Federal State Autonomous Educational Institution of Higher Education "Moscow Institute of Physics and Technology (National Research University)"

APPROVED
Head of the Phystech School of Applied Mathematics and Informatics
A.M. Raygorodskiy

Work program of the course (training module)

course:	Mathematical Thinking/Математическое мышление
major:	Information Science and Computer Engineering
specialization:	Computer Science/Информатика
	Phystech School of Applied Mathematics and Informatics
	Chair of Discrete Mathematics
term:	1
qualification:	Bachelor

Semester, form of interim assessment: 1 (fall) - Grading test

Academic hours: 60 AH in total, including:
lectures: 30 AH.
seminars: 30 AH.
laboratory practical: 0 AH.

Independent work: 75 AH.

In total: 135 AH, credits in total: 3

Number of course papers, tasks: 2

Author of the program: E.V. Dashkov, candidate of physics and mathematical sciences, associate professor

The program was discussed at the Chair of Discrete Mathematics 04.06.2020

Annotation

The course covers mainly traditional issues: the foundations of set theory, propositional logic and first-order logic, elements of model theory, proof theory and the theory of algorithms. Considerable space is devoted to the most rigorous presentation of the set-theoretic formalism as the language of subsequent mathematical courses, as well as for didactic purposes. In connection with the focus on discrete mathematical sciences, the course examines and substantiates in detail various types of recursion, inductive definitions of sets, as well as the foundations of the theory of formal languages, the system of natural inference, lambda calculus.

1. Study objective

Purpose of the course

- mastering general mathematical terminology (sets, relationships, functions).

Tasks of the course

- Develop the skill of structured logical thinking;
- learn to give formal definitions and give examples of defined objects;
- learn to build formal records of mathematical statements and their proofs and work with these records;
- learn to conduct mathematical reasoning, not based on the specific properties of the objects under consideration.

2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

Code and the name of the competence	Competency indicators
Gen.Pro.C-1 Apply fundamental knowledge acquired in the physical and mathematical fields and/or natural sciences and use it in professional settings	Gen.Pro.C-1.1 Analyze the task in hand, outline the ways to complete it
Pro.C-1 Assign, formalize, and solve tasks, develop and research mathematical models of the studied phenomena and processes, systematically analyze scientific problems, obtain new scientific outcomes	Pro.C-1.2 Make hypotheses, build mathematical models of the studied phenomena and processes, evaluate the quality of the developed model

3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

- fundamental concepts, laws, theories of a part of discrete mathematics;
- modern problems of the corresponding sections of discrete mathematics;
- concepts, axioms, methods of proofs and proofs of the main theorems in the sections included in the basic part of the cycle;
- basic properties of the corresponding mathematical objects.

be able to:

- understand the task at hand;
- use your knowledge to solve fundamental and applied problems;
- evaluate the correctness of the problem setting;
- strictly prove or disprove the statement;
- independently find algorithms for solving problems, including non-standard ones, and analyze them;
- independently see the consequences of the results obtained;
- accurately present mathematical knowledge in the field orally and in writing.

master:

- skills of mastering a large amount of information and solving problems (including complex ones);
- skills of independent work and mastering new disciplines;
- culture of formulation, analysis and solution of mathematical and applied problems that require the use of mathematical approaches and methods for their solution;
- the subject language of discrete mathematics and the skills of competently describing the solution of problems and presenting the results obtained.

4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

4.1. The sections of the course (training module) and the complexity of the types of training sessions

№	Topic (section) of the course	Types of training sessions, including independent work			
		Lectures	Seminars	Laboratory practical	Independent work
1	Statements and logical connectives	5	5		10
2	Induction Principle: equivalent forms and applications	5	5		10
3	Sets and elements	5	5		10
4	Binary relations	5	5		10
5	Pigeonhole Principle	5	5		10
6	Special binary relations	3	3		10
7	Formal propositional logic	2	2		15
AH in total		30	30		75
Exam preparation		0 AH.			
Total complexity		135 AH., credits in total 3			

4.2. Content of the course (training module), structured by topics (sections)

Semester: 1 (Fall)

1. Statements and logical connectives

Truth tables, tautologies, logical equivalence. Predicates and quantifiers. Vacuous truths. Structural induction for lists (strings, words). Inductive and recursive definitions.

