Placement and Sizing of Series Compensation (SC) and Static Var Compensation (SVC) devices in Transmission Grids: The Case of AC Power Flows

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Abstract

Aiming to relieve transmission grid congestion, improve or extend feasibility domain of the operations and reduce overall energy generation cost by supporting transport of renewable energy and energy from cheap generation facilities to the demand places, we build optimization heuristics, generalizing standard AC Optimal Power Flow (OPF) procedure, for placement and sizing of Flexible Alternating Current Transmission System (FACTS) devices of the Series Compensation (SC) and Static VAR Compensation (SVC) type. We model devices which generally represent a way to compensate lines or loads respectively and the compensation level can be adjusted according to the installed capacity. Main effect of an SC device consists in modifying line inductance, while an SVC device injects or consumes reactive power. One use of these devices is in resolving the case when the AC OPF solution is not feasible because of congestion. Another application of interest is related to developing a long-term investment strategy for placement and sizing of the SC and SVC devices to reduce operational cost and improve power system operation where one also takes into account many scenarios, for example various load configurations, availability of generators and line connections. We develop optimization framework which accounts for the most general AC case and works with multiple scenarios. It allows us to find sparse and non-local solutions such that number of installed devices is relatively small and problems at a particular location of the system can be better resolved by installing compensation devices in other locations. We find one optimal placement and sizing of FACTS devices for multiple scenarios and optimal device settings for each scenario simultaneously. Our solution of the nonlinear and non-convex generalized AC-OPF problem consists of building a convergent sequence of convex optimizations containing only linear constraints. The approach, which scales well for large/realistic systems, is illustrated on single and multi-scenario examples of the Matpower case-30 model.
Contents

1 Introduction 5
  1.1 Power system layout: high level description 5
  1.2 Introduction to the project 7
  1.3 Motivation 9
  1.4 Solution strategy 10
  1.5 Research tools 11
    1.5.1 Matlab 11
    1.5.2 Matpower 12
    1.5.3 Mosek 12
    1.5.4 Web visualization 12

2 Literature review 16
  2.1 Power system modelling 16
  2.2 Power Flow (PF) and Optimal Power Flow (OPF) problems 19
  2.3 Steady-state regimes, load-duration curves 21
  2.4 FACTS devices 24
    2.4.1 Series Compensation devices 24
    2.4.2 Static Var Compensation devices 25
  2.5 Existing installation approaches 26

3 Methodology 28
  3.1 Optimization model 28
  3.2 Problem statement 30
  3.3 Optimization algorithm 31
  3.4 Solvers functionality 34
    3.4.1 Single scenario solver 34
    3.4.2 Multiple scenarios solver 36

4 Results 37
  4.1 Solver validation 38
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1.1</td>
<td>Comparison of the solutions for 9-bus system</td>
<td>38</td>
</tr>
<tr>
<td>4.1.2</td>
<td>Comparison of the solutions for 30-bus system</td>
<td>42</td>
</tr>
<tr>
<td>4.2</td>
<td>Single scenario optimization</td>
<td>44</td>
</tr>
<tr>
<td>4.2.1</td>
<td>OPF feasible operation</td>
<td>45</td>
</tr>
<tr>
<td>4.2.2</td>
<td>OPF infeasible operation</td>
<td>51</td>
</tr>
<tr>
<td>4.3</td>
<td>Multiple scenarios optimization</td>
<td>53</td>
</tr>
<tr>
<td>4.3.1</td>
<td>All OPF feasible scenarios</td>
<td>54</td>
</tr>
<tr>
<td>4.3.2</td>
<td>General case</td>
<td>57</td>
</tr>
<tr>
<td>4.3.3</td>
<td>Analysis of fluctuations</td>
<td>59</td>
</tr>
<tr>
<td>4.4</td>
<td>Investment planning</td>
<td>62</td>
</tr>
<tr>
<td>5</td>
<td>Research relevance and applications</td>
<td>64</td>
</tr>
<tr>
<td>5.1</td>
<td>Research relevance and novelty</td>
<td>64</td>
</tr>
<tr>
<td>5.2</td>
<td>Implementation of the research, applications</td>
<td>65</td>
</tr>
<tr>
<td>6</td>
<td>Conclusions</td>
<td>66</td>
</tr>
<tr>
<td>6.1</td>
<td>Project summary</td>
<td>66</td>
</tr>
<tr>
<td>6.2</td>
<td>Future work</td>
<td>67</td>
</tr>
<tr>
<td>7</td>
<td>Acknowledgements</td>
<td>68</td>
</tr>
<tr>
<td>8</td>
<td>Appendix</td>
<td>69</td>
</tr>
<tr>
<td>8.1</td>
<td>Equations for general optimization problem</td>
<td>70</td>
</tr>
<tr>
<td>8.2</td>
<td>Equations for linearized optimization problem</td>
<td>72</td>
</tr>
<tr>
<td>8.3</td>
<td>Multiple-scenarios optimization generalization</td>
<td>76</td>
</tr>
<tr>
<td>8.4</td>
<td>Convergence of the algorithm</td>
<td>76</td>
</tr>
<tr>
<td>References</td>
<td></td>
<td>81</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

1.1 Power system layout: high level description

The subject of our research is power system and we are going to start highlighting important special features of the power systems. Any other infrastructures where a commodity is transported from supply to demand (for example food chain, crude oil or natural gas systems) are characterized by ability to store the commodity. However, the power systems are different - storage, even though available in theory, is expensive and not wide spread, thus imposing a significant feasibility requirement to maintain in operations an almost real time balance of supply and demand.

Schematic structure of a power systems is illustrated in the Fig. 1.1.

Figure 1.1: Illustration of the typical power system layout, schematic structure and components. Energy is produced at the generation facilities (black) then it is transported through the high-voltage transmission system (blue) to low-voltage distribution systems (green) where it finally gets to consumers.

It represents the way how power flows from producers to customers. System Operator is
the entity responsible for keeping the lights on, i.e. making sure that the power is delivered at the specified frequency (e.g. 50Hz in Russia and 60Hz in US) and the voltages stay sufficiently close to their nominal values (allowed voltage deviations in transmission are normally set to 5\%). At the same time system operator is set to operate the system in the way that none of the components are overloaded and the overall power generation cost is minimized.

Power systems across the globe undergo tremendous changes, in particular in terms of new components, such as renewable sources of energy, added. From the economical and ecological point of view the renewables are desirable. However, the renewables also create complications for system operators, e.g. related to their inherited uncertainty and necessity to control related fluctuations by some other means. These fluctuations, e.g. induced at large wind farms, need to be dealt with at the transmission level. However, effect of fluctuations is not limited to the transmission system only. Small scale loads at the distributions, e.g. houses equipped with solar panels and/or small-scale gas-fired co-generation turbines connected to the grid at the distribution level, can also be sources of hidden fluctuations, which are as difficult to deal with. All of the "new" fluctuating effects emerge on the top of the standard economic growth pushing any system in the world (even these designed originally with enough of a safety buffer) closer to the margin. In the Fig. 1.2 regional electricity consumption and total values is illustrated. It can be clearly seen that globally, and in Asia in particular, a steady growth of the electricity consumption is observed.

![Global electricity consumption](image)

**Figure 1.2**: Global electricity consumption with years. Different colors illustrate consumption for different regions [1].

Fig. 1.3 shows annual electricity generation is US by sectors. Since 2007 there is a huge
growth of wind generation and from 2009 again overall generation increase can be observed. In summary, standard economic growth combined with increasing role/contribution of new producers and consumers prone to fluctuations has pushed many transmission systems to the limits. The change is principal, making improvements of the transmission system reliability is a pressing necessity which need to be dealt with and cannot be ignored (e.g. in favor of extracting profit).

New ways of planning the development of the system, regime and emergency control should be implemented in order to reduce number and size of power outages and extend feasibility domain of the operations of power system to prevent the equipment overloading, transmission lines congestion or operational limits violation.

Figure 1.3: Annual electricity generation in US by sectors [2].

1.2 Introduction to the project

In addition to traditional ways to balance AC-flows and meet the demand through generation dispatch, a number of new technological solutions are now available. In particular, installation of the so-called Flexible Alternating Current Transmission System (FACTS) adds an important new option to the mix of other available control options, see e.g. [3, 4, 5, 6] and references therein.

Serial Compensation (SC) and Static VAR Compensation (SVC) are FACTS devices of new type [7, 8, 9, 10] which generally represent a way to compensate lines or loads respectively. Main effect of an SC device consists in modifying line inductance, while an SVC device injects or consume reactive power. It is more reasonable to use such devices in transmission grids
because they usually have loops and power has more than one way to go to customers. When the inductance of the given line increases, active power "flows" to other path and the loading of line reduces. But the effect is non-local and when combined with generation redispatch it may lead to significant modification of the state. While SC devices are used to reroute power, SVC devices are used typically to resolve local voltage problems (through injection or consumption of reactive power).

Planning installation of new FACTS devices with sufficient capacity and flexibility to meet requirements of multiple demand scenarios, e.g. accounting for seasonal variations and growth of demands, various network configurations and different generators on-line, is a challenging optimization problem discussed in the past by many other authors, e.g. [3, 4, 5, 6], and we also address it here.

Our project has started from resolving the challenge within the paradigm of the DC approximation in [11, 12]. We have developed an approach for placement and sizing of SC devices and have explored the effect of installation on the overall performance of the grid. However, we have also discovered in this study that application of the DC approximation to large systems leads to significant inconsistency, as the approximation error (from the linearization embedded into DC) causes significant and important difference from the actual AC-modelled regimes when power system is highly loaded.

In this thesis we generalize the previously developed optimization framework of FACTS placement, sizing and operational optimality to the general AC case. The approach is also adopted for handling multiple configurations of loads and generations within single optimization. Our approach consists in posing an optimization problem that extends the standard AC OPF by introducing new degrees of freedom accounting for the flexibility in line inductances and reactive power corrections provided by SC and SVC devices. In addition to discovering optimal investment strategy in FACTS devices our optimization problem also outputs optimal settings for these devices which are specific for every configuration/scenario (of injection consumption) included in the formulation.

The problem is stated as a network optimization problem, which is generally non-convex and nonlinear. To overcome the difficulties we build efficient optimization heuristics which constructs a convergent and carefully controlled sequence of convex optimizations with linear constraints. These convex optimizations are solved efficiently with the state-of-the-art Mosek solver [13]. Importantly, our approach is scalable, i.e. it allows extension to large system with polynomial grows of complexity with increase in the system size. This aspect of our approach and developed methodology is extremely important for practical applications.
1.3 Motivation

FACTS devices of SC and SVC types can be utilized for multiple purposes. Series Compensation devices can be used in order to [14]:

- Reduce line voltage drops
- Limit load-dependent voltage drop
- Influence load flow in parallel transmission lines
- Increase transfer capability
- Reduce transmission angle
- Increase system stability

Functions of Static Var Compensation devices are [14]:

- Voltage control
- Reactive power control
- Damping of power oscillations
- Unbalance control

In principle, some of the modern (and expensive) FACTS devices can be used on a fast time scales to mitigate dynamical problems. Even though dynamic applications are important, they are still exotic (lacking practical applications). Therefore we limit our study and analysis solely to the static setting(s). In this study we analyze how to use FACTS devices of both types to:

- Reduce overall generation cost

Each available generator in the power system has some cost function for power generation and there is a procedure called Optimal Power Flow (OPF) which system operator uses to define generation levels for generators to minimize the overall cost. And there is a possibility that for highly loaded configurations of the power system some "cheap" generators will not be able to produce maximum amount of electric power due to line congestion around them. As the devices can influence the distribution of power flows by modification of inductance of transmission lines we can use that ability in order to increase system throughput and support "cheap" power to go to the demand places. This can be renewable energy or just energy from some modern or really large generators.
• Improve power system reliability

This means that we can use FACTS devices to reinforce power system to withstand the disturbances and to reduce the risk of power outages. An emergency situation occurs when some elements (for example transmission lines) are overloaded. If one considers huge power outages in US (for example Northeast blackout of 2003) they started from simple line faults [15]. When the OPF solution is infeasible the only way to resolve that is to use load scheduling or turn of the customers. We aiming to find a way of extending feasibility region for the system operation by using FACTS devices.

