Report on the content of the dissertation

Joel Spencer
(name of the member of the committee)

Andrey Borisovich Kupavskii
(name of the candidate for the degree)

Families of Sets with Forbidden Configurations and Applications to Discrete Geometry
(Title of the dissertation, degree, specialization)

doctor of physical and mathematical sciences,

05.13.17 – theoretical foundations of computer science

Date of the defense: *18.05.2019*

The evaluation of the dissertation in accordance with the Regulations on the award of scientific degrees of candidates and doctors of sciences at MIPT (in what follows, referred to as Regulations):

1. Relevance of the topic of the dissertation:

The combinatorial aspect of the extremal problems considered has a strong Hungarian flavor, dating back to results of Turán and others. Simultaneously there is a key geometric flavor as many of the graphs are derived from random subsets of sets in d-space. The joining of combinatorial, probabilistic and geometric arguments gives a powerful methodology that is central to Kupavskii's work.

2. Scientific novelty of the results:

The opening sections give an insightful discussion of the powerful methodologies available. The discussion of shifting and its cousin, the Katona circle method, were particularly strong.

Kneser graphs are geometric graphs with strong combinatorial properties. Their study from a combinatorial perspective – particularly the chromatic number - was based on a remarkable early result of Lovasz. Kupavskii studies random Kneser graphs. He shows that over a surprisingly wide range of parameters the chromatic number of the random Kneser graph varies only by a small term. The more precise results are found in Theorems 4.1, 4.2. The results are surprising and require a juxtaposition of combinatorial, probabilistic and geometric arguments. This notion of the robustness of the Kneser graph has had a strong influence on work in this field.

Kupavskii gives applications of his results to classic distance problems. Borsuk's conjecture has been known for some decades to be false and asymptotically very false. All of the previously known counterexamples had basically used points of R^d lying on spheres with radii close to $2^{-1/2}$. This appeared to be a natural limitation. However, Kupavskii shows that for any $r > 1/2$, for sufficiently high dimension $d > d(r)$, there exists a subset of the d-sphere of radius r with diameter 1 which cannot be partitioned into $d + 1$ parts of smaller diameter. Indeed, there are more precise results given in Theorem 6.5 and 6.6. Again the arguments require a mixture of probabilistic, combinatorial and geometric reasoning.

3. Theoretical and practical importance of the dissertation:

The dissertation is of theoretical importance.

4. Are the main results of the dissertation adequately represented in the publications in refereed journals, according to the Regulations?

The results of the dissertation are adequately represented in 27 publications in refereed journals, 21 out of which are indexed by Scopus.

5. Questions and remarks (according to part 4.13 of the Regulations, the candidate addresses the questions and remarks formulated below during the defense):

No questions or remarks.

6. General evaluation of the dissertation (excluding the introductory part):

Kupavskii already has an international reputation and so it comes as no surprise that this work "Families of Sets ... Discrete Geometry" is an excellent piece of work. I enthusiastically recommend acceptance of this dissertation for the degree of doctor of physical and mathematical sciences.

Date 30 APRIL 2019

Signature Joel Spencer / Joel Spencer