Federal State Autonomous Educational Institution of Higher Education "Moscow Institute of Physics and Technology (National Research University)"

Work program of the course (training module)

course:	Combinatorics and Graphs/Комбинаторика и графы
major:	Information Science and Computer Engineering
specialization:	Computer Science/Информатика
	Phystech School of Applied Mathematics and Informatics
	Chair of Discrete Mathematics
term:	2
qualification:	Bachelor

Semester, form of interim assessment: 3 (fall) - Grading test

Academic hours: 60 AH in total, including:
    lectures: 30 AH.
    seminars: 30 AH.
    laboratory practical: 0 AH.

Independent work: 75 AH.

In total: 135 AH, credits in total: 3

Author of the program: A.B. Daynyak, candidate of physics and mathematical sciences, associate professor, associate professor

The program was discussed at the Chair of Discrete Mathematics 05.03.2020

<div align="center">**Annotation**</div>

One of the main objectives of the course is to teach students the methods of thinking characteristic of discrete mathematics, the basic concepts of combinatorics and graph theory, as well as the development of algorithmic thinking skills. The manual covers such areas of discrete mathematics as the foundations of set theory, combinatorics and graph theory.

1. Study objective

Purpose of the course

- mastering the basic concepts of combinatorics and graphs

Tasks of the course

- students mastering basic knowledge (concepts, concepts, methods and models) in the field of combinatorics and graphs;
- acquisition of theoretical knowledge and practical skills in the field of combinatorics and graphs;
- providing advice and assistance to students in conducting their own theoretical research in the field of combinatorics and graphs.

2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

Code and the name of the competence	Competency indicators
Gen.Pro.C-1 Apply fundamental knowledge acquired in the physical and mathematical fields and/or natural sciences and use it in professional settings	Gen.Pro.C-1.2 Build mathematical models, make quantitative measurements and estimates
Gen.Pro.C-4 Collect and process scientific and technical and/or technological data for fundamental and applied problem-solving	Gen.Pro.C-4.2 Search for primary sources of scientific and technical and/or technological information in professional settings
Pro.C-2 Conduct scientific research and testing independently or as a member (leader) of a small research team	Pro.C-2.1 Apply the principles of scientific work, methods of collecting and analyzing the obtained data and ways of argumentation

3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

- Fundamental concepts, laws, theories of combinatorics and graphs;
- current problems of the relevant sections of combinatorics and graphs;
- concepts, axioms, methods of proof and proof of the main theorems in the sections included in the basic part of the cycle;
- basic properties of the corresponding mathematical objects;
- analytical and numerical approaches and methods for solving typical applied problems of combinatorics and graphs.

be able to:

- Understand the task;
- use your knowledge to solve fundamental and applied problems of combinatorics and graphs;
- evaluate the correctness of the problem statements;
- strictly prove or disprove the statement;
- independently find algorithms for solving problems, including non-standard ones, and conduct their analysis;
- independently see the consequences of the results;
- accurately represent mathematical knowledge in the field of complex computing in oral and written form.

master:

- Skills of mastering a large amount of information and solving problems (including complex ones);
- skills of independent work and mastering new disciplines;
- the culture of the formulation, analysis and solution of mathematical and applied problems requiring the use of mathematical approaches and methods of combinatorics and graphs for their solution;
- the subject language of complex calculations and the skills of competent description of problem solving and presentation of the results.

4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

4.1. The sections of the course (training module) and the complexity of the types of training sessions

№	Topic (section) of the course	Types of training sessions, including independent work			
		Lectures	Seminars	Laboratory practical	Independent work
1	Formal power series and generating functions. Generating functions for linear recurrences	6	6		12
2	Generating functions (continued). Derivation of a formula for Catalan numbers. Generating functions and integer partitions	6	6		15
3	Combinatorics of permutations. Cyclic structure of permutations. Permutation groups. Graph automorphism	6	6		16
4	Other examples of groups. Cayley's theorem. Counting w.r.t. group actions	6	6		16
5	Counting w.r.t. group actions (continued). Cauchy—Frobenius—Burnside lemma and Redfield—Polya counting framework. Counting graph colorings revisited	6	6		16
AH in total		30	30		75
Exam preparation	0 AH.				
Total complexity	135 AH., credits in total 3				

4.2. Content of the course (training module), structured by topics (sections)

Semester: 3 (Fall)

  1. Formal power series and generating functions. Generating functions for linear recurrences

  Extremal problems on graphs. Turán's theorem. More extremal problems: paths, trees and 4-cycles.

  2. Generating functions (continued). Derivation of a formula for Catalan numbers. Generating functions and integer partitions

  Bounds for Ramsey numbers: a lower bound via counting, an upper bound by binomial coefficient.