• Reduce congestion

Again, if some line is overloaded we can try to use compensation devices to resolve that. The question is how to do that because generation redispatch is also one of the options. We need to develop a procedure how to choose from all the possibilities the right one or a number of them.

To summarize, there are three main research areas covered in this thesis. The first one is FACTS investment planning for improvement of feasible operations of the power system, e.g. in what concerns moving operations further away from the dangerous limits when the regime becomes (or can become) infeasible while also reducing operational cost. Our second focus concerns investing in FACTS devices so that it would be optimal economically while also improving reliability, reinforce the system by extending feasibility domain of the operations in the situation when load and fluctuations/uncertainty increase causes regime infeasibility. And the third one area is optimal control of existing FACTS devices that have been already installed in the system.

1.4 Solution strategy

In this study we aim to optimally place and size Series Compensation and Static Var Compensation devices accounting for multiple load and generation configurations in a way which allows to minimize overall capital investment and system operational costs. We work with the exact/complete AC model of the power system.

We discuss main objectives and steps of the study below.

• State optimization problem

The first step is to construct the basic model and then to transform it into a mathematically sound optimization formulation. The general optimization problem is non-convex and sufficiently complex, thus prompting us to develop special, new and powerful tools to solve it.
• Formulate solution methodology
  The crack of our solution method consists in reducing the difficult non-convex problem to a sequence of convex problems, where each problem in the sequence can be solved with the standard methods thus producing input parameters for the next one, till convergence.

• Develop solution algorithm for a single load/generation scenario
  For simplicity of the coding and validation we start working with single scenario.

• Validate the algorithm
  Algorithm which works with a single scenario is tested to obey known asymptotics. When validation of the single scenario case is complete we shift to the next step.

• Develop solver for 1000+ scenarios
  Here we develop a new solver which allows computation and efficient visualization of multiple load/generation scenarios accounted for simultaneously.

• Explore solutions and get results
  When the optimization framework is built we use it for actual analysis described in Section 1.3.

• Develop custom, fast and convenient visualization
  Importance of this step is easy to underestimate. Operating with the problem as difficult as ours, one should have a powerful and convincing method for testing and debugging. This newly developed comprehensive visualization methodology has allowed us to overcome these profound challenges stressfully and efficiently.

1.5 Research tools
This section describes tools and software that is used in this project. Our solvers and algorithms are coded manually but also relies on public and commercial environments and software.

1.5.1 Matlab
MATLAB (matrix laboratory) is a multi-paradigm numerical computing environment and fourth-generation programming language. Developed by MathWorks, MATLAB allows matrix manipulations, plotting of functions and data, implementation of algorithms, creation of user interfaces, and interfacing with programs written in other languages, including C, C++, Java, Fortran and Python [16].
Although MATLAB is intended primarily for numerical computing, an optional toolbox uses the MuPAD symbolic engine, allowing access to symbolic computing capabilities. An additional package, Simulink, adds graphical multi-domain simulation and Model-Based Design for dynamic and embedded systems [16].

Matlab is used for actual solvers implementation, it is not the fastest among other languages but during the developing stage we need an ability to easily debug the code and connect some additional libraries, use various optimization toolboxes and special power system software.

1.5.2 Matpower

Matpower is used as a power system solver, it helps us solve power flow and optimal power flow problems. It can work with various power system models (if it is converted to the right input format) and actually have some open source systems included.

By the description from the home page MATPOWER is a package of MATLAB M-files for solving power flow and optimal power flow problems. It is intended as a simulation tool for researchers and educators that is easy to use and modify. MATPOWER is designed to give the best performance possible while keeping the code simple to understand and modify. It was initially developed as part of the PowerWeb project [17].

1.5.3 Mosek

Our algorithm performs Quadratic Programming (QP) optimization and we need a solver for that. Matlab has its own optimization toolbox but after first experiments we decided to use some specialized optimization software. Mosek is high performance software for large-scale LP, QP, SOCP, SDP and MIP including interfaces to C, Java, MATLAB, .NET, R and Python [13]. It is quite popular and accepted by a scientific community, it has a free academic licence and it just worked fine so we decided to use it.

1.5.4 Web visualization

In Section 1.4 it was mentioned that good visualization was very important for success of the project. We developed a web visualization which works separately from a solvers using data stored in a database. The architecture of the system is illustrated in the Fig. 1.4.

Solution algorithm is coded in Matlab, it produces the data and the results are sent to MySQL database. We do not use Matlab visualizations as no appropriate libraries for graph illustrations (for large graphs especially) were found/available. Instead JavaScript vis.js library is used for that [18]. So, after data is collected, php script takes it from database, prepares data structures and send these to JavaScript part of the code via json interface where illustration is finally constructed with the help of vis.js library.
Vis.js is a dynamic, browser based visualization library. The library is designed to be easy to use, to handle large amounts of dynamic data, and to enable manipulation of and interaction with the data. The library consists of the components DataSet, Timeline, Network, Graph2d and Graph3d [18]. It allows to illustrate graphs or networks with various possible marks, colorcoding and size or shape coding. And JavaScript (js) allows to build dynamic web pages, for example we can follow how the state of the power system change during the iterations of the solver and see how actually everything works.
In the Figs. 1.5-1.6 exemplary visualizations of a 30 node power grid model are shown.

Figure 1.5: The example of the illustrated Optimal Power Flow solution for 30 nodes system.
Figure 1.6: The example of solution given by the single scenario solver.

In the next chapters we describe details of our solution methods, show research results and then formulate future plans.
Chapter 2

Literature review

In this chapter we describe steady-state power system modelling and formulate Power Flow (PF) and Optimal Power Flow (OPF) problems. We also discuss long-term operations of the power transmission system, introduce SC and SVC devices and discuss their roles and opportunities in operations and planning of the system.

2.1 Power system modelling

First we describe AC model and then the simplified DC model. Main components of the power system are:

- Branches or power lines
- Generators
- Loads

Matpower [17] is the open source package which allows to test/run all of the standard steady-state models typically used for power flow analysis. According to Matpower’s documentation all transmission lines, transformers and phase shifters are modeled with a common branch model, consisting of a standard $\pi$ transmission line model, with series impedance $z = r + jx$ and total charging susceptance $b$, in series with an ideal phase shifting transformer [19]. Each line has two sides one called from and another one is to. The transformer, whose tap ratio has magnitude $\tau$ and phase shift angle $\theta_{shift}$, is located at the from end of the branch, as shown in Fig. 2.1

The complex current injections $i_f$ and $i_t$ at the from and to ends of the branch, respectively, can be expressed in terms of the $2 \times 2$ branch admittance matrix $Y_{br}$ and the respective terminal voltages $v_f$ and $v_t$ (from [19]):
With the series admittance element in the $\pi$ model denoted by $y = 1/z$, the branch admittance matrix can be written as follows [19]:

$$ Y_{br} = \begin{bmatrix} \left(y + \frac{j b}{2}\right) \frac{1}{\tau^2} & -y \frac{1}{\tau \exp(-j \theta_{shift})} \\ -y \frac{1}{\tau \exp(j \theta_{shift})} & \left(y + \frac{j b}{2}\right) \end{bmatrix} $$ (2.1.2)

For simplicity we do not take into account phase shifters and tap-changers for now (this can be easily added to the model). Thus setting $\tau = 1$ and $\theta_{shift} = 0$ we get the following complex current flowing through the line $a - b$ (in the direction from $a$ to $b$):

$$ i_{ab} = \left(\frac{1}{r + jx} + j \frac{b}{2}\right) * v_a - \frac{1}{r + jx} * v_b $$ (2.1.3)

Taking into account the fact that complex voltage $v_a = V_a e^{j \theta_a}$, the apparent power flowing through the line satisfies:

$$ S_{ab} = v_a^* i_{ab}^* $$ (2.1.4)

$$ S_{ab} = \frac{r V_a^2 - r V_a V_b \cos(\theta_a - \theta_b) + x V_a V_b \sin(\theta_a - \theta_b)}{r^2 + x^2} + \frac{j x V_a^2 - r V_a V_b \sin(\theta_a - \theta_b) - x V_a V_b \cos(\theta_a - \theta_b)}{r^2 + x^2} - j \frac{b}{2} V_a^2 $$ (2.1.5)
This is the result of the $\pi$ model for branches. While lines transfer active and reactive power through the system, there are two other players - generators and loads which produce or consume electricity.

A generator is modeled as a complex power injection at a specific bus. For generator $i$, the injection is:

$$S_i^g = P_i^g + Q_i^g \quad (2.1.6)$$

where, $P_i^g$ is active power positive injection and $Q_i^g$ - reactive power injection which can be both positive or negative.

Constant power loads are modeled as a specified quantity of real and reactive power consumed at a bus. For bus $i$, the load is

$$S_i^l = P_i^l + Q_i^l \quad (2.1.7)$$

where, $P_i^l$ is active power negative consumption and $Q_i^l$ - reactive power negative consumption.

Eq. 2.1.5 defines active and reactive powers flowing through an edge of the system. It can be seen that the function is complex and depends on many variables (predefined constants for the system are just line parameters $x$, $r$ and $b$ if there are no any adjustable compensation devices used). We are going to adjust $x$ - inductance of the transmission line, then fixed parameters will be only $r$ and $b$. When an optimization problem is formulated one includes these non-linear and non-convex functions in the constraints for power system state definition by Kirchhoff’s circuit laws [20].

Thus, during our initial research we simplified the task and worked within reasonable for transmission systems DC-approximation which is defined according to the following three assumptions:

- Branches can be considered lossless. In particular, branch resistances $r$ and charging capacitances $b$ are negligible:

- All bus voltage magnitudes are close to their nominal value or 1 p.u.

- Voltage angle differences across branches are small enough that $\sin(\theta_f - \theta_l) \approx \theta_f - \theta_l$

In DC-approximation the apparent power $S_{ab}$ will be:
\[ S_{ab} = v_a i_a^* = V_a e^{j\theta_a} \ast ((\frac{1}{r + jx} + j \ast \frac{b}{2}) \ast V_a e^{j\theta_a} - \frac{1}{r + jx} \ast V_b e^{j\theta_b})^* \approx \]
\[ \approx e^{j\theta_a} \ast (\frac{1}{jx} \ast e^{j\theta_a} - \frac{1}{jx} \ast e^{j\theta_b})^* = \frac{j}{x} (1 - e^{j(\theta_a - \theta_b)}) = \]
\[ = \frac{j}{x} (1 - \cos(\theta_a - \theta_b) - jsin(\theta_a - \theta_b)) \approx \frac{\theta_a - \theta_b}{x} \tag{2.1.8} \]

Which is much simpler than Eq. 2.1.5. Only active power flows in the system and for each line is defined by phase difference between the ends and line inductance, function still nonlinear (if x is adjusted).

2.2 Power Flow (PF) and Optimal Power Flow (OPF) problems

The standard Power Flow (PF) problem outputs voltages and flows in a network for a given profile of consumption/injections at loads and generators [19]. For known structure of the network, branch parameters and values of generation and consumption of power solving PF means finding voltages, phases and active in reactive power flows in the system.

By convention a single largest generator is typically chosen as a reference bus also called the slack bus. The reference/slack bus is assumed providing a slack of generation, thus input/output of active and reactive power at the slack bus is a parameter to allow consistency of PF equations in the balancing the global power budget. Other generators are typically classified as PV buses, with the values of voltage magnitude and generator real power injection given [19]. Loads are specified as PQ buses with given power consumption and unknown voltage and phase values.

The solution of the AC-PF problem is a solution of the PF equations representing the Kirchhoff’s laws. Stated in their standard form the PF equations are usually split into active and reactive parts. At each node of the system actual active and reactive power injection or consumption is equal to the sum of flows going to lines connected to the given node:

\[ P_i = \sum_{j \sim i} \frac{r_{ij} V_i^2 - r_{ij} V_i V_j \cos(\theta_i - \theta_j) + x_{ij} V_i V_j \sin(\theta_i - \theta_j)}{r_{ij}^2 + x_{ij}^2} \tag{2.2.1} \]
\[ Q_i = \sum_{j \sim i} \left( \frac{x_{ij} V_i^2 - r_{ij} V_i V_j \sin(\theta_i - \theta_j) - x_{ij} V_i V_j \cos(\theta_i - \theta_j)}{r_{ij}^2 + x_{ij}^2} - \frac{b_{ij}}{2} V_i^2 \right) \tag{2.2.2} \]

We use Matpower toolbox to solve the AC PF equations, and when needed for embedding in higher level (optimization) problem develop our own solver (computational framework).