2. Induction Principle: equivalent forms and applications

Graphs. Vertex degree. Isomorphism. Bipartite graphs. Matchings. Connected components. Trees. Spanning tree. Directed graphs.

3. Sets and elements

Specifying new sets. Russell’s paradox. Algebra of sets. Ordered pair and Cartesian product. Tuples and Cartesian power.

4. Binary relations

Algebra of binary relations. Special binary relations. Functions. Set equivalence and embedding. Cantor's theorem. Cantor-Schr¨oderBernstein theorem. The equivalence N 2 ˜ N. 'Basic' equivalences like (AB) C ˜ AC×B. Cardinalities of sets N2, Z,Q, R2, NN, R N. Indicator function. The equivalences 2A ˜ P(A) and An ˜ An.

5. Pigeonhole Principle

Finite and countable sets. Rules of sum and product. Counting functions, injections, bijections, subsets. Binomial coefficients and their properties. Inclusion–exclusion principle. Various applications.

6. Special binary relations

Partial orders. Maxima and minima. Suprema and infima. Equivalence relations. Quotient set and partitions. Counting partitions. Boolean functions and circuits. Clones. Functional completeness. Counting functions of various classes.

7. Formal propositional logic

Natural deduction. Completeness and compactness. Algorithms. Computable functions, decidable and recursively enumerable sets. Turing machines. Halting problem. Untyped lambda calculus. Programming therein

5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)

Necessary equipment for lectures and practical exercises: classroom, computer, projector.

6. List of the main and additional literature, that is necessary for the course (training module) mastering

Main literature

1. Введение в математическую логику. Множества и отношения [Текст], учеб. пособие /Е. В. Дашкова; М-во науки и высш. образования РФ, Моск. физ.-техн. ин-т (нац. исслед. ун-т). М., МФТИ, 2019

2. Вводный курс математической логики [Текст] : [учеб. пособие для вузов] / В. А. Успенский, Н. К. Верещагин, Н. К. Плиско .— 2-е изд. — М. : Физматлит, 2002, 2007 .— 128 с. - На обл. авт. не указаны. - Библиогр.: с. 122. - Предм. указ.: с. 123-125. - 2000 экз. - ISBN 978-5-9221-0278-0 .— Полный текст (Доступ из сети МФТИ / Удаленный доступ).

3. Задачи по теории множеств, математической логике и теории алгоритмов [Текст] : [учеб. пособие для вузов] / И. А. Лавров, Л. Л. Максимова .— 5-е изд., испр. — М. : Физматлит, 2004, 2006 .— 256 с. - Библиогр.: с. 248-249. - Предм. указ.: с. 250-255.- ISBN 5-9221-0026-2 .— Полный текст (Доступ из сети МФТИ / Удаленный доступ).

Additional literature

1. Математическая логика [Текст] : учеб. пособие для вузов / А. Н. Колмогоров, А. Г. Драгалин ; Моск. гос. ун-т им. М. В. Ломоносова .— 3-е изд., стереотип. — М. : КомКнига, 2006 .— 240 с.

7. List of web resources that are necessary for the course (training module) mastering

http://dm.fizteh.ru

8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)

Not provided.

9. Guidelines for students to master the course

1. It is recommended to successfully pass test papers, as this simplifies the final certification in the subject.
2. To prepare for the final certification in the subject, it is best to use the lecture materials.

SUPPLEMENT

Assessment funds for course (training module)

major:	Information Science and Computer Engineering
specialization:	Computer Science/Информатика Phystech School of Applied Mathematics and Informatics Chair of Discrete Mathematics
term:	1
qualification:	Bachelor

Semester, form of interim assessment: 1 (fall) - Grading test

Author: E.V. Dashkov, candidate of physics and mathematical sciences, associate professor

1. Competencies formed during the process of studying the course

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2. Competency assessment indicators

As a result of studying the course the student should:

know:

- fundamental concepts, laws, theories of a part of discrete mathematics;
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- skills of independent work and mastering new disciplines;
- culture of formulation, analysis and solution of mathematical and applied problems that require the use of mathematical approaches and methods for their solution;
- the subject language of discrete mathematics and the skills of competently describing the solution of problems and presenting the results obtained.