3. Combinatorics of permutations. Cyclic structure of permutations. Permutation groups. Graph automorphism

Set families (hypergraphs). Systems of distinct representatives (connection with bipartite graph matchings), systems of common representatives, witnesses.

4. Other examples of groups. Cayley's theorem. Counting w.r.t. group actions

Intersecting set families. Theorems of Erdos—Ko—Rado (with proof) and Ahlswede—Khachatrian (statement only).

5. Counting w.r.t. group actions (continued). Cauchy—Frobenius—Burnside lemma and Redfield—Polya counting framework. Counting graph colorings revisited

Combinatorics on words. De Bruijn sequences via eulerian walks.

5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)

A standard classroom.

6. List of the main and additional literature, that is necessary for the course (training module) mastering

Main literature
1. Random Graphs /B. Bollobas ; University of Memphis, Trinity College. Cambridge, Cambridge University Press, 2001
2. Дискретный анализ. Комбинаторика. Алгебра логики. Теория графов [Текст] : учеб. пособие для вузов / Ю. И. Журавлев, Ю. А. Флеров, О. С. Федько ; М-во образования и науки РФ, Моск. физ.-техн. ин-т (гос. ун-т .— М. : МФТИ, 2012 .— 248 с.
3. Комбинаторика [Текст], [учеб. пособие для вузов] /Н. Я. Виленкин, А. Н. Виленкин, П. А. Виленкин. -М., ФИМА : МЦНМО, 2015

Additional literature
1. Extremal Combinatorics. With Applications in Computer Science /Stasys Jukna. Berlin Heidelberg, Springer, 2011
2. Задачи по программированию. Математическая логика. Теория алгоритмов. Рекурсия. Сортировка. Графы [Текст] : метод. указания к практикуму по курсу "Основы информатики" (для студентов 1 курса) / М-во образования Рос. Федерации, Моск. физ.-техн. ин-т (гос. ун-т), Каф. вычислит. математики ; сост. В. В. Прут .— М. : МФТИ, 2002 .— 32 с.

7. List of web resources that are necessary for the course (training module) mastering

http://dm.fizteh.ru/

8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)

Multimedia technologies can be employed during lectures and practical lessons, including presentations.

9. Guidelines for students to master the course

It is recommended to successfully pass the test papers, as this simplifies the final certification in the subject. To prepare for the final certification on the subject, it is best to use lecture materials.

Assessment funds for course (training module)

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	Phystech School of Applied Mathematics and Informatics
	Chair of Discrete Mathematics
term:	2
qualification:	Bachelor

Semester, form of interim assessment: 3 (fall) - Grading test

Author: A.B. Daynyak, candidate of physics and mathematical sciences, associate professor, associate professor

1. Competencies formed during the process of studying the course

Code and the name of the competence	Competency indicators
Gen.Pro.C-1 Apply fundamental knowledge acquired in the physical and mathematical fields and/or natural sciences and use it in professional settings	Gen.Pro.C-1.2 Build mathematical models, make quantitative measurements and estimates
Gen.Pro.C-4 Collect and process scientific and technical and/or technological data for fundamental and applied problem-solving	Gen.Pro.C-4.2 Search for primary sources of scientific and technical and/or technological information in professional settings
Pro.C-2 Conduct scientific research and testing independently or as a member (leader) of a small research team	Pro.C-2.1 Apply the principles of scientific work, methods of collecting and analyzing the obtained data and ways of argumentation

2. Competency assessment indicators

As a result of studying the course the student should:

know:
- Fundamental concepts, laws, theories of combinatorics and graphs;
- current problems of the relevant sections of combinatorics and graphs;
- concepts, axioms, methods of proof and proof of the main theorems in the sections included in the basic part of the cycle;
- basic properties of the corresponding mathematical objects;
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- the subject language of complex calculations and the skills of competent description of problem solving and presentation of the results.

3. List of typical control tasks used to evaluate knowledge and skills

Examples of home tasks:

1. What is the maximum number of edges in a graph with 12 vertices with 3 connected components?
2. How many pairwise non-isomorphic 2-regular bipartite graphs on 16 vertices exist?
3. How many different 2-regular graphs on 16 vertices with fixed parts $\{x1, ..., x8\}$ and $\{y1, ..., y8\}$ exist? 4. What is the maximum number of vertices in a graph in which there is neither an independent set on three vertices nor odd cycles?
5. Find the smallest possible independence number of a graph on 6 vertices without triangles.

