Federal State Autonomous Educational Institution of Higher Education "Moscow Institute of Physics and Technology (National Research University)"

APPROVED
Head of the Phystech School of Applied Mathematics and Informatics
A.M. Raygorodskiy

Work program of the course (training module)

course:	Basics of Mathematical logic I/Основы математической логики I
major:	Information Science and Computer Engineering
specialization:	Computer Science/Информатика
	Phystech School of Applied Mathematics and Informatics
	Chair of Discrete Mathematics
term:	1
qualification:	Bachelor

Semester, form of interim assessment: 2 (spring) - Exam

Academic hours: 60 AH in total, including:
- lectures: 30 AH.
- seminars: 30 AH.
- laboratory practical: 0 AH.

Independent work: 90 AH.

Exam preparation: 30 AH.

In total: 180 AH, credits in total: 4

Number of course papers, tasks: 2

Author of the program: E.V. Dashkov, candidate of physics and mathematical sciences, associate professor

The program was discussed at the Chair of Discrete Mathematics 04.06.2020

Annotation

The course covers mainly traditional issues: the foundations of set theory, propositional logic and first-order logic, elements of model theory, proof theory and the theory of algorithms. Considerable space is devoted to the most rigorous presentation of the set-theoretic formalism as the language of subsequent mathematical courses, as well as for didactic purposes. In connection with the focus on discrete mathematical sciences, the course examines and substantiates in detail various types of recursion, inductive definitions of sets, as well as the foundations of the theory of formal languages, the system of natural inference, lambda calculus.

1. Study objective

Purpose of the course

- mastering general mathematical terminology (sets, relationships, functions).

Tasks of the course

- Develop the skill of structured logical thinking;
- learn to give formal definitions and give examples of defined objects;
- learn to build formal records of mathematical statements and their proofs and work with these records;
- learn to conduct mathematical reasoning, not based on the specific properties of the objects under consideration.

2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

Code and the name of the competence	Competency indicators
Gen.Pro.C-1 Apply fundamental knowledge acquired in the physical and mathematical fields and/or natural sciences and use it in professional settings	Gen.Pro.C-1.1 Analyze the task in hand, outline the ways to complete it
Pro.C-1 Assign, formalize, and solve tasks, develop and research mathematical models of the studied phenomena and processes, systematically analyze scientific problems, obtain new scientific outcomes	Pro.C-1.2 Make hypotheses, build mathematical models of the studied phenomena and processes, evaluate the quality of the developed model

3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

- fundamental concepts, laws, theories of a part of discrete mathematics;
- modern problems of the corresponding sections of discrete mathematics;
- concepts, axioms, methods of proofs and proofs of the main theorems in the sections included in the basic part of the cycle;
- basic properties of the corresponding mathematical objects.

be able to:

- understand the task at hand;
- use your knowledge to solve fundamental and applied problems;
- evaluate the correctness of the problem setting;
- strictly prove or disprove the statement;
- independently find algorithms for solving problems, including non-standard ones, and analyze them;
- independently see the consequences of the results obtained;
- Accurately present mathematical knowledge in the field orally and in writing.

master:

 skills of mastering a large amount of information and solving problems (including complex ones);
 skills of independent work and mastering new disciplines;
 culture of formulation, analysis and solution of mathematical and applied problems that require the use of mathematical approaches and methods for their solution;
 the subject language of discrete mathematics and the skills of competently describing the solution of problems and presenting the results obtained.

4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

4.1. The sections of the course (training module) and the complexity of the types of training sessions

№	Topic (section) of the course	Types of training sessions, including independent work			
		Lectures	Seminars	Laboratory practical	Independent work
1	Methods of forming sets	4	4		12
2	Isomorphism and arithmetic on VUM	3	3		9
3	Lemma Zorn.	2	2		6
4	Disjunctive normal forms. The logic of statements. Boolean function classes	2	2		6
5	Set operations	2	2		6
6	Predicates	2	2		6
7	Algorithms	2	2		6
8	Properties of bijections. Set embedding	2	2		6
9	Equivalence classes	3	3		9
10	Mathematical induction. Recursion. Counting	2	2		6
11	Axioms of countable and dependent choice. Formal languages	2	2		6
12	Universal computable function	2	2		6
13	Turing machines. Lambda calculus	2	2		6
AH in total		30	30		90
Exam preparation		30 AH.			
Total complexity		180 AH., credits in total 4			

4.2. Content of the course (training module), structured by topics (sections)

Semester: 2 (Spring)

1. Methods of forming sets

Intuitive concept of a set. Elements of sets. Inclusion and equality of sets. The main ways of forming new sets are: enumeration of all elements, when there are certainly many of them; allocation of a subset by a property; degree (set of subsets) of a set; union of the set. Empty set, Russell's paradox, set intersection

2. Isomorphism and arithmetic on VUM

Isomorphism of structures. Any two countable dense linear orders without the least and largest element are isomorphic

3. Lemma Zorn.

Chains in a partially ordered set. Zorn's lemma and Zermelo's theorem. Their equivalence to the axiom of choice. Examples of application of Zorn's lemma. A theorem on the comparability of sets in terms of cardinality. The cardinalities of the union and product of two infinite sets.

