the dissertation:

Dissertation considers extremal problems for families of subsets of finite sets. In a number of cases the families considered are connected with geometry. Therefore, several results have interesting consequences in geometry and Computer Science. This direction of research is important from several points of view: it is in the focus of the contemporary extremal combinatorics and is closely related to several open problems in Computer Science and geometry. Therefore, the dissertation is a significant contribution to several areas of mathematics.

2. Scientific novelty of the results:

The dissertation contains several significant contributions to our knowledge. In particular, in the dissertation the following results are proved:

- (a) The exact bound on the diversity of an intersecting family of  $k$ -element subsets of an  $n$ -element set is proved.
- (b) It is shown that the typical intersecting family of  $k$ -element subsets of an  $n$ -element set is a star when  $n > (2 + o(1))k$ .
- (c) A classical old result of Kleitman on the size of a maximum family of subsets of an  $n$ -element set with no  $s$  pairwise disjoint sets is extended.
- (d) New bounds on the chromatic number of random subgraphs of the Kneser graph  $KG(n, k)$  are proved.
- (e) The order of magnitude of a minimum  $\varepsilon$ -net in important range spaces is determined.
- (f) Counterexamples to Borsuk's Conjecture contained on the spheres of diameter arbitrarily close to 1 are constructed.
- (g) Exact bounds on the size of largest intersecting families of  $n$ -dimensional vectors with coordinates 0, 1, and -1 are proved.

These results are new and strong.

3. Theoretical and practical importance of the dissertation:

The dissertation is theoretical. The results are important in development of discrete mathematics, discrete geometry and some theoretical aspects of computer science. The results on  $\epsilon$ -nets may help in constructing and analysis of efficient algorithms of small  $\epsilon$ -nets.

4. Are the main results of the dissertation adequately represented in the publications in refereed journals, according to the Regulations?

The main results of the dissertation are published in leading international refereed journals in the area, in particular, in Journal of Combinatorial Theory A (4 papers), Doklady Math. (2 papers), Journal of the London Math. Soc., Moscow Journal of Combinatoric and Number Theory (3), European J. of Combinatorics (2) and others. Altogether, 27 papers in refereed journals. They also were reported at many international conferences and at seminars in leading mathematical departments of the world.


5. Questions and remarks (according to part 4.13 of the Regulations, the candidate addresses the questions and remarks formulated below during the defense):

There are very minor misprints in the dissertation and the abstract, but nothing serious. The level of the dissertation is very high.

6. General evaluation of the dissertation (excluding the introductory part):

The dissertation of Andrey Borisovich Kupavskii deals with interesting and important problems of extremal combinatorics, discrete geometry and theoretical computer science. It is a high level scientific study. The topic of the dissertation satisfies the conditions of the specialization 05.13.17 – theoretical foundations of computer science. The dissertation fully satisfies the conditions of the Regulations on the award of scientific degrees of candidates and doctors of sciences at MIPT.

Date 01.04.19

Signature  / A. V. Kostochka