Another important standard for power systems is Optimal Power Flow problem (OPF). This task involves searching for the optimal active and reactive power dispatch of generation
minimizing the value of the power generation cost function. The optimization vector $x$ for the standard AC OPF problem consists of the $n_b \times 1$ ($n_b$ - number of nodes or buses) vectors of voltage angles $\Theta$ and magnitudes $V$ and the $n_g \times 1$ ($n_g$ - number of generators) vectors of generator real and reactive power injections $P_{\text{gen}}$ and $Q_{\text{gen}}$.

$$x = \begin{bmatrix} \Theta \\ V \\ P_{\text{gen}} \\ Q_{\text{gen}} \end{bmatrix} \quad (2.2.3)$$

The objective function in the OPF is a sum of costs of active and reactive power injections at the generators $C^i_p$ and $C^i_q$.

Finally, the AC-OPF problem is formulated as follows:

$$\min_x \sum_{i=1..n_g} C^i_p(p^i_g) + C^i_q(q^i_g)$$

s.t.

$$V_{\text{min}} \leq V \leq V_{\text{max}}$$

$$Q_{\text{min-gen}} \leq Q_{\text{gen}} \leq Q_{\text{max-gen}}$$

$$P_{\text{min-gen}} \leq P_{\text{gen}} \leq P_{\text{max-gen}}$$

$$\sqrt{(P_{ij})^2 + (Q_{ij})^2} \leq S_{ij}^{ij} \quad \forall i \sim j$$

$$P_i + iQ_i = \sum_{j \sim i} (S_{ij}) \quad \forall i = 1..n_b \quad (2.2.4)$$

The optimization constraints above have the following meaning:

- voltage amplitude at each node is within appropriate range defined by minimum and maximum levels
- reactive power generation is limited
- active power generation is limited
- lines thermal limits for each pair of connected nodes
- active and reactive power balance at each node (network equations)

This problem is very important and has a significant value in power systems research. In our study we extend this problem formulation to include placement and sizing of FACTS devices which will be described in the following sections.
2.3 Steady-state regimes, load-duration curves

If all input characteristics of a power system would change slowly and predictably then operating it according to guidance provided by solving OPF is not a problem. However, these assumptions do not always hold in practice. Even without discussion of rapid changes of demand and supply and without taking into account power system emergency situations when dynamics is involved there is a variety of steady-state regimes related to uncertainty in the system operation and planning which make solving the OPF in a way aware of the uncertainty a challenging problem.

If we look at the power system during the day and during the year we observe emergence of the generation/demand patterns. Schematic illustration of the so-called power demand curve during the day is shown in the Fig. 2.2.

![Figure 2.2: Schematic illustration of a typical "grid load profile" forecast, the actual power consumption over the course of a day, and the amount of power generation resources that are online to maintain reliability. Power system operators use this type of forecast to allocate resources [21].](image)

Electricity demand starts to grow in the morning, it is more or less flat during the day, usually there is an evening peak of consumption and then during the night demand decreases. Fig. 2.3 shows monthly generation profile of US for the last three years.
Figure 2.3: Monthly US energy generation profile for the last three years [2].

We also observe peaks during summer and winter and decrease during autumn and spring. This suggests to account for a number of regimes representing winter and summer typical situations, e.g. peak and off-peak profiles. Another tool which allows a somehow more accurate modeling is provided by the so-called load duration curves. The load curves can be found in published power market reports [22]. An example of the load duration curve for one quarter is given in the Fig. 2.4.

Figure 2.4: Examples of hourly load duration curves for quarter from [23].

It has the following meaning. For each operating hour peak load level is stored and a list of hours and load levels is constructed, then the list is sorted from low loads to high loads. The whole amount of operating hours is taken to be 100\% and this way the load duration plot is
organized. One takes from a sorted load list values from high to low and add them to the plot in the descending order representing all loading states up to 100%.

Fig. 2.5 illustrates how a load duration curve can be approximated so that the system operations will be represented with less loading levels. We use this approximation in our simulations.

Let us define some parameters here:

- $\alpha$ - uniform loading scaling parameter. Supposing that one base load configuration is given it can be uniformly rescaled by multiplication of all active and reactive loads to a given value $\alpha$, $\alpha = 1$ corresponds to base loading configuration.
- $M$ - number of chunks representing load duration curve.
- $p_i = w_i$ - occurrence probability of chunk is equal to its width.
- $\alpha_{\text{min}}, \alpha_{\text{max}}$ - minimum and maximum loading levels.
- $N$ - number of scenarios representing each chunk, totally load duration curve is described by $N \times M$ loading configurations.

For each chunk $i = 1..M$:

$$\text{loads}_i^0 = \alpha_i \times \text{loads}^0$$ (2.3.1)
For each chunk \( i \) and for each \( j = 1..N \) we generate loading configuration by modification of initial \( l_0^i \) which defines the chunk adding Gaussian correction to each load with zero expected value and specified standard deviation:

\[
    l_i^j = l_0^i + \text{normrd}(0, \text{stddev} \times l_0^i) \quad (2.3.2)
\]

\[
    p_i^j = w_i / N \quad (2.3.3)
\]

This scheme allows to simulate power system behavior during long period of time and represent it accounting for variations in the distribution of loads. We aim to optimize power system taking into account multiple scenarios and one of the applications is optimizing for a given load duration curve.

2.4 FACTS devices

According to [24] Flexible AC Transmission System (FACTS) is a generic term for a group of technologies that dramatically increase the capacity of the existing transmission network - by as much as 50 percent or more - while maintaining or improving voltage stability, grid reliability and energy security. FACTS technologies have a small footprint and minimal impact on the environment. Project implementation times are considerably faster and investment costs substantially lower than the alternative of building more transmission lines or new power generation facilities. FACTS technologies are traditionally divided into the two categories of series compensation and shunt compensation. All four technologies that make up the two categories are fixed series compensation and thyristor-controlled series compensation in the former, SVC and STATCOM/SVC Light in the latter [24].

2.4.1 Series Compensation devices

Series compensation is defined as insertion of reactive power elements into transmission lines [14]. Series compensation is a well established technology that is primarily used to reduce transfer reactances, most notably in bulk transmission corridors. The result is a significant increase in the transient and voltage stability in transmission systems [24]. The main principle of SC devices is to modify reactance of a transmission line by series connection of a capacitator. There are two types of the devices - Fixed SCs and Thyristor controlled SCs (TCSC). For us more interesting is a second, adjustable type of SC devices which can do both - increase or decrease the overall inductance of a power line. As TCSC enables rapid dynamic modulation of the inserted reactance we can use them for improving of power system operations globally by adding setting of the installed device to the OPF problem for instance. Example of the installed
TCSC is shown in the Fig. 2.6.

![TCSC](image)

Figure 2.6: TCSC for stable transmission of surplus power from Eastern to Western India from [24].

2.4.2 Static Var Compensation devices

A SVC is a high voltage system that controls dynamically the network voltage at its coupling point. Its main task is to keep the network voltage constantly at a set reference value [14]. SVC is connected in parallel to a consumer in a transmission system and injects the reactive power. The SVC consists of a number of fixed or switched branches, so it can be dynamically adjustable. At least one branch includes thyristors, and the combination of branches can be varied a lot depending on requirements. An SVC typically includes a combination of at least two of the given items below (most common topologies for SVCs are: TCR/FC or TCR/TSC/FC):

- Thyristor controlled reactor (TCR)
- Thyristor switched capacitor (TSC)
- Harmonic filter (FC)
- Mechanically switched capacitor bank (MSC) or reactor bank (MSR)

Functionality of the devices was discussed in Section 1.3. Example of an installed SVC is given in the Fig. 2.7.
The devices are large, complicated and expensive. The installation time is 1-2 years, and thus usage decisions should be well planned and reasoned.

2.5 Existing installation approaches

The approach to device installation discussed in this manuscript is more of a forward looking "theoretical" type. However, let us first discuss current industry practice.

As of now SVC and SC devices are mainly installed to resolve support and transmission congestion problems respectively. Guidance for installation is typically local - related to historical observations at the given localized portion of the grid. In [26] the installation of fixed SCs is discussed and the authors state that the main impact on the system is improved voltage and angular stability of a power lines, shown in Fig. 2.8.

Figure 2.8: The impact of SC on a) voltage and b) angular stability [26].
In Fig. 2.8 two curved lines illustrate the stability region with and without SC installation and dashed line shows the operational state. It is clearly seen that SC devices improve both voltage and angular stability of a power line, thus allowing more power to be transported through the line without loss of steady-state stability.

Producers of FACTS devices also give number of examples of successful installations of SVCs for reactive power support and voltage control [27].

Overview of FACTS devices [28], effect of the installed FACTS devices [29], local control problems and effect of coordinated control [6], planning of placement of the devices with appropriate characteristics and suitable for various loading configurations problem [30] and economical effect caused by compensation [31],[32] are among subjects broadly discussed in research papers representing scientific point of view on the FACTS installation.
Chapter 3

Methodology

In this chapter we discuss our path to resolving problems outlined in the preceding Sections. Our first objective here is to construct an optimization model for the problem of SCs and SVCs placement and sizing in the power system taking into account multiple scenarios when the regimes of the system should be described in complete AC case. Then we focus on the optimization algorithm and finally discuss functionality of the developed tools.

3.1 Optimization model

We begin by describing the network setting and introducing notations. As it was discussed previously power network is modelled as a graph defined by the set of nodes which are either loads or generators and a set of edges which are transmission lines with some inductance $x$, resistance $r$ and charging capacitance $b$.

- Layout of the power transmission network, $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ and $\mathcal{E}$ represent the set of nodes and edges of the network/graph, with line characteristics such as inductances, resistances and shunt capacitances is known.

- List of projected scenarios, i.e. different load configurations, $a = 1, \cdots, N$. The scenarios may include sampled (typical) configurations and/or contingency (rare) configurations projected for different level of loading.

- Each scenario is characterized by:
  - Occurrence probability
  - State of the network – energized network is the subgraph of the complete network
  - State of generators – list of generators on-line
  - Configuration of loads

28
List of fixed parameters characterizing scenario $a$ is as follows:

- $T^{(a)}$ - temporal rate (frequency) of scenario occurrence
- $G^{(a)} = (\mathcal{V}^{(a)}, \mathcal{E}^{(a)}) \subseteq G$ - energized subgraph of the full network, $G$
- $x_0^{(a)} = (r_{ij}^{(a)})_{\{i,j\} \in \mathcal{V}^{(a)}}$ - vector of initial inductances of energized lines
- $r^{(a)} = (r_{ij}^{(a)})_{\{i,j\} \in \mathcal{E}^{(a)}}$ - vector of resistances of lines
- $b^{(a)} = (b_{ij}^{(a)})_{\{i,j\} \in \mathcal{E}^{(a)}}$ - vector of shunt capacitances of lines
- $P_{\text{min,max}}^{(a)} = (P_i^{(a)}|i \in \mathcal{V}_g^{(a)} \subset \mathcal{V}^{(a)})$ - vectors of minimum (maximum) active power outputs of energized generators
- $Q_{\text{min,max}}^{(a)} = (Q_i^{(a)}|i \in \mathcal{V}_g^{(a)})$ - vectors of minimum (maximum) reactive power outputs of energized generators
- $P^{(a)}_{\text{load}} = (P_i^{(a)}|i \in \mathcal{V}_l^{(a)} \subset \mathcal{V}^{(a)})$ - vector of active power consumptions at loads
- $V_{\text{min,max}}^{(a)} = (V_i^{(a)}|i \in \mathcal{V}^{(a)})$ - vectors of minimum (maximum) allowed voltages
- $S_{\text{max}}^{(a)} = (S_{ij,\text{max}}^{(a)}|\{i,j\} \in \mathcal{E}^{(a)} \subset \mathcal{E})$ - vector of the apparent power limits of energized lines

