Federal State Autonomous Educational Institution of Higher Education "Moscow Institute of Physics and Technology (National Research University)"

APPROVED
Vice Rector for Academic Affairs

A.A. Voronov

Work program of the course (training module)

course:	Mathematical Analysis – Sequences and Series of Functions, Functions of Several Variables/Математический анализ – функциональные последовательности и ряды, функции нескольких переменных
major:	Biotechnology
specialization:	Biomedical Engineering/Биомедицинская инженерия
	Phystech School of Biological and Medical Physics
	Chair of Higher Mathematics
term:	2
qualification:	Bachelor

Semester, form of interim assessment: 3 (fall) - Exam

Academic hours: 120 AH in total, including:
lectures: 60 AH.
seminars: 60 AH.
laboratory practical: 0 AH.

Independent work: 120 AH.

Exam preparation: 30 AH.

In total: 270 AH, credits in total: 6

Authors of the program:

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O.V. Besov, doctor of physics and mathematical sciences, full professor, professor
B.I. Golubov, doctor of physics and mathematical sciences, full professor, professor
V.Z. Sakbaev, doctor of physics and mathematical sciences, associate professor, associate professor
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O.E. Orel, candidate of physics and mathematical sciences, associate professor, associate professor

The program was discussed at the Chair of Higher Mathematics 20.05.2020

Annotation

Discipline belongs to the basic part of the educational program. Mastering the discipline is aimed at developing the ability to acquire new scientific and professional knowledge using modern educational and information technologies. Topics covered include Differential calculus of functions of many variables, Numerical series, Functional sequences and series, Power series.

1. Study objective

Purpose of the course

further familiarization of students with the methods of mathematical analysis, the formation of their evidence-based and logical thinking.

Tasks of the course

- formation of students' theoretical knowledge and practical skills in the problems of searching for unconditional and conditional extrema of a function of many variables, measure and integral theory, field theory;
- preparing students for the study of related mathematical disciplines;
- acquisition of skills in the application of methods of mathematical analysis in physics and other natural science disciplines.

2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

Code and the name of the competence	Competency indicators
UC-1 Search and identify, critically assess, and synthesize information, apply a systematic approach to problem-solving	UC-1.1 Analyze problems, highlight the stages of their solution, plan the actions required to solve them
	UC-1.2 Find, critically assess, and select information required for the task in hand
	UC-1.3 Consider various options for solving a problem, assess the advantages and disadvantages of each option
	UC-1.4 Make competent judgments and estimates supported by logic and reasoning
UC-6 Use time-management skills, apply principles of self-development and lifelong learning	UC-6.2 Plan independent activities in professional problem-solving; critically analyze the work performed; find creative ways to use relevant experience for self-development

3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

- The implicit function theorem;
- determination of the extremum of a function of many variables and the conditional extremum of a function of many variables in the presence of connections, necessary and sufficient conditions in the problems of finding an unconditional, as well as a conditional extremum in the presence of connections;
- signs of convergence of functional sequences and series.

be able to:

- To investigate functions of many variables for extremum;
- to solve problems on conditional extremum by the method of Lagrange multipliers;
- to investigate functional sequences and series for convergence;
- find the region of convergence of the power series;
- expand regular functions into power series.

master:

- Logical thinking, methods of proving mathematical statements;
- the skills of calculating integrals and the skills of applying field theory theorems in mathematical and physical applications;
- ability to use the necessary literature to solve problems.

4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

4.1. The sections of the course (training module) and the complexity of the types of training sessions

№	Topic (section) of the course	Types of training sessions, including independent work			
		Lectures	Seminars	Laboratory practical	Independent work
1	Metric space and topology	6	6		12
2	Limit and continuity of a function of several variables	4	4		8
3	Differential calculus of functions of several variables	12	12		24
4	Implicit functions	10	10		20
5	Extrema of functions of several variables	6	6		12
6	Number series	4	4		8
7	Functional sequences and ranks	10	10		20
8	Power series	8	8		16
AH in total		60	60		120
Exam preparation		30 AH.			
Total complexity		270 AH., credits in total 6			

4.2. Content of the course (training module), structured by topics (sections)

Semester: 3 (Fall)

1. Metric space and topology

1.1. Point n-dimensional Euclidean space. Distance between points, its properties. Limit of a sequence of points in n-dimensional Euclidean space. Bolzano-Weierstrass theorem and Cauchy's criterion for convergence of a sequence. Internal, limit, isolated points of the set; points of contact. Open and closed sets, their properties. Interior, closure and boundary of a set.

