Report on the content of the dissertation

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Families of Sets with Forbidden Configurations and Applications to Discrete Geometry
(Title of the dissertation, degree, specialization)

doctor of physical and mathematical sciences,

05.13.17 – theoretical foundations of computer science

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The evaluation of the dissertation in accordance with the Regulations on the award of scientific degrees of candidates and doctors of sciences at MIPT (in what follows, referred to as Regulations):

1. Relevance of the topic of the dissertation:

   Extremal set theory is an important branch of combinatorial mathematics that has applications to many other areas of mathematics, as well as to other disciplines such as computer science, coding theory and information theory. The major development of this subject began in the 1960s with the problems and results of Paul Erdos, and today it is a vibrant area of mathematics, with many famous open problems that are intensively studied by the best researchers in the world. The dissertation makes a positive contribution to several of these important questions and hence it is highly relevant in relation to current mainstream research in mathematics.

2. Scientific novelty of the results:

   There are three main strands of research in the dissertation so let me briefly describe the contributions of each of them.

   (a) Structure and size of intersecting families
   Here the dissertation focuses on extensions of the celebrated Erdos-Ko-Rado theorem which determines the maximum size of intersecting families of sets. The extremal example for this theorem is the full star, namely the family of all sets that contain a particular point, and there has been much research on intersecting families that do not have this structure. The diversity of a set system is the difference between its size and the size of its largest star. Several new results about the size of an intersecting family in terms of its diversity are given. The most difficult case of intersecting theorems is when the size of the sets is a constant multiple of the number of vertices, and a result is proved about typical intersecting families in this range.

(b) Families without large matchings
The most important problem in this area is the Erdos Matching Conjecture, which posits a conjecture about the maximum size of a family of k-elements sets on n elements that contains no s disjoint members. The same problem has also been considered (starting with Kleitman in the 1960s) when there is no restriction on the set sizes. A variety of new results surrounding these questions are obtained using novel methods.

(c) Geometric Problems
Several results are proved about epsilon nets which is itself one of the most important concepts in discrete geometry. Following the celebrated disproof of the half century old "Borsuks conjecture" by Kahn and Kalai, there have been many related constructions for the problem and here another construction is provided that has a nice geometric structure. In a different direction, a construction of geometrically defined graphs with large girth and chromatic number is provided.

3. Theoretical and practical importance of the dissertation:

The dissertation is of theoretical importance. The findings of the author may be applied in different problems in extremal combinatorics and discrete geometry. In fact, there have already been some applications of the results.

4. Are the main results of the dissertation adequately represented in the publications in refereed journals, according to the Regulations?

The results of the dissertation are adequately represented in 27 publications in refereed international journals, 21 out of which are indexed by Scopus. The results of the dissertation were presented at numerous international conferences.

5. Questions and remarks (according to part 4.13 of the Regulations, the candidate addresses the questions and remarks formulated below during the defense):

A general question/comment: there were several recent works that study the finer structure of intersecting families (and related questions) well below the bound guaranteed by even the Hilton-Miler theorem, (Kohayakawa-Han, Kostochka-Mubayi). A general formulation was provided by Jackowska-Polcyn-Rucinski, in "Ramsey numbers and restricted Turan numbers for the loose 3-uniform path of length *Electron. J. Combin.* 24(3) (2017) \#P3.5" where a hierarchy of Turan number is formulated.

Can the diversity result (Theorem 1.7) be taken even further in the sense described by these papers, i.e., if the family is not a subset of some structured family (in this case H_2 from the Hilton-Milner theorem), can one prove even better bounds?

One can ask the very same question for Theorem 1.10: if we do not allow stars, can we still get good bounds on the number of intersecting families? What if we go one step further and do not allow subsets of Hilton-Miler configurations?

In fact, the same question can be asked for virtually any extremal Turan type question where the extremal configurations have some sort of well defined structure (even perhaps for cross intersecting theorems). Eg, for the Kruskal-Katona theorem, one could restrict the maximum clique size (or forbid some other configuration) and ask how small the shadow can be -- here we would get larger and larger values for the minimum shadow until, eventually, perhaps we get shadows of the same order of magnitude as the original family given by a matching. Frankl has some related results here, but the point is to formulate and begin a systematic investigation of extremal set theory problems (where possible) as indicated by the Jackowska-Polcyn-Rucinski paper.

6. General evaluation of the dissertation (excluding the introductory part):

   The dissertation of Andrey Borisovich Kupavskii deals with interesting and important problems of extremal combinatorics and is a sound scientific investigation. It not only proves new results on problems that have been researched for a while, but also develops new methods that should have impact well into the future. The topic of the dissertation corresponds to the specialization 05.13.17 – theoretical foundations of computer science. The dissertation is in accordance with the Regulations on the award of scientific degrees of candidates and doctors of sciences at MIPT.

Date ____________4-24-2019__________

Signature ____________________ / Dhruv Mubayi