The following are scenario-indexed degrees of freedom which are optimized over:

- $x^{(a)} = (x_{ij}^{(a)})_{\{i,j\} \in \mathcal{V}^{(a)}}$ - vector of line inductances (modified by SC devices)
- $Q^{(a)} = (Q_i^{(a)}|i \in \mathcal{V}^{(a)})$ - vector of reactive power injection/consumption at loads and generators (modified by SVC devices)
- $P_{\text{g}}^{(a)} = (P_i^{(a)}|i \in \mathcal{V}_g^{(a)})$ - vector of active power injections at the generators (operational cost for each scenario is cost of active power generation)
- $V^{(a)} = (V_i^{(a)}|i \in \mathcal{V}^{(a)})$ - vector of operational voltages
- $\theta^{(a)} = (\theta_i^{(a)}|i \in \mathcal{V}^{(a)})$ - vector of operational phases

Cost of the device placement and related service period:

- $C_{\text{SC}}$ - SC capacity placement cost (per 1 Ohm)
- $C_{\text{SVC}}$ - SVC capacity placement cost (per 1 MVar)
- $N_y$ - service period of the system

Finally, global (i.e. scenario independent) optimization degrees of freedom are:
\[
\Delta x = (\Delta x_{ij} | \{i, j\} \in \mathcal{E}) - \text{vector of SC device capacities (positive values, allowed up and down regulated intervals are assumed equal)}
\]

\[
\Delta Q = (\Delta Q_i | i \in \mathcal{V}_l) - \text{vector of SVC device capacities (positive values, regulated up and down intervals are assumed equal)}
\]

To account for operational flexibility of the devices we use as optimization variables scenario independent capacities and independently actual correction values for the devices contained within the capacity limits. We utilize the standard \(\pi\)-model for lines, however without tap changers and phase shifters for simplicity (they can be easily added to the model). Branch modelling is described in the Section 2.1.

### 3.2 Problem statement

The problem is to place and size FACTS devices of SC and SVC types taking into account multiple scenarios and define actual settings of the devices in a way that the combination of the cost of the upgrade and the cost of operations (load configurations), will be minimized:

\[
\min_{\Delta x, \Delta Q, \text{state}^{(a)}}, \forall a \quad \text{COST} (\Delta x, \Delta Q; \text{state}^{(a)})
\]

\[
\text{COST} \equiv (C_{SC} \sum_{\{i, j\} \in \mathcal{E}} \Delta x_{ij} + C_{SVC} \sum_{i \in \mathcal{V}_l} \Delta Q_i)
\]

\[+ N_y \sum_{a=1..N} T_a * C_a(P^{(a)}) \]

\[
\text{state}^{(a)} \equiv (x^{(a)}, v^{(a)}, \theta^{(a)}, Q^{(a)}, P^{(a)}), \forall a
\]

\[
x^{(a)} = x_0^{(a)} + \Delta x^{(a)}, \forall a
\]

\[
Q_{\text{load}}^{(a)} = Q_{\text{load}, 0}^{(a)} + \Delta Q_{\text{load}}^{(a)}, \forall a
\]

\[-\Delta x \leq \Delta x^{(a)} \leq \Delta x, \forall a
\]

\[-\Delta Q \leq \Delta Q_{\text{load}}^{(a)} \leq \Delta Q, \forall a
\]

\[
V_{\text{min}}^{(a)} \leq V^{(a)} \leq V_{\text{max}}^{(a)}, \forall a
\]

\[
Q_{\text{min-gen}}^{(a)} \leq Q_{\text{gen}}^{(a)} \leq Q_{\text{max-gen}}^{(a)}, \forall a
\]

\[
P_{\text{min-gen}}^{(a)} \leq P_{\text{gen}}^{(a)} \leq P_{\text{max-gen}}^{(a)}, \forall a
\]

\[
\sqrt{(P^{(a)}_{ij})^2 + (Q^{(a)}_{ij})^2} \leq S_{\text{max}}^{(a)} \quad \forall a; \forall \{i, j\} \in \mathcal{E}^{(a)}\]

\[
P_i^{(a)} + iQ_i^{(a)} = \sum_{j; (i, j) \in \mathcal{E}^{(a)}} (S^{(a)}_{ij}), \forall i \in \mathcal{V}^{(a)}, \forall a
\]

where all the inequalities containing vectors are considered component-wise; \(C_a(P^{(a)})\) stands for the function representing the cost of generation for scenario \(a\), and \(\forall a\) is a shortcut for, \(\forall a =
The objective function (8.1.7) represents capital investment cost of the installation of two types of FACTS devices (taking linear in the installation capacities and thus promoting sparseness, see [11, 12] for related discussion) plus operational cost summed over all the scenarios with their probabilities taken into account and multiplied by the number of years (service period). The optimization constraints above have the following meaning:

- state for each scenario is defined by vectors of line inductances, voltages, phases, active and reactive power injections at nodes
- actual line inductance is equal to its initial value plus SC correction adjusted to a scenario, however maintained within the installed capacity bounds
- actual reactive power demand for a load is equal to its initial value plus SVC adjusted to a scenario, however maintained within the installed capacity bounds
- limits for reactive power generation
- limits for active power generation
- line thermal limits
- active and reactive power balance at nodes

Thermal and power balance constraints are non-linear and non-convex. In order to resolve this complication we develop the iterative heuristic approach to solve Eq. (8.1.6) described in the next Section.

It can be seen that the formulated problem is a generalization of the AC-OPF. If we choose just one scenario for optimization and predefine capacities of the devices to be zero we will get standard AC-OPF formulation.

3.3 Optimization algorithm

The idea of the algorithm is to proceed sequentially. At each step within the sequence we linearize the constraints around a current state (found at the preceding step) and then use a standard convex optimization solver to evaluate the resulting quadratic programming (QP) optimization (for the generation cost modelled as a quadratic function) with linear constraints. We also control each step so that the resulting correction to the current state is sufficiently small to justify the linearization. The algorithm is terminated when preset target precision/accuracy is reached. Flowchart of the algorithm is shown in Fig. 3.1.
Each step of the solution procedure has the following meaning:

1* Each load configuration (each scenario) is given.

2* Here for each scenario we solve OPF with thermal limits removed (standard OPF can be infeasible for some load configurations and this could be resolved with the FACTS placement).

3* Initial state for each scenario is taken from step 2*.

1. Linearization of the thermal and power balance constraint over the current state for each scenario is applied (see Appendix for the details).

2. QP is evaluated. Here we artificially restrict change of reactive power injections on generators. (The restriction is caused by empirical observation that for a system with multiple alternative loops for power flows and reactive power assumed injected/consumed at no cost, there may be multiple solutions with close costs and similar active power generation, however showing rather distinct profiles of reactive power injection/consumption.)

3. This step assumes that a feasible solution of the preceding optimization problem is found. The solution outputs active powers and voltages at the generators, active and reactive...
powers at loads which we then use to run AC Power Flow (AC-PF) solver (in our experiments we use Mathpower solver) to update the remaining parameters of the current state.

4. One arrives at this step if some constraints of the preceding optimization problem are violated. One checks if the number of iterations is still less than the maximum allowed and then proceed to the next iteration, or exit (declaring infeasibility) otherwise.

In this scheme linearization of the constraints is straightforward (detailed in Appendix). We consider apparent power squared flowing through the line \( \{i, j\} \) under the scenario, \( a \), \( (P_{ij}^{(a)})^2 + (Q_{ij}^{(a)})^2 \), as well as real and reactive powers injected/consumed at a node, \( i \), under scenario, \( a \), \( P_i^{(a)} \) and \( Q_i^{(a)} \), as functions of the state variables \( (x, v, \theta) \), and then linearize around the current state by computing partial derivatives over state variables explicitly as it is shown in Appendix.

We have developed two separate solvers, each supplied with its own visualization tool. Our first solver works with single scenario. We use the solver to validate the approach and the algorithm. Then we applied the experience gained constructing the single scenario solver to build the multiple scenarios solver. In constructing the solvers we have resolved multiple technical problems. See Appendix for details. Next section describes functionality of both solvers.
3.4 Solvers functionality

3.4.1 Single scenario solver

The example of the input for single scenario solver is illustrated in the Fig. 3.2.

```
Configuration Parameters:

  case_cur: case30
  Nyears: 15
  LineXsFixed: 0
  LoadsQsFixed: 0
  ActivePowerDispatchFixed: 0
  ReactivePowerDispatchFixed: 0
  SCmult [S/Ohm]: 50000
  SVCmult [S/MVar]: 50000
  PQ_steps_opt: 0
  Pfixed_steps: 2
  Limited_step_size: 0
  max_step_ratio: 0.8
  limited_Q_step: 1
  Eps_Q: 1
  Q_step_reduction: 0.7
  LimitDistance: 0.99999
  ACPF_Update: 1
  alpha: 1.15
  nIterations: 50
  X_step_ratio: 1
  P_step_ratio: 1
  Q_step_ratio: 1
```

Figure 3.2: Example of the input of the single scenario solver.

It uses the following list of parameters (in the same order as in the Fig. 3.2), they are important for the following solver validation and simulations as they show how solver can be used.

Modelling parameters:

- model of the power system, which includes initial lines, loads and generators properties and network structure.
- service period of the system (number of years)

Optimization flags:

- if we maintain line inductances constant (1) or not (0)
• if we maintain reactive power demand for loads constant (1) or not (0)
• if active power dispatch for generators is fixed (1) or not (0)
• if reactive power dispatch for generators is fixed (1) or not (0)

Installation costs:
• price of SCs capacity
• price of SVCs capacity

Optimization by fixing P or Q on generators on each iteration:
• 1 if use this type of optimization
• how many iterations are performed with fixed active dispatch

Optimization by artificial limitation of the step size on the 2nd stage of the iterations (QP):
• 1 if use this type of optimization
• proportional step reduction parameter

Optimization by artificial limitation of the step size on the 2nd stage of the iterations (QP) for reactive powers on generators:
• 1 if use this type of optimization
• initial maximum step in MVA
• proportional step reduction parameter

Other parameters:
• multiplier for decrease of thermal limits
• 1 if forced to update the dispatch during state update step even for fixed dispatch
• uniform load scaling parameter for active and reactive loads
• maximum number of iterations

Step reduction:
• ratio of inductance change after QP optimization \( x_{new} = x_{prev} + \text{ratio} \times (x_{QP} - x_{prev}) \)
• ratio of active power generation change after QP optimization \( P_{new} = P_{prev} + \text{ratio} \times (P_{QP} - P_{prev}) \)
• ratio of reactive power generation change after QP optimization

\[ Q_{\text{new}} = Q_{\text{prev}} + \text{ratio} \times (Q_{\text{QP}} - Q_{\text{prev}}) \]

As an output solver gives a solution for the problem if it was feasible, we do not give examples or description of the output here, one will become familiar with it in Section 4.2

3.4.2 Multiple scenarios solver

Multiple scenarios solver uses similar input which is illustrated in the Fig. 3.3. The only difference is that user can specify number of scenarios to generate and choose if show information about convergence of the variables during the iterations or not.

```
Configuration Parameters:

case_cur: case30
Nyears: 0.000114155

LineXsFixed: 0
LoadsQsFixed: 0
ActivePowerDispatchFixed: 0
ReactivePowerDispatchFixed: 0

SCmult [$/x(pu)$]: 50000
SCVmult [$$/MVAr]: 50000

PQ_steps_opt: 0
Pfixed_steps: 2

Limited_step_size: 0
max_step_ratio: 0.8

limited_Q_step: 1
Eps_Q: 2
Q_step_reduction: 0.6

LimitDistance: 0.099
ACPF_Update: 1
alpha: 1
niterations: 25

X_step_ratio: 1
P_step_ratio: 1
Q_step_ratio: 1

N_scenarios: 10
ShowDebugging_LP_PF: 0
```

Figure 3.3: Example of the input of the multiple scenario solver.