4. Evaluation criteria

The list of control questions for delivery of the differentiated offset:

1. Metrics on graphs. Graph diameter, radius, center. Various notions of vertex centrality.
2. Extremal problems on graphs. Turán's theorem. More extremal problems: paths, trees and 4-cycles.
3. Ramsey-type theorems in graph theory and combinatorics (a brief overview). Basic Ramsey theorem for graphs.
4. Bounds for Ramsey numbers: a lower bound via counting, an upper bound by binomial coefficient.
5. Posets. Sperner's lemma, LYM-inequality. Yet another example of combinatorial duality: chain/antichain decomposition theorems of Mirsky and Dilworth.
6. Set families (hypergraphs). Systems of distinct representatives (connection with bipartite graph matchings), systems of common representatives, witnesses.
7. Covers of graphs and hypergraphs. Vertex covers vs. matchings in graphs. Greedy covers.
8. Intersecting set families. Theorems of Erdos—Ko—Rado (with proof) and Ahlswede—Khachatrian (statement only).
9. Introduction to combinatorial designs (incidence systems). Steiner triple. Finite geometries.
10. Combinatorics on words. De Bruijn sequences via eulerian walks.
11. Formal power series and generating functions. Generating functions for linear recurrences.
12. Generating functions (continued). Derivation of a formula for Catalan numbers. Generating functions and integer partitions.
13. Combinatorics of permutations. Cyclic structure of permutations. Permutation groups. Graph automorphisms.
14. Other examples of groups. Cayley's theorem. Counting w.r.t. group actions.
15. Counting w.r.t. group actions (continued). Cauchy—Frobenius—Burnside lemma and Redfield—Polya counting framework. Counting graph colorings revisited.

Assessment "excellent (10)" is given to a student who has displayed comprehensive, systematic and deep knowledge of the educational program material, has independently performed all the tasks stipulated by the program, has deeply studied the basic and additional literature recommended by the program, has been actively working in the classroom, and understands the basic scientific concepts on studied discipline, who showed creativity and scientific approach in understanding and presenting educational program material, whose answer is characterized by using rich and adequate terms, and by the consistent and logical presentation of the material;

Assessment "excellent (9)" is given to a student who has displayed comprehensive, systematic knowledge of the educational program material, has independently performed all the tasks provided by the program, has deeply mastered the basic literature and is familiar with the additional literature recommended by the program, has been actively working in the classroom, has shown the systematic nature of knowledge on discipline sufficient for further study, as well as the ability to amplify it on one's own, whose answer is distinguished by the accuracy of the terms used, and the presentation of the material in it is consistent and logical;

Assessment "excellent (8)" is given to a student who has displayed complete knowledge of the educational program material, does not allow significant inaccuracies in his answer, has independently performed all the tasks stipulated by the program, studied the basic literature recommended by the program, worked actively in the classroom, showed systematic character of his knowledge of the discipline, which is sufficient for further study, as well as the ability to amplify it on his own;

Assessment "good (7)" is given to a student who has displayed a sufficiently complete knowledge of the educational program material, does not allow significant inaccuracies in the answer, has independently performed all the tasks provided by the program, studied the basic literature recommended by the program, worked actively in the classroom, showed systematic character of his knowledge of the discipline, which is sufficient for further study, as well as the ability to amplify it on his own;

Assessment "good (6)" is given to a student who has displayed a sufficiently complete knowledge of the educational program material, does not allow significant inaccuracies in his answer, has independently carried out the main tasks stipulated by the program, studied the basic literature recommended by the program, showed systematic character of his knowledge of the discipline, which is sufficient for further study;

Assessment "good (5)" is given to a student who has displayed knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, who while not being sufficiently active in the classroom, has nevertheless independently carried out the main tasks stipulated by the program, mastered the basic literature recommended by the program, made some errors in their implementation and in his answer during the test, but has the necessary knowledge for correcting these errors by himself;

Assessment "satisfactory (4)" is given to a student who has discovered knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, who while not being sufficiently active in the classroom, has nevertheless independently carried out the main tasks stipulated by the program, learned the main literature but allowed some errors in their implementation and in his answer during the test, but has the necessary knowledge for correcting these errors under the guidance of a teacher;

Assessment "satisfactory (3)" is given to a student who has displayed knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, not showed activity in the classroom, independently fulfilled the main tasks envisaged by the program, but allowed errors in their implementation and in the answer during the test, but possessing necessary knowledge for elimination under the guidance of the teacher of the most essential errors;

Assessment "unsatisfactory (2)" is given to a student who showed gaps in knowledge or lack of knowledge on a significant part of the basic educational program material, who has not performed independently the main tasks demanded by the program, made fundamental errors in the fulfillment of the tasks stipulated by the program, who is not able to continue his studies or start professional activities without additional training in the discipline in question;

Assessment "unsatisfactory (1)" is given to a student when there is no answer (refusal to answer), or when the submitted answer does not correspond at all to the essence of the questions contained in the task.

5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience

During examination the student are allowed to use the program of the discipline.