4. Disjunctive normal forms. The logic of statements. Boolean function classes

Propositional formulas (i.e., over the set of connectives $\wedge$, $\vee$, $\neg$, $\rightarrow$, $\leftrightarrow$,>, $\perp$). The logic of statements. Equivalence of propositional formulas. Substitutions and their preservation of equivalence. Basic equivalences. Tautologies and satisfiable formulas. Semantic (logical) consequence and its properties

5. Set operations

Operations of union, intersection and complement of sets. Basic identities of the algebra of sets. Set relations. Types of binary relations. Operations of inversion and composition of relations

6. Predicates

The theorem on the completeness of the predicate calculus (without equality) in various formulations. Extension of the theory by Genkin's axioms; Lindenbaum's lemma; model construction; the power of the signature and the power of the model. Completeness theorem for predicate calculus with equality. Compactness theorem

7. Algorithms

An intuitive concept of an algorithm. Computable functions. Decidable and enumerable sets. The connection between finiteness, decidability and enumerability. Solvable and enumerable sets under the action of the operations of the algebra of sets, Cartesian product and projection. Post's theorem. T-predicate for the algorithm and its intuitive meaning. Computable function graph theorem. Enumerability of the image and inverse image of a set under the action of a computable function. Semi-characteristic function. Equivalence of different definitions of an enumerable set

8. Properties of bijections. Set embedding

Properties of functionality, injectivity, surjectivity and totality of a relation. ... Injections, surjections and bijections. The criterion for the bijectivity of the relationship. Equal cardinality of sets. ABOUT

9. Equivalence classes

Equivalence ratio. Equivalence classes and quotient set. Partitioning a set.

10. Mathematical induction. Recursion. Counting

Power properties of finite and countable sets. Fundamental orders. Induction principle. Equivalence of Funding Conditions, Finiteness of Decreasing Chains, and the Induction Principle

11. Axioms of countable and dependent choice. Formal languages

Words and formal languages. Concatenation of words, empty word. Prefixes and Suffixes. The "prefix" relation as a partial order. Operations on languages. Examples of inductive language definitions. Prefix-free languages. P

12. Universal computable function

Universal computable function (u.v.f .; in the class of computable functions N p → N). Unsolvable problems of self-applicability and stopping. Examples of enumerable undecidable and non-enumerable sets. An example of a computable function that has no computable totally extension. The domain of any such function is enumerable but undecidable. An example of disjoint enumerable sets that are not separated by any decidable set. Main universal computable function. Computable bijective coding of pairs of natural numbers. Building the main y. at. f. using arbitrary y. at. f. Kleene fixed point theorem. The infinity of the set of fixed points. Recursion theorem. Computability of the composition index of computable functions. Joint recursion; solution of "systems of equations"

13. Turing machines. Lambda calculus

Equivalence of model of partially recursive functions and lambda calculus. Specific computation models. Turing machines; examples of Turing computable functions. Primitive recursive and partially recursive functions; examples of such

5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)

Necessary equipment for lectures and practical exercises: classroom, computer, projector.

6. List of the main and additional literature, that is necessary for the course (training module) mastering

Main literature

1. Введение в математическую логику. Множества и отношения [Текст], учеб. пособие /Е. В. Дашкова; М-во науки и высш. образования РФ, Моск. физ.-техн. ин-т (нац. исслед. ун-т). М., МФТИ, 2019
2. Вводный курс математической логики [Текст] : [учеб. пособие для вузов] / В. А. Успенский, Н. К. Верещагин, Н. К. Плиско .— 2-е изд. — М. : Физматлит, 2002, 2007 .— 128 с. - На обл. авт. не указаны. - Библиогр.: с. 122. - Предм. указ.: с. 123-125. - 2000 экз. - ISBN 978-5-9221-0278-0 .— Полный текст (Доступ из сети МФТИ / Удаленный доступ).
3. Задачи по теории множеств, математической логике и теории алгоритмов [Текст] : [учеб. пособие для вузов] / И. А. Лавров, Л. Л. Максимова .— 5-е изд., испр. — М. : Физматлит, 2004, 2006 .— 256 с. - Библиогр.: с. 248-249. - Предм. указ.: с. 250-255.- ISBN 5-9221-0026-2 .— Полный текст (Доступ из сети МФТИ / Удаленный доступ).

Additional literature

1. Математическая логика [Текст] : учеб. пособие для вузов / А. Н. Колмогоров, А. Г. Драгалин ; Моск. гос. ун-т им. М. В. Ломоносова .— 3-е изд., стереотип. — М. : КомКнига, 2006 .— 240 с.