Both solvers show status of a QP optimizer for each iteration. This is important thing for first judgement of the obtained solution.
Chapter 4

Results

This chapter describes the results which were obtained. We present examples of how the developed approach can be used to solve the problems of interest listed in Section 1.3. We perform validation of the single scenario solver and then explore investment strategy for feasible and infeasible load configurations. After that we explore multiple scenario optimization algorithm and apply it for FACTS investment planning in cases when all the scenarios are feasible or there are a number of infeasible cases among them. Finally we discuss optimization over given load-duration curve representing long-term power system behavior.
4.1 Solver validation

In order to check if solver works correctly we compare it against the solver included to Matpower AC-OPF package. As previously mentioned we have built a generalized OPF type optimization problem and if we choose the installed capacities of the devices to be zeros we should obtain just the same OPF solution as provided by other/existing solvers/software (if it is feasible).

4.1.1 Comparison of the solutions for 9-bus system

Here we take single configuration of 9-bus model and artificially overload one line by reducing its thermal limit. Then we run OPF to find a feasible solution which is illustrated in the Fig. 4.1. Initial overload of the line means that on the 3* step of our algorithm the resulting power flow through the line is higher than the thermal limit. All additional optimization tricks are not used here (see discussion of Section 3.4).

Figure 4.1: OPF solution of 9-bus system. Generators are blue, slack bus is white and loads are marked in green. Numbers next to each bus are id, voltage and phase for the given bus.
In the Fig. 4.2 the solution which is obtained by solver is shown in the same way but with voltage levels coded in colors from white to yellow. It can be seen that nodes have similar voltages to the OPF solution resulting in no observed overloads.

Figure 4.2: Solver solution for the same 9-bus system and the same scenario. Initially overloaded line marked in red.

Then in the Figs. 4.3-4.4 we show how the solution progresses with iterations. In this simple case with small initial overload about 5 iterations are needed to actually converge to a solution.
Figure 4.3: Dependence of number of overloaded lines on iteration number.

Figure 4.4: Dependence of the total overload value for all overloaded lines on iteration number.
The following Figs. 4.5-4.6 compares initial active and reactive generation dispatch with final one and with the solution given by the OPF procedure. It can be seen that solver works fine and gives the same solution as the Matpower solver.

Figure 4.5: Active power generation dispatch comparison. Blue bars - initial dispatch on the 1st iteration of algorithm, orange bars - final solution, purple - OPF dispatch.

Figure 4.6: Reactive power generation dispatch comparison. Blue bars - initial dispatch on the 1st iteration of algorithm, orange bars - final solution, purple - OPF dispatch.
4.1.2 Comparison of the solutions for 30-bus system

Then we perform the same experiment for more complicated 30-bus power system which is shown in the Fig. 4.9. And surprisingly solver does not converge (number of line overloads is in the Fig. 4.7).

Figure 4.7: Dependence of number of overloaded lines on iterations. There is no convergence.

Figure 4.8: Dependence of reactive power output for each generator on iteration number.

Having tried various possibilities and optimization tricks we have finally discovered that the problem is caused by the reactive power dispatch for generators which have grid connections with loops. There is no cost for reactive power and the solution space is degenerated. For each
active generation dispatch there are many appropriate configurations of reactive power dispatch and solver "cannot choose" between them. It can be seen in the Fig. 4.8 which illustrates progress of reactive power outputs for each active generator with iterations. Total value of reactive power injected to the system remains approximately the same but the dispatch always changes from one to another.

One possible solution for this problem is to limit maximum change of reactive power output for the generator during the iteration. We also decrease this limit with iterations to obtain convergence of the solution and manage to get the same regime, shown in Fig. 4.9, as OPF procedure does.

Figure 4.9: Solver solution for the 30-bus system and the same scenario. Initially overloaded line marked in red. Generators represented by squares and loads by circles, the color of the node represents voltage level (white - low, yellow - high). Generators are numbered in the same way as in the Figs. 4.10- 4.11.
Comparison of initial, final and OPF generation despatch is given in the Figs. 4.10 - 4.11.

Figure 4.10: Active power generation dispatch comparison. Blue bars - initial dispatch on the 1st iteration of algorithm, orange bars - final solution, purple - OPF dispatch.

Figure 4.11: Reactive power generation dispatch comparison. Blue bars - initial dispatch on the 1st iteration of algorithm, orange bars - final solution, purple - OPF dispatch.

We conclude that our algorithm discovers right solutions for known asymptotics.

4.2 Singe scenario optimization

Once the single scenario solver which shows reasonable solutions for the test cases is constructed, we use it to explore optimization of feasible and infeasible single scenarios for 30-bus model considered previously.
4.2.1 OPF feasible operation

For the first simulation the service period is taken to be 1 hour. Figs. 4.12-4.13 illustrates how the cost of installed SC and SVC devices depends on iteration number in this case.

Figure 4.12: Dependence of SCs capital cost on iteration number.

Figure 4.13: Dependence of SVCs capital cost on iteration number.

We conclude that it is not reasonable to invest money into the system for 1 hour service period, capital investment cost is approximately zero, OPF solution is feasible in this case and the optimal decision is to operate in OPF dispatch regime. Solver returns OPF dispatch solution in this simulation as it is shown in the Fig. 4.14.
For the second simulation the service period is chosen to be 15 years. One observes in the Fig. 4.15 that it is optimal to place one SVC device in the system and generate reactive power locally instead of transporting it from generators. This solution helps to relieve line overload which occurs in OPF without thermal limits regime and reduces operational cost for the system and saves money.

The operational cost data is the following:

- OPF Cost: $576.893/hour (75803800 $ total for 15 years)
- Solver Cost: $574.549/hour (75632500 $ total cost for 15 years which is capital investment plus operational)

In this non-overloaded case the developed approach allows to find an investment that saves about 2 dollars per hour for this system. That is not a large saving, but the point is that as it was discussed in Section 2.3 real power systems operates in various regimes and if we going to plan the investment we should consider many of them, the biggest economy is achievable for expensive overloaded regimes. Most importantly - we conclude from this test that the approach works.
Figure 4.15: Optimal solution for the 30-bus system for 15 years service period and base load level (uniform load scaling factor is 1). Initially overloaded line marked in red. Generators represented by squares and loads by large circles, the color of the node represents voltage level (white - low, yellow - high). Generators are enumerated. SVC compensated load is shown by a small circle, SVC installed capacity is given - for one scenario installed capacity is equal to actual compensation setting.

The following Figs. 4.16-4.17 illustrate convergence of the algorithm and show comparison of the initial active generation dispatch with final one and with the solution given by the OPF procedure. It can be seen that here in comparison with 9-bus model about 25 iteration is needed
to get the solution. That is because the system is bigger and we also limit the maximum change of reactive power generation.

Figure 4.16: Convergence of the SVC capital investment cost for 15 years service period and $\alpha = 1$ (uniform load scaling factor).

Figure 4.17: Active power dispatch comparison for 15 years service period and $\alpha = 1$. Blue bars - initial dispatch on the 1st iteration of algorithm, orange bars - final solution, purple - OPF dispatch.
For the next simulation we will uniformly overload the system till the point when OPF is still feasible, that will take place at $\alpha = 1.03$, and then choose service period to be 30 years. The goal of the experiment is to see how much saving one can gain by devices installation in the 30-bus system model. Load configuration should be OPF-feasible, as otherwise we would not be able to compare the costs.

The solution is shown in the Fig. 4.18.

Figure 4.18: Optimal solution for the 30-bus system for 30 years service period and $\alpha = 1.03$ (uniform load scaling factor is 1.03). Initially overloaded line marked in red. Generators represented by squares and loads by large circles, the color of the node represents voltage level (white - low, yellow - high). Generators are enumerated. SVC compensated load is shown by a small circle, SVC installed capacity is given - for one scenario installed capacity is equal to actual compensation setting.

The structure of the solution is similar to the case when 15 years service period was considered. That is because the structure of the system is also similar, all initial parameters except load values are the same and load scaling factor is very close to 1. For multiple scenarios we
introduce an approach to generating different scenarios which describes given load level.

- alpha: 1.03
- OPF Cost: 622.481 $/hour (163588000 $ total for 30 years, no investments were made)
- Solver Cost: 596.598 $/hour (157032000 $ total cost for 30 years which is capital investment plus operational)

Here the operational saving is about 26 $/hour which is more than 4 percent in comparison with OPF cost. For the real systems this small percent can translate into a very significant saving. However let us restate the warning - these savings should be calculated based on many operational scenarios and we will do that in the following Sections.

Next let us analyse the structure of active and reactive power dispatch to see how exactly the savings emerge.

In the Fig. 4.19 it can be seen that final active power generation dispatch (which defines the generation cost) is approximately the same as the dispatch at the first iteration which is OPF without thermal limits.

![Active Power Dispatch Comparison](image)

Figure 4.19: Active power dispatch comparison for 30 years service period and $\alpha = 1.03$. Blue bars - initial dispatch on the 1st iteration of algorithm, orange bars - final solution, purple - OPF dispatch.

Power flow limitations do not allow to use cheaper energy, because it cannot be transported due to line congestion and the developed approach use the ability of FACTS devices to reroute power in order to optimally support cheap energy to go to demand places. As a result ideally we can get the same price for generation which is given by non-constrained OPF, so if it is
significantly less than OPF cost, it signalize that the system is congested and one can try to use FACTS devices in order to resolve that.

Final reactive power dispatch in the Fig. 4.20 is also similar to initial one, but overall less power is produced by generators (each generator produces less reactive power than OPF without thermal limits procedure shows). In this case SVC compensator do two things - it relieves the congestion of a line and produces reactive power at a place. Less power is transported around the network which reduces losses.

![Reactive Power Dispatch Comparison (Q)](image)

Figure 4.20: Reactive power dispatch comparison for 30 years service period and $\alpha = 1.03$. Blue bars - initial dispatch on the 1st iteration of algorithm, orange bars - final solution, purple - OPF dispatch.

### 4.2.2 OPF infeasible operation

If the system is uniformly overloaded by $\alpha > 1.03$ then OPF solution is not feasible, the system cannot operate for this load configurations, but we can try to optimally reinforce the system with FACTS placement in order to extend the feasibility domain and prevent lines overloading caused by increased consumption levels.

We perform 4 simulations with fixed load level and different service periods in order to see how the solution progresses. For each simulation $\alpha$ is taken to be 1.07 and service periods will be 1 hour, 1 year, 15 and 30 years.

We do not show actual final states here, they are similar with one SVC placed in the same node as previously (see Fig. 4.18 for example). The installed capacities of the SVC are 2.8MVar,
4.24MVar, 6.92MVar and 7.28MVar - each time when service period increases we can invest more money in order to get better economy on operational cost.

Fig. 4.21 shows the comparison of active power dispatch solutions, it can be observed that when going to 30 years service period the dispatch becomes closer to OPF without thermal limits.

Figure 4.21: Active power dispatch comparison for four solutions of OPF infeasible scenario with $\alpha = 1.07$ and service period in the range from 1 hour to 30 years. Blue bars - initial dispatch on the 1st iteration of algorithm, orange bars - final solution, purple - OPF dispatch.

The last experiment in this Section is performed for OPF-infeasible single load configuration which was created by overloading the feasible base case of the Mathpower uniformly by 15%. We consider the single-scenario optimization over 15 years of the planning horizon. The obtained solution is shown in Fig. 4.22. We observe that the optimal correction is achieved with investment in one SC device and one SVC device, so both types of devices were used in this case.
Figure 4.22: Visualization of the solver output. Circles and squares mark consumers and generators. Voltage profile is shown in color (transitioning from yellow for maximum voltage to white for the minimal voltage). Line marked blue was initially overloaded and it was also selected by the solver for SC correction/placement. Number, shown next to the blue line, shows correction (in percentage). Node which was chosen for the (only) SVC correction is shown as a white dot. Bold numbers which appear next to the dot show level of the voltage and corrected/installed reactive power provided at the optimal solution.

Both devices have similar effect on the state of the power system but they do it in the different way by line inductance adjustment or reactive power injection. The developed approach allows to find optimal configuration taking in account installation prices and scenario settings and choose between the devices or find good combination of installed FACTS.