3. List of typical control tasks used to evaluate knowledge and skills

Current control consists of two tests per semester, as well as oral delivery of tasks for independent solution. Evaluation criteria are attached. Also attached is an example of a test assignment and several tasks for independent solution on various topics at the end of the program.

4. Evaluation criteria

1. Elementary set theory.

2. The concepts of sets and subsets, the simplest operations on sets. Ordered pairs and tuples, Cartesian product.

3. Mappings and matching. The concepts of the image and the prototype. Injections, surjections and bijections. Composition and reverse mapping.

4. Comparison of capacities and the concept of equal power. Cantor-Bernstein theorem. Countable and uncountable sets, their properties.

5. Cantor's theorem. Relations on sets. Properties of binary relations. Equivalence relations, theorem on equivalence classes.

6. Relations of partial and linear order. Minimum / maximum and smallest / largest elements. Properties of ordered sets. Operations on ordered sets. Isomorphisms of ordered sets.

7. Logic of statements. Boolean variables and functions. Construction of propositional formulas.

8. Calculation of the formula value on a set of variable values. Truth tables.

9. Tautologies and contradictions. Reduction of formulas to CNF and DNF. Zhegalkin polynomials.

10. Complete systems of connectives, Post's theorem.

11. Propositional calculus. Axioms and rules for inference of the propositional calculus. The correctness of the calculus of statements.

12. Lemma on deduction. Completeness of the propositional calculus. Consistent and consistent families of formulas.

- the mark "excellent (10)" is given to a student who has shown comprehensive, systematized, in-depth knowledge of the curriculum of the discipline and the ability to confidently apply them in practice when solving specific problems, free and correct justification of the decisions made

- the mark "excellent (9)" is given to a student who has shown comprehensive, systematized, in-depth knowledge of the curriculum of the discipline and the ability to apply them in practice in solving specific problems, free and correct justification of the decisions

- the mark "excellent (8)" is given to a student who has shown comprehensive systematized, deep knowledge of the curriculum of the discipline and the ability to apply them in practice in solving specific problems, and the correct justification of the decisions

- the mark "good (7)" is given to a student if he firmly knows the material, expresses it competently and to the point, knows how to apply the acquired knowledge in practice, but makes some inaccuracies in the answer or in solving problems;

- the mark "good (6)" is given to the student if he knows the material, presents it competently and in essence, knows how to apply the knowledge gained in practice, but makes some inaccuracies in the answer or in solving problems;

- the mark "good (5)" is given to the student if he knows the material, and essentially expounds it, knows how to apply the knowledge gained in practice, but makes some inaccuracies in the answer or in solving problems;

- the mark "satisfactory (4)" is given to a student who has shown a fragmented, scattered nature of knowledge, insufficiently correct formulations of basic concepts, a violation of the logical sequence in the presentation of the program material, but at the same time he owns the main sections of the curriculum necessary for further education and can apply the obtained knowledge by model in a standard situation;

- the mark "satisfactory (3)" is given to a student who has shown a fragmentary, scattered nature of knowledge, insufficiently correct formulations of basic concepts, violation of the logical sequence in the presentation of program material, but at the same time he has fragmentary knowledge of the main sections of the curriculum necessary for further education and can apply the knowledge gained by the model in a standard situation;

- the mark "unsatisfactory (2)" is given to a student who does not know most of the main content of the curriculum of the discipline, makes gross mistakes in the formulation of the basic concepts of the discipline and does not know how to use the knowledge gained in solving typical practical problems

- the mark "unsatisfactory (1)" is given to a student who does not know the wording of the basic concepts of the discipline

5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience

It is allowed to use literature during the differentiated test.