2. Limit and continuity of a function of several variables

2.1. Limit of a numeric function of several variables. Definitions according to Heine and Cauchy, their equivalence. Repeated limits and directional limits. Investigation of the limit of a function of two variables using the transition to polar coordinates. Limit of a function over a set.

2.2. Continuity of a function of several variables at a point and over a set. Continuity of a complex function. The properties of functions that are continuous on a compactum are boundedness, achieving exact upper and lower bounds, uniform continuity. A theorem on intermediate values of a function that is continuous in a domain.

3. Differential calculus of functions of several variables

3.1. Partial derivatives of functions of several variables. Differentiability of a function of several variables at a point, differential. Necessary conditions for differentiability, sufficient conditions for differentiability. Differentiability of a complex function. Invariance of the form of the differential under a change of variables. Gradient, its independence from the choice of a rectangular coordinate system. Directional derivative.

3.2. Partial derivatives of higher orders. Independence of the mixed partial derivative from the order of differentiation. Differentials of higher orders, lack of invariance of their form under a change of variables. Taylor's formula for functions of several variables with a remainder in Lagrange and Peano forms.

4. Implicit functions

4.1. System of implicit functions.
4.2. Differentiable mappings.

5. Extrema of functions of several variables

5.1. Local extremum.
5.2. Conditional local extremum.

6. Number series

6.1. Number series. Cauchy's criterion for convergence of a series. Sign-constant series: criteria for comparing convergence, d'Alembert and Cauchy criteria, integral criterion. Alternating series: absolute and conditional convergence. Dirichlet and Abel signs. The independence of the sum of an absolutely convergent series on the order of the terms. Riemann's theorem on the permutation of the terms of a conditionally convergent series. The product of absolutely converging series.

7. Functional sequences and ranks

7.1. Uniform convergence of functional sequences and series. Cauchy's criterion for uniform convergence. Continuity of the sum of a uniformly converging series of their continuous functions. Term-by-term integration and differentiation of functional series. Weierstrass test for uniform convergence of functional series. Dirichlet and Abel signs.

8. Power series

8.1. Power series with complex members. Abel's first theorem. Circle and radius of convergence. The nature of the convergence of a power series in the circle of convergence. Cauchy-Hadamard formula for the radius of convergence. Abel's second theorem. Continuity of the sum of a complex power series.
8.2. Power series with full members. Preservation of the radius of convergence in term-by-term integration and differentiation of a power series. Infinite differentiability of the sum of a power series in the circle of convergence. Uniqueness of the expansion of a function in a power series; Taylor series. Taylor's formula with remainder in integral form. An example of an infinitely differentiable function that does not expand in a power series. Taylor series expansion of basic elementary functions. Power series expansion of a complex function.

5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)

Classroom equipped with a multimedia projector, screen and microphone.

6. List of the main and additional literature, that is necessary for the course (training module) mastering

Main literature

1. Mathematical analysis II /V. A. Zorich. Berlin, Springer, 2016

Additional literature

1. Advanced calculus, A. Friedman ; The Ohio State University. Mineola ; New York, Dover publications, inc., 2016

7. List of web resources that are necessary for the course (training module) mastering

1. http://lib.mipt.ru/catalogue/1195/?page=0
2. http://www.exponenta.ru
3. http://mathnet.ru
4. http://www.edu.ru
5. http://benran.ru
6. http://www.i-exam.ru

8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)

The lectures use multimedia technologies, including the demonstration of presentations.
In the process of independent work of students, it is possible to use software such as Mathcad, Scilab, etc.

9. Guidelines for students to master the course

Provided in the annually developed homework assignments.

SUPPLEMENT

Assessment funds for course (training module)

major:	Biotechnology
specialization:	Biomedical Engineering/Биомедицинская инженерия
	Phystech School of Biological and Medical Physics
	Chair of Higher Mathematics
term:	2
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O.E. Orel, candidate of physics and mathematical sciences, associate professor, associate professor

1. Competencies formed during the process of studying the course

Code and the name of the competence	Competency indicators
UC-1 Search and identify, critically assess, and synthesize information, apply a systematic approach to problem-solving	UC-1.1 Analyze problems, highlight the stages of their solution, plan the actions required to solve them
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UC-6 Use time-management skills, apply principles of self-development and lifelong learning	UC-6.2 Plan independent activities in professional problem-solving; critically analyze the work performed; find creative ways to use relevant experience for self-development

2. Competency assessment indicators

As a result of studying the course the student should:

know:

- The implicit function theorem;
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be able to:

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- to investigate functional sequences and series for convergence;
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- expand regular functions into power series.

master:

- Logical thinking, methods of proving mathematical statements;
- the skills of calculating integrals and the skills of applying field theory theorems in mathematical and physical applications;
- ability to use the necessary literature to solve problems.