4.3 Multiple scenarios optimization

Based on our experience gained working on single scenario solver we develop a general version which finds one optimal investment for multiple scenarios and optimal settings of the installed devices for each load configuration simultaneously. In this Section we illustrate the performance of multiple scenario solver and explore the effect of FACTS devices on the system in the OPF feasible and infeasible regimes.
4.3.1 All OPF feasible scenarios

Firstly, we work with multiple all OPF feasible scenarios when it is possible to compare operational costs with OPF solutions. Scenario generation procedure is described previously in Section 2.3.

10 different scenarios are considered on equal footing, i.e. each occurring with the probability of 10%. Each of the ten configurations is generated from a base feasible case of the Matpower with subsequent addition of random correction to initial power consumption at nodes. Corrections are sampled from the Gaussian zero mean distribution with standard deviation equal to the 3% of the load positioned at the node. That is rather small level of fluctuations, the problem is that for the given 30 bus model due to line congestion in many cases we obtain some OPF infeasible load configurations.

In this case all 10 scenarios are similar from the loads distribution point of view and the result of the optimization is to place one SVC device and use it at approximately 100% compensation level for each scenario which is similar to previous results for single scenario solver and the reasons are the same - preference to local generation of reactive power and relieve of line contingency.

The behavior and performance of solver in this simulation is illustrated in the Figs. 4.23 - 4.26.

![SVCs Cost vs Iterations (Capital Investment Cost, $):](image)

Figure 4.23: SVCs cost convergence for multiple scenarios solver, 10 different load configurations are used for simulation.
Figure 4.24: Average over 10 load configurations generation cost convergence with iterations.

Figure 4.25: Convergence of each of 10 operational costs corresponding to the generated scenarios.
One can observe the difference of multiple scenario optimization from simple single scenario case. The state which represents each given loading configuration progresses independently but uses joint for all the scenarios installed capacity of FACTS devices for that. Finally line overloads are relieved and optimal generation dispatch is found for each operating regime. If we compare average over scenarios OPF cost with the solution when investment is made we will observe some economy:

- Average OPF cost: 580.6 $/hour
- Average scenarios cost: 579.3 $/hour
- Economy: 1.3 $/hour
- Economy percentage: 0.23%

When the scenarios are OPF feasible, solver do not use the ability of FACTS devices to reinforce the system, there is no need for that, but when some of the cases are infeasible and loads are distributed around the system in different ways we observe interesting results of how actually FACTS help to improve the regimes by cost reduction and reinforce the system in the same time.
4.3.2 General case

Next we illustrate performance of our multiple-scenario solver with the example accounting for 10 configuration scenarios and some of them can be OPF infeasible. Here we cannot calculate the economy in comparison to OPF solutions but it is interesting to observe what kind of solutions do we get and how the devices are adjusted for various load configurations and if we actually use the flexibility of the devices or not.

The scenarios are considered on equal footing, i.e. each occurring with the probability of 10%. Each of the ten configurations is generated overloading uniformly by 10% a feasible case of the Matpower with subsequent addition of random correction (sampled from the Gaussian zero mean distribution with standard deviation equal to the 10% of the load positioned at the node as it is discussed in Section 2.3).

The system is optimized over the year-long time horizon. In this case some of the scenarios are originally OPF-feasible and others are not. Fig. 4.28 shows optimal voltage profiles and optimal settings for the installed devices over two (out of ten) exemplary scenarios. We observe that the multi-scenario algorithm discovers distinct feasible solutions for each of the scenario. The resulting optimal placement is sparse. Moreover, once a device is installed it is not utilized at its maximum in all the scenarios.

Progress of the algorithm is shown in Fig. 4.27.

![Figure 4.27: Illustration of the algorithm dynamics: we show dependence of line overloads in MVA (lines numbered on the right) and dependence of the total cost in M$/years (blue dotted line shadowed from above) on the number of iterations.](image)

Left y-axis of Fig. 4.27 illustrates convergence of the total cost (value of the objective func-
We find that the multi-scenario solver requires approximately 30 iterations to converge. Right y-axis portion of Fig. 4.27 illustrates gradual (and generally non-monotonic) reduction of line overloads.

Figure 4.28: Visualization of the final states for two scenarios (out of 10 considered). Marking and color-coding (yellow-to-white) is the same as given in the captions of Fig. 4.22. Additionally we show "actual setting of the device"/"capacity of the device". Initially overloaded and compensated line is marked blue, initially overloaded line which was relieved without placement of a SC devise on it is shown red, and line which is chosen for compensation even though it was not overloaded prior to the correction is shown green.
4.3.3 Analysis of fluctuations

We also explore how capital investment cost and relative operational savings (in comparison with OPF solution) depends on the value of the loading level $\alpha$ and on the level of fluctuations $\text{stddev}$ (see Section 2.3 for details). We work with general multiple scenarios and use 20 of them for each simulated point on the graph. Service period is taken to be 15 years. Previously we have stated that if OPF is infeasible we cannot compare operational costs, actually the OPF solver outputs a value of generation cost if line overloads are neglected, these values are taken as the OPF cost estimations. Figs. 4.29 - 4.30 show investment cost and economy dependence on loading level for various levels of fluctuations.

![Graph showing investment cost and economy dependence on loading level](image)

Figure 4.29: Illustration of the capital investment cost dependence on $\alpha$ for optimal solution given by multiple scenarios solver for various fluctuations levels.

It can be concluded from the Fig. 4.29 that for a given level of fluctuations overloading of the system increases capital investment cost to make it operable and/or improve the system. When fluctuations level is increased this effect becomes even more pronounced. High fluctuations at a level described by $\text{stddev} = 0.2$ provoke to make an investment for underloaded system operations. Some improvement of base OPF feasible load configuration is possible (see $\alpha = 1$ and $\text{stddev} = 0$).
With regards to calculating savings we first of all confirm that properly installed FACTS devices are not only capable of resolving the local voltage or line congestion problems but their installation also saves money on power generation. Depending on the overloading and fluctuations levels we observe up to 15 % economy which is really incredible number. We see two distinct areas with comparably low and high fluctuation levels. For high fluctuation values (stddev = 0.1, stddev = 0.2) scenarios can become OPF infeasible even without additional overloading and flexibility of the installed devices allows to gain an economy while resolving congestions. In case of low fluctuation levels system is not congested without extra overloading and OPF procedure finds feasible solutions with similar generation costs. An economy is gained when lines cannot transport cheap energy through the system.
Similar results can be observed in the Figs. 4.31 - 4.32 for fixed loading levels and growing fluctuations of power consumption.

Figure 4.31: Illustration of the capital investment cost dependence on $\text{stddev}$ for optimal solution given by multiple scenarios solver for various loading levels.

When uncertainty of loading distribution increases for each demand level more and more investments should be made to reinforce the system. Overloading just amplifies the effect.
In case of relative economy dependencies we observe again two cases representing the scenarios when power system has been already congested (high overloading) or the effect of FACTS becomes more obvious just when uncertainty level increases for non-overloaded configurations.

We also suggest that in the case of high fluctuation and overloading more scenarios should be sampled to maintain an accuracy corresponding to base loading neighborhood (\( \alpha \approx 1, \text{stddev} \approx 0 \)).

In the next section we discuss an approach on how to plan investment in FACTS devices for a given service period, known load duration curve and loading growth prediction.

4.4 Investment planning

Suppose that for a given power system a load duration curve for a previous year is given. Also let us assume that a predicted value of loading growth during a year - \( \beta \%/\text{year} \) is available. In order to place FACTS devices in the system for a given service period we suggest to do the following:

- Construct a piece-wise step function approximation of a load duration curve.
- Construct predicted load-duration curves for each year of the service period by scaling initial one with \( \beta \)
- For each chunk of constructed load duration curves generate reasonable amount of sampled scenarios as it was described in the Section 2.3. It is reasonable to generate more
configurations for highly loaded cases or in case of large fluctuations values.

- Give all the generated scenarios with their probabilities as an input to multiple scenarios solver and run it to find an optimal placement solution if it is feasible.

If each load duration curve is represented by 10 chunks and 20 samples is generated for each chunk we need 200 scenarios to describe one load duration curve. Then we should multiply this number to amount of service period years which is 15 for example. The total number of scenarios to resolve will be 3000. Now we work on the performance and convergence improvements which will allow us to resolve that in reasonable time on a standard PC.
Chapter 5

Research relevance and applications

In this chapter we justify relevance of the research and discuss possible useful and valuable applications.

5.1 Research relevance and novelty

FACTS devices are used to improve power system operations for years, but the existing approaches are local in nature thus not exploring multiple large scale and longer terms benefits of the installations. Instead of solving problems such as lack of reactive power at a place or line stability or congestion locally we suggest an approach which generalizes the way how power system is controlled through the standard AC-OPF procedure.

Novelty of the research is that we consider two types of devices simultaneously and choose the best combination of them to improve power system. Our consideration is accurate as not relying on assumptions and working within the most general AC paradigm. The generality of the approach is very important in case of high loading levels. We take into account multiple operational scenarios with their probabilities which means that the solution we find applies to all considered cases. If power system operation is represented with appropriate scenarios our approach will find valuable solution which improves given power system using installed capacities of the devices in each case/regime. One particularly important observation is that we find not just installed capacities of the devices but optimal settings for each scenario also, thus we can apply the approach for planning and for adjustments of installed devices as well. The solution which solver gives is sparse, which means that small number of devices are required to resolve voltage and line congestion problems and reinforce power system. We have also shown that installed devices allow to reduce operational cost for the system and save money. Flexibility of the devices helps to operate power system better in modern uncertain conditions when the behavior of generators and loads in transmission system becomes more stochastic instead of deterministic as it was in the past.
Our algorithm itself is scalable and we are planning to demonstrate its application to large real power systems with thousands of nodes.

5.2 Implementation of the research, applications

Applications of the developed optimization framework are very important. Using our solvers we place and size FACTS devices in the power system in order to address main aspects of power transmission:

- Improve reliability. Power system is reinforced to withstand the disturbances and the risk of power outages is reduced.

- Reduce congestion. Throughput of the transmission system is increased, lines do not reach their capacity limits.

- Reduce overall generation cost. Renewable energy and energy from cheap generation facilities is supported to go to the demand places.

We consider application of the research in developing a transmission system service which aims to resolve three important problems:

- 1. FACTS investment planning for feasible power system operations.
- 2. FACTS investment planning for emergencies and grid reinforcement (equipment failures, lines’ overloads, voltage problems, N-k improvements).
- 3. Optimal control of the installed FACTS devices.
Chapter 6

Conclusions

In this chapter we summarize results which contribute this thesis and draw plans for future work.

6.1 Project summary

The project goal was to solve a problem of optimal placement of FACTS devices in the transmission systems. We explored how this is usually done in industry and concluded that the procedure can be significantly improved.

In this project we have developed new optimization framework for placement and sizing of FACTS devices in the power transmission systems. We worked within the exact AC PF paradigm and accounted for many properly weighted load configurations. Our problem formulation can be considered as generalizing standard AC OPF approach. The generalization consists of (a) modifying cost of the OPF to account, in addition to the standard cost of the generation dispatch, for the cost of FACTS installation (also promoting sparsity of solution); (b) allowing operational FACTS controls, different for individual loads but all within the limits of the state of installation. We have constructed efficient heuristics for solving this nonlinear and non-convex optimization. Our solver builds a convergent sequence of convex optimizations with linear constraints. Each constraint is represented explicitly through exact analytical linearization of the originally nonlinear constraints (e.g. representing power flows and apparent power line limits) over all the degrees of freedoms (including FACTS corrections) around the current operational point. Performance of the solver is illustrated on our enabling example of the 30 node Matpower model.

Armed with the newly developed computational tool we utilize it to improve power system operations, to reinforce the system to withstand the disturbances and to reduce the risk of power outages. We have developed a technique of FACTS investment planning applicable for a given power system based on its load duration curve and loading predictions.
We also have developed convenient web visualization which allows to get information about solver status at each iteration, see and compare our solutions with OPF. We can detect congested lines and voltage problems in the system, visualize a solution process and monitor how the state of the system changes after each iteration. With the help of the visualization tool we were able to derive capacities of installed devices and optimal locations for their (devices) installation. We also were able to discover optimal settings of the devices for each considered scenario.