7. List of web resources that are necessary for the course (training module) mastering

http://dm.fizteh.ru

8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)

Not provided.

9. Guidelines for students to master the course

1. It is recommended to successfully pass test papers, as this simplifies the final certification in the subject.
2. To prepare for the final certification in the subject, it is best to use the lecture materials.

SUPPLEMENT

Assessment funds for course (training module)

major: Information Science and Computer Engineering
specialization: Computer Science/Информатика
Phystech School of Applied Mathematics and Informatics
Chair of Discrete Mathematics
term: 1
qualification: Bachelor

Semester, form of interim assessment: 2 (spring) - Exam

Author: E.V. Dashkov, candidate of physics and mathematical sciences, associate professor

1. Competencies formed during the process of studying the course

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2. Competency assessment indicators

As a result of studying the course the student should:

know:

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3. List of typical control tasks used to evaluate knowledge and skills

Current control consists of two tests per semester, as well as oral delivery of tasks for independent solution. Evaluation criteria are attached. Also attached is an example of a test assignment and several tasks for independent solution on various topics at the end of the program.

4. Evaluation criteria

Checklist for passing the exam:

1. Compactness theorem for propositional formulas.
2. Languages of the first order. Signature concept. Construction of first-order formulas: theorems, atomic formulas, logical connectives and quantifiers.
3. Formula parameters. The concept of a closed formula. Interpretation of the signature. The validity of the formula in the given interpretation on the given assessment.
4. Satisfaction and general validity of first-order formulas. Replacing a bound variable.
5. Preceded normal form. Expression of predicates in this interpretation by first-order formulas.
6. Isomorphisms and automorphisms of interpretations. Examples of ineffable predicates.

7. Method of elimination of quantifiers. Elementary equivalence of interpretations. Ehrenfeucht Games.
8. Predicate calculus and model theory. Axioms and rules for the derivation of the predicate calculus.
9. Generalization rule. A deduction lemma for predicate calculus. Correctness of the predicate calculus.
Tasks:
1. Consistent and consistent theories.
2. Theories and models. Complete and existentially complete theories. Gödel's theorem on the completeness of the predicate calculus. Semantic following.
3. Maltsev's compactness theorem.

Ticket 1:
1. Comparison of capacities and the concept of equal power. Cantor-Bernstein theorem. Countable and uncountable sets, their properties .;
2. The logic of statements. Boolean variables and functions. Construction of propositional formulas.

Ticket 2:
1. Elementary set theory;
2. Propositional calculus. Axioms and rules for inference of the propositional calculus. The correctness of the calculus of statements.

- the mark "excellent (10)" is given to a student who has shown comprehensive, systematized, in-depth knowledge of the curriculum of the discipline and the ability to confidently apply them in practice when solving specific problems, free and correct justification of the decisions made

- the mark "excellent (9)" is given to a student who has shown comprehensive, systematized, in-depth knowledge of the curriculum of the discipline and the ability to apply them in practice in solving specific problems, free and correct justification of the decisions

- the mark "excellent (8)" is given to a student who has shown comprehensive systematized, deep knowledge of the curriculum of the discipline and the ability to apply them in practice in solving specific problems, and the correct justification of the decisions

- the mark "good (7)" is given to a student if he firmly knows the material, expresses it competently and to the point, knows how to apply the acquired knowledge in practice, but makes some inaccuracies in the answer or in solving problems;

- the mark "good (6)" is given to the student if he knows the material, presents it competently and in essence, knows how to apply the knowledge gained in practice, but makes some inaccuracies in the answer or in solving problems;

- the mark "good (5)" is given to the student if he knows the material, and essentially expounds it, knows how to apply the knowledge gained in practice, but makes some inaccuracies in the answer or in solving problems;

- the mark "satisfactory (4)" is given to a student who has shown a fragmented, scattered nature of knowledge, insufficiently correct formulations of basic concepts, a violation of the logical sequence in the presentation of the program material, but at the same time he owns the main sections of the curriculum necessary for further education and can apply the obtained knowledge by model in a standard situation;

- the mark "satisfactory (3)" is given to a student who has shown a fragmentary, scattered nature of knowledge, insufficiently correct formulations of basic concepts, violation of the logical sequence in the presentation of program material, but at the same time he has fragmentary knowledge of the main sections of the curriculum necessary for further education and can apply the knowledge gained by the model in a standard situation;

- the mark "unsatisfactory (2)" is given to a student who does not know most of the main content of the curriculum of the discipline, makes gross mistakes in the formulation of the basic concepts of the discipline and does not know how to use the knowledge gained in solving typical practical problems

- the mark "unsatisfactory (1)" is given to a student who does not know the wording of the basic concepts of the discipline

5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience

It is allowed to use literature during the differentiated test.