6.2 Future work

There are several following extensions of this work in our plans. The first path is algorithm improvement which includes both performance and convergence, for example evaluating constraints only when needed (cutting plane) and implementation of Bender’s decomposition [33] (way to approach very large optimization problems that have a natural separation property) to solve problem faster and developing tricks for limiting change of parameters on each iteration step, for example dividing overloads to an intervals and resolving them smoothly or starting from small service periods when approaching large times and then take new solution and increase service period. For now we can successfully resolve reasonable number of scenarios for small modelling systems but when we switch to real model we need to have huge reserve of performance. The second is demonstration of scalability (for thousands-node large systems). For now we are able to resolve from 2 to 4 configurations of real large systems, but we suggest that our improvements will allow us to work with hundreds of scenarios on a standard PC. Finally, we plan to work on model generalizations, which includes accounting for other installation and control options like these related to phase-shifters, line switching and also improving modeling of costs. We will improve power system modelling by adding support of tap changers and various types of loads.

We also plan to apply developed approach. We think about exact problems which can be resolved by us and how actually deal with them. We plan to develop a service which will work aiming to improve operations of the power system online or plan investments to a power system for the future.
Chapter 7

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Chapter 8

Appendix

In this chapter we show mathematical derivations and equations used. We construct optimization problem using introduced model and notations and then approach it by introducing another optimization problem with linear constraints which can be consequently resolved in order to find a solution of a general one.
8.1 Equations for general optimization problem

All the notations are introduced in Section 3.1 and power system modelling is discussed in Section 2.1.

Here again a - number of the scenario, i,j - numbering of nodes in the grid. We use π-model of the transmission power line with charge capacitance taken into account. For each line \( r \) - resistance, \( x \) - inductance and \( b \) - capacitance are known.

Using π-model of the line (no tap changers and phase shifters), the dependence between the current on the ends and voltages on the ends can be derived:

\[
\begin{align*}
  i_{ij} &= \left( \frac{1}{r + jx} + j \frac{b}{2} \right) v_i - \frac{1}{r + jx} v_j \\
  \text{(8.1.1)}
\end{align*}
\]

Then apparent power flowing at each end can be calculated:

\[
\begin{align*}
  S_{ij} &= \frac{r V_i^2 - r V_i V_j \cos(\theta_i - \theta_j) + x V_i V_j \sin(\theta_i - \theta_j)}{r^2 + x^2} + \\
  &+ j \frac{x V_i^2 - r V_i V_j \sin(\theta_i - \theta_j) - x V_i V_j \cos(\theta_i - \theta_j)}{r^2 + x^2} - \frac{b}{2} V_i^2 \\
  \text{(8.1.2)}
\end{align*}
\]

We introduce three parameters in order to write equations for thermal limits limitations for the lines and network equations (power balance at each node):

- \( F_{ij} = |\sqrt{(P_{ij}^{(a)})^2 + (Q_{ij}^{(a)})^2}| \) - squared absolute value of the power flow from the node i to the node j
- \( P_i \) - active power injection/consumption at the node i
- \( Q_i \) - reactive power injection/consumption at the node i

This can be expressed in terms of state variables \((V, \theta, P, Q, x)\) as it is shown below:

\[
\begin{align*}
  F_{ij} &= \frac{V_i^4 + V_i^2 V_j^2 - 2 V_i^3 V_j \cos(\theta_i - \theta_j)}{r_{ij}^2 + x_{ij}^2} - \\
  &- \frac{b(x_{ij} V_i^4 - x_{ij} V_i^3 V_j \cos(\theta_i - \theta_j) - r_{ij} V_i^3 V_j \sin(\theta_i - \theta_j))}{r_{ij}^2 + x_{ij}^2} + \\
  &+ \frac{b_{ij}^2 V_i^4}{4} \\
  \text{(8.1.3)}
\end{align*}
\]
This equations define non-linear constraints in the general optimization problem for FACTS placement which is formulated in Section 3.2. For the convenience we also rewrite it here:

\[
P_i = \sum_{j \sim i} \frac{r_{ij} V_i^2 - r_{ij} V_i V_j \cos(\theta_i - \theta_j) + x_{ij} V_i V_j \sin(\theta_i - \theta_j) + b_{ij} V_i^2}{r_{ij}^2 + x_{ij}^2}
\]

\[
Q_i = \sum_{j \sim i} \left( \frac{x_{ij} V_i^2 - r_{ij} V_i V_j \sin(\theta_i - \theta_j) - x_{ij} V_i V_j \cos(\theta_i - \theta_j)}{r_{ij}^2 + x_{ij}^2} - \frac{b_{ij}}{2} V_i^2 \right)
\]

\[
\text{min}_{\Delta x, \Delta Q; \text{state}} \quad \text{COST} (\Delta x, \Delta Q; \text{state})
\]

\[
\text{COST} = (C_{SC} \sum_{(i,j) \in E} \Delta x_{ij} + C_{SVC} \sum_{i \in V} \Delta Q_i + N_y \sum_{a=1}^{T_a} \sum_{C_a} (P^{(a)}))
\]

\[
\text{state} = (x^{(a)}, v^{(a)}, \theta^{(a)}, Q^{(a)}, P^{(a)}), \forall a
\]

\[
x^{(a)} = x_0^{(a)} + \Delta x^{(a)}, \forall a
\]

\[
Q_{\text{load}}^{(a)} = Q_{\text{load}}^{(a)} - 0 + \Delta Q_{\text{load}}^{(a)}, \forall a
\]

\[
-\Delta x \leq \Delta x^{(a)} \leq \Delta x, \forall a
\]

\[
-\Delta Q \leq \Delta Q_{\text{load}}^{(a)} \leq \Delta Q, \forall a
\]

\[
V_{\text{min}}^{(a)} \leq V^{(a)} \leq V_{\text{max}}^{(a)}, \forall a
\]

\[
Q_{\text{min-gen}}^{(a)} \leq Q_{\text{gen}}^{(a)} \leq Q_{\text{max-gen}}^{(a)}, \forall a
\]

\[
P_{\text{min-gen}}^{(a)} \leq P_{\text{gen}}^{(a)} \leq P_{\text{max-gen}}^{(a)}, \forall a
\]

\[
\sqrt{(P_{ij}^{(a)})^2 + (Q_{ij}^{(a)})^2} \leq S_{ij}^{(a)}, \forall i, j \in E^{(a)}
\]

\[
P_i + i Q_i = \sum_{j : (i,j) \in E^{(a)}} (S_{ij}^{(a)}), \forall i \in V^{(a)}, \forall a
\]

We develop an iterative algorithm which finds a solution for a given problem starting from a specified initial states for the scenarios and improves it consequently by solving Quadratic Programming (QP) with linear constraints on each step. The linearization of constraints is done analytically, on each step we calculate Jacobian matrices and use them to define constraints for the QP solver.
8.2 Equations for linearized optimization problem

We illustrate the linearization procedure for one scenario, for multiple scenarios it is generalized by constructing block matrices out of Jacobian matrices calculated for each scenario. Firstly we calculate partial derivatives, used for the Jacobian matrices calculations:

\[
\frac{\partial F_{ij}}{\partial x_{ij}} = -b_{ij}(V_i^4 - V_i^3 V_j cos(\theta_i - \theta_j)) - 2x_{ij} \frac{V_i^4 + V_i^2 V_j^2 - 2V_i^3 V_j cos(\theta_i - \theta_j)}{(r_{ij}^2 + x_{ij}^2)^2} - 2x_{ij} b_{ij} \frac{(x_{ij} V_i^4 - x_{ij} V_i^3 V_j cos(\theta_i - \theta_j) - r_{ij} V_i^3 V_j sin(\theta_i - \theta_j))}{(r_{ij}^2 + x_{ij}^2)^2}
\]

\[
\frac{\partial F_{ij}}{\partial V_i} = b^2_{ij} V_i^2 + 4V_i^3 + 2V_j^2 - 6V_i^2 V_j cos(\theta_i - \theta_j) - b_{ij} V_i^3 x_{ij} cos(\theta_i - \theta_j) - 3r_{ij} V_i^2 V_j sin(\theta_i - \theta_j)
\]

\[
\frac{\partial F_{ij}}{\partial V_j} = 2V_i^2 V_j - 2V_i^3 cos(\theta_i - \theta_j) - b_{ij} V_i^3 x_{ij} cos(\theta_i - \theta_j) - r_{ij} V_i^3 V_j sin(\theta_i - \theta_j)
\]

\[
\frac{\partial F_{ij}}{\partial \theta_i} = \frac{2V_i^3 V_j sin(\theta_i - \theta_j)}{r_{ij}^2 + x_{ij}^2} - b_{ij} \frac{-r_{ij} V_i^3 V_j cos(\theta_i - \theta_j) + x_{ij} V_i^3 V_j sin(\theta_i - \theta_j)}{r_{ij}^2 + x_{ij}^2}
\]

\[
\frac{\partial F_{ij}}{\partial \theta_j} = \frac{-2V_i^3 V_j sin(\theta_i - \theta_j)}{r_{ij}^2 + x_{ij}^2} - b_{ij} \frac{r_{ij} V_i^3 V_j cos(\theta_i - \theta_j) - V_i^3 V_j x_{ij} sin(\theta_i - \theta_j)}{r_{ij}^2 + x_{ij}^2}
\]

Important thing here is that each line has two ends and then two different values of apparent power on each end, there is a list of partial derivatives which is calculated for $F_{ji}$ in the same way.

$DFm$ is matrix of corresponding partial derivatives calculated in the "previous" point for power flows, it is constructed in the way that multiplication on the change of state vector $\Delta y$ gives a vector representing $\Delta F$ for every line (two values for each for both ends). Linearized thermal limit inequalities around the "previous" point for each given scenario will be written in
the following way:

\[ F_\text{Prev} + DFm(y - y_{prev}) \leq F^{\text{max}} \]  \hfill (8.2.6)

Here \( y \) is vector of the variables, \( y = (x, V, \theta, Q, P) \) (again for one given scenario)

Partial derivatives for \( P \) and \( Q \) at the nodes will be:

\[
\frac{\partial P_i}{\partial x_{ij}} = \sum_{j \sim i} \left[ \frac{V_i V_j \sin(\theta_i - \theta_j)}{r_{ij}^2 + x_{ij}^2} - 2x_{ij} \frac{r_{ij} V_i^2 - r_{ij} V_i V_j \cos(\theta_i - \theta_j) + V_i V_j x_{ij} \sin(\theta_i - \theta_j)}{(r_{ij}^2 + x_{ij}^2)^2} \right]
\]  \hfill (8.2.7)

\[
\frac{\partial P_i}{\partial V_i} = \sum_{j \sim i} \left[ \frac{2r_{ij} V_i - r_{ij} V_j \cos(\theta_i - \theta_j) + V_i x_{ij} \sin(\theta_i - \theta_j)}{r_{ij}^2 + x_{ij}^2} \right]
\]  \hfill (8.2.8)

\[
\frac{\partial P_i}{\partial V_j} = \sum_{j \sim i} \left[ -r_{ij} V_i \cos(\theta_i - \theta_j) + V_i x_{ij} \sin(\theta_i - \theta_j) \right] \frac{1}{r_{ij}^2 + x_{ij}^2}
\]  \hfill (8.2.9)

\[
\frac{\partial P_i}{\partial \theta_i} = \sum_{j \sim i} \left[ V_i V_j x_{ij} \cos(\theta_i - \theta_j) + r_{ij} V_i \sin(\theta_i - \theta_j) \right] \frac{1}{r_{ij}^2 + x_{ij}^2}
\]  \hfill (8.2.10)

\[
\frac{\partial P_i}{\partial \theta_j} = \sum_{j \sim i} \left[ -V_i V_j x_{ij} \cos(\theta_i - \theta_j) - r_{ij} V_i \sin(\theta_i - \theta_j) \right] \frac{1}{r_{ij}^2 + x_{ij}^2}
\]  \hfill (8.2.11)

\[
\frac{\partial P_i}{\partial P_i} = 1 \hfill (8.2.12)
\]

\[
\frac{\partial Q_i}{\partial x_{ij}} = \sum_{j \sim i} \left[ \frac{V_i^2 - V_i V_j \cos(\theta_i - \theta_j)}{r_{ij}^2 + x_{ij}^2} - 2x_{ij} \frac{x_{ij} V_i^2 - x_{ij} V_i V_j \cos(\theta_i - \theta_j) - r_{ij} V_j \sin(\theta_i - \theta_j)}{(r_{ij}^2 + x_{ij}^2)^2} \right]
\]  \hfill (8.2.13)
\[
\frac{\partial Q_i}{\partial V_i} = \sum_{j \sim i} \left[ -b_{ij}V_i + \frac{2x_{ij}V_i - x_{ij}V_j\cos(\theta_i - \theta_j) - V_j r_{ij}\sin(\theta_i - \theta_j)}{r_{ij}^2 + x_{ij}^2} \right]
\]  
(8.2.14)

\[
\frac{\partial Q_i}{\partial V_j} = \sum_{j \sim i} \left[ -x_{ij}V_i\cos(\theta_i - \theta_j) - V_j r_{ij}\sin(\theta_i - \theta_j) \right]
\]  
(8.2.15)

\[
\frac{\partial Q_i}{\partial \theta_i} = \sum_{j \sim i} \left[ -V_j r_{ij}\cos(\theta_i - \theta_j) + x_{ij}V_j\sin(\theta_i - \theta_j) \right]
\]  
(8.2.16)

\[
\frac{\partial Q_i}{\partial \theta_j} = \sum_{j \sim i} \left[ V_j r_{ij}\cos(\theta_i - \theta_j) - x_{ij}V_j\sin(\theta_i - \theta_j) \right]
\]  
(8.2.17)

\[
\frac{\partial Q_i}{\partial Q_i} = 1
\]  
(8.2.18)

If i-load node, then \( P_i \) is fixed (\( P_\text{Prev}_i = P_i \)) and around the previous point, for each \( j \sim i \):

\[
0 = \left( \frac{\partial P_i}{\partial x_{ij}}, \frac{\partial P_i}{\partial V_i}, \frac{\partial P_i}{\partial V_j}, \frac{\partial P_i}{\partial \theta_i}, \frac{\partial P_i}{\partial \theta_j} \right) \ast (x_{ij} - x_\text{Prev}_{ij}, V_i - V_\text{Prev}_{i}, V_j - V_\text{Prev}_{j}, \theta_i - \theta_\text{Prev}_{i}, \theta_j - \theta_\text{Prev}_{j})
\]  
(8.2.19)

\[
1 \ast (Q_i - Q_\text{Prev}_i) = \left( \frac{\partial Q_i}{\partial x_{ij}}, \frac{\partial Q_i}{\partial V_i}, \frac{\partial Q_i}{\partial V_j}, \frac{\partial Q_i}{\partial \theta_i}, \frac{\partial Q_i}{\partial \theta_j} \right) \ast (x_{ij} - x_\text{Prev}_{ij}, V_i - V_\text{Prev}_{i}, V_j - V_\text{Prev}_{j}, \theta_i - \theta_\text{Prev}_{i}, \theta_j - \theta_\text{Prev}_{j})
\]  
(8.2.20)

If i-generator node, then around the previous point, for each \( j \sim i \):

\[
(P_i - P_\text{Prev}_i) = \left( \frac{\partial P_i}{\partial x_{ij}}, \frac{\partial P_i}{\partial V_i}, \frac{\partial P_i}{\partial V_j}, \frac{\partial P_i}{\partial \theta_i}, \frac{\partial P_i}{\partial \theta_j} \right) \ast (x_{ij} - x_\text{Prev}_{ij}, V_i - V_\text{Prev}_{i}, V_j - V_\text{Prev}_{j}, \theta_i - \theta_\text{Prev}_{i}, \theta_j - \theta_\text{Prev}_{j})
\]  
(8.2.21)

\[
(Q_i - Q_\text{Prev}_i) = \left( \frac{\partial Q_i}{\partial x_{ij}}, \frac{\partial Q_i}{\partial V_i}, \frac{\partial Q_i}{\partial V_j}, \frac{\partial Q_i}{\partial \theta_i}, \frac{\partial Q_i}{\partial \theta_j} \right) \ast (x_{ij} - x_\text{Prev}_{ij}, V_i - V_\text{Prev}_{i}, V_j - V_\text{Prev}_{j}, \theta_i - \theta_\text{Prev}_{i}, \theta_j - \theta_\text{Prev}_{j})
\]  
(8.2.22)

The same linearized equations for active and reactive power in vector notation:

\[
0 = \left( \frac{\partial P_i}{\partial x_{ij}}, \frac{\partial P_i}{\partial V_i}, \frac{\partial P_i}{\partial V_j}, \frac{\partial P_i}{\partial \theta_i}, \frac{\partial P_i}{\partial \theta_j}, -\frac{\partial P_i}{\partial x_{ij}} \right) \ast (x_{ij} - x_\text{Prev}_{ij}, V_i - V_\text{Prev}_{i}, V_j - V_\text{Prev}_{j}, \theta_i - \theta_\text{Prev}_{i}, \theta_j - \theta_\text{Prev}_{j}, P_i - P_\text{Prev}_i)
\]  
(8.2.19)

\[
0 = \left( \frac{\partial Q_i}{\partial x_{ij}}, \frac{\partial Q_i}{\partial V_i}, \frac{\partial Q_i}{\partial V_j}, \frac{\partial Q_i}{\partial \theta_i}, \frac{\partial Q_i}{\partial \theta_j}, -\frac{\partial Q_i}{\partial x_{ij}} \right) \ast (x_{ij} - x_\text{Prev}_{ij}, V_i - V_\text{Prev}_{i}, V_j - V_\text{Prev}_{j}, \theta_i - \theta_\text{Prev}_{i}, \theta_j - \theta_\text{Prev}_{j}, Q_i - Q_\text{Prev}_i)
\]  
(8.2.20)
Then we can rewrite all the linearized active and reactive power balance equations in the matrix form:

\[ DP_m(y - y_{prev}) = 0 \]  

(8.2.23)

Here \( y \) is vector of the variables, \( y = (x, V, \theta, Q, P) \).

Cost functions for each generator are usually polynomial functions of the 2nd order. Optimization over sum of quadratic form and linear function will be used in order to solve the following problem on each iteration step from \( y_{Prev} \) to new state \( y \):

\[
\begin{align*}
\min_{\Delta x, \Delta Q} & \quad C_{SC} \sum_{(i,j) \in E} \Delta x_{ij} + C_{SVC} * \sum_{i \in V_l} \Delta Q_i + N_y \sum_{a=1..N} T_a * C_a(P^{(a)}) \\
\text{s.t.} & \quad \text{state}^{(a)} = y^{(a)} = (x^{(a)}, V^{(a)}, \theta^{(a)}, Q^{(a)}, P^{(a)}) \quad \forall a = 1, \ldots, N \\
& \quad x^{(a)} = x_0^{(a)} + \Delta x^{(a)} \quad \forall a = 1, \ldots, N \\
& \quad Q_{load}^{(a)} = Q_{load}^{(a)} - 0 + \Delta Q_{load}^{(a)} \quad \forall a = 1, \ldots, N \\
& \quad -\Delta x \leq \Delta x^{(a)} \leq \Delta x \quad \forall a = 1, \ldots, N \\
& \quad -\Delta Q \leq \Delta Q_{load}^{(a)} \leq \Delta Q \quad \forall a = 1, \ldots, N \\
& \quad V_{\min}^{(a)} \leq V^{(a)} \leq V_{\max}^{(a)} \quad \forall a = 1, \ldots, N \\
& \quad Q_{\min-gen}^{(a)} \leq Q_{gen}^{(a)} \leq Q_{\max-gen}^{(a)} \quad \forall a = 1, \ldots, N \\
& \quad P_{\min-gen}^{(a)} \leq P_{gen}^{(a)} \leq P_{\max-gen}^{(a)} \quad \forall a = 1, \ldots, N \\
& \quad F_{Prev}^{(a)} + DFm^{(a)}(y^{(a)} - y_{prev}^{(a)}) \leq F^{max}_{(a)} \\
& \quad DPm^{(a)}(y^{(a)} - y_{prev}^{(a)}) = 0 \\
\end{align*}
\]  

(8.2.24)

where \( C_a(P^{(a)}) \) stands for the function representing the cost of generation for scenario \( a \), and we use the following notation for the complex voltage/potential: \( v_i^{(a)} = V_i^{(a)} \exp(i\theta_i^{(a)}) \), \( \forall a = 1, \ldots, N \), \( i \in V^{(a)} \).
8.3 Multiple-scenarios optimization generalization

In case of multiple scenario optimization each scenario is defined by occurrence probability, grid structure, load configuration and uniform loading parameter $\alpha$. We optimize over all input scenarios simultaneously, so, on each iteration we linearize constraints for each scenario as it is described above. But capacity variables are the same for all the scenarios. There are various way how to deal with it, we implement a solution when solver operates with variables vector $X$:

$$
X = (\Delta Q, \Delta x|x^1, V^1, \Theta^1, Q^1, P^1|...|x^N, V^N, \Theta^N, Q^N, P^N)'\;.
$$

We construct block matrices $DF_{m\_mult}$ and $DP_{m\_mult}$ corresponding to $X$ vector, so that linear approximations of lines thermal constraints and active and reactive power balances at each node written in a matrix form are valid for all the scenarios simultaneously.

8.4 Convergence of the algorithm

In Section 3.3 we discuss optimization algorithm which allows us to find a solution of the general optimization problem formulated previously. One observes that during each iteration we operate with two states. The first is before step 2 - previous state before solving QP and the second state is given by QP solver (solution of the linearized optimization problem). We call the first state preopt and the solution of QP - qpsol state. On the step 3 we update qpsol to gen new preopt state and perform another linearization. Overall during one iteration cycle we have two states (preopt and qpsol) and two changes of states - QP difference, which is equal to qpsol-preopt, and PF difference caused by state update with PF solver. In order to our linearization to be valid QP difference should be relatively small. And a signal that approximation works fine is that PF difference is small (if it is zero that means QP solver found some existing state which satisfies power network equations).

We include visualizations of QP difference and PF difference to our specially developed visualization procedure. The state of the power system in our case is defined by a vector $y = (x, V, \theta, Q, P)$, we explore QP and PF differences for each of that parameters for each line or node respectively.
The examples of QP and PF differences of line inductances $x$ for single OPF infeasible case are shown in the Figs. 8.1 - 8.2:

**Figure 8.1:** Illustration of the QP difference of line inductance for all the lines of the modelled system. Change of inductance is pretty high (initial is 7 Ohms approximately), but for now we do not artificially limit it.

Solver places SC devise to adjust line inductance.

**Figure 8.2:** Illustration of the PF difference of line inductance for all the lines of the modelled system. It is zero, PF solver does not adjust line inductances and use input values.

Line inductance does not change during PF state update step then PF difference of $x$ is
equal to zero. The examples of QP and PF differences of active power for all generators in the system P for single OPF infeasible case are shown in the Figs. 8.3 - 8.4:

Figure 8.3: Illustration of the QP difference of active power inputs for all the generators of the modelled system. Change of P is small in comparison with actual values.

QP step of algorithms can correct all power injections, there is a cost of generation which automatically limits the adjustments.

Figure 8.4: Illustration of the PF difference of active power inputs for all the generators of the modelled system. Change of P is equal to zero for PV nodes and has some small value for slack bus.
PF solver can adjust only power output of slack bus generator, the other generators are PV nodes. PF adjustment is much less than QP one. The examples of QP and PF differences of reactive power for all nodes in the system Q for single OPF infeasible case are shown in the Figs. 8.5 - 8.6:

Figure 8.5: Illustration of the QP difference of reactive powers for all the nodes of the modelled system. Change of Q is artificially limited for generators and small in comparison with actual value for SVC compensated load.

Figure 8.6: Illustration of the PF difference of reactive power inputs for all the nodes of the modelled system. PF corrections are small.
Solver does both adjustments - of reactive power output of active generators and of SVC compensator placed in the system. SVC compensation is not limited on QP step and in this case on each QP step it changes more than generators outputs. Finally the process converges.

PF corrections are small which indicates correct work of a solver. We observe similar results for voltages and phases in the nodes and do not show them here. PF difference for voltages and phases in case of convergence always much less than QP difference.
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