3. List of typical control tasks used to evaluate knowledge and skills

Current control is carried out on the basis of a point-rating system (BRS) for assessing knowledge in the discipline being studied. BRS takes into account the implementation by students of a set of homework and tests in accordance with the curriculum. Data on attendance and current performance are entered by teachers in special journals and recorded in the BRS.

Current control based on homework is carried out during the academic semester within the timeframe established by the Educational Department, in accordance with the curriculum.

To pass the assignment, the student must provide a solution to the homework problem in writing, answer the teacher's questions and write a test on the assignment, which tests the knowledge of concepts and statements on the topics of the assignment being handed over and the ability to solve problems.

During the execution of the test, you cannot use the help of other persons, computers and mobile phones.

* Attached by BRS on the subject under study.

4. Evaluation criteria

Certification in the discipline "Mathematical Analysis – Sequences and Series of Functions, Functions of Several Variables" is carried out in the form of an exam.

The exam is conducted taking into account the control tasks previously completed by the students.

Control tasks:

1. Theorem about an implicit function given by one equation. A theorem on implicit functions given by a system of equations.
2. Necessary conditions for a local extremum, sufficient conditions for a local extremum.
3. Conditional extremum. Lagrange's method for finding the points of a conditional extremum: necessary conditions, sufficient conditions.
4. The independence of the sum of an absolutely convergent series on the order of the terms.
5. Riemann's theorem on the permutation of the terms of a conditionally convergent series.
6. Product of absolutely converging series.
4. Uniform convergence of functional sequences and series. Cauchy criterion.
5. Dirichlet and Abel tests for uniform convergence of series.
6. Properties of uniformly converging series.
7. Properties of power series.
8. Analytical functions.
9. Decomposition of functions in a Taylor series.

Examples of exam tickets:

Ticket number 1
1. Theorem about an implicit function given by one equation. A theorem on implicit functions given by a system of equations.
2. Dirichlet and Abel tests for uniform convergence of series.

Ticket number 2
1. The measure of the graph of a function of several variables, the measure of the subgraph of a non-negative function.
2. Power series and their properties.

Grade "excellent (10)" is given to a student who has exhibited extensive and deep knowledge of the course and ability to apply skills when solving specific tasks;

Grade "excellent (9)" is given to a student who has exhibited extensive and deep knowledge of the course and ability to apply skills when solving specific tasks, but he has made minor errors that were independently found and corrected;

Grade "excellent (8)" is given to a student who has exhibited extensive and deep knowledge of the course and ability to apply skills when solving specific tasks, but he has made minor errors that were independently corrected after the instructions of an examiner;

Grade "good (7)" is given to a student who has a good command of the course and is able to apply skills when solving specific tasks, but has made minor mistakes when answering questions or solving problems;

Grade "good (6)" is given to a student who has a good command of the course and is able to apply skills when solving specific tasks, but has made rare mistakes when answering questions or solving problems;

Grade "good (5)" is given to a student who has a good command of the course and is able to apply skills when solving specific tasks, but has made mistakes when answering questions or solving problems;

Grade "satisfactory (4)" is given to a student who has exhibited fragmented knowledge, has made inaccurate formulation of the basic concepts, but understands the subject well, is able to apply the knowledge in standard situations and possesses skills necessary for the future study;

Grade "satisfactory (3)" is given to a student who has exhibited fragmented knowledge, has made inaccurate formulation of the basic concepts, has inconsistencies in understanding the course, but is able to apply the knowledge in standard situations and possesses skills necessary for the future study;

Grade "unsatisfactory (2)" is given to a student who does not possess knowledge of the essential concept of the course, has made gross mistakes in formulations of basic concepts and cannot use the knowledge in solving typical tasks;

Grade "unsatisfactory (1)" is given to a student who has exhibited total lack of knowledge of the course.

5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience

The time of the written exam is 4 astronomical hours.

When conducting an oral exam, the student is given 1 astronomical hour to prepare. The student's survey on the ticket for the oral exam should be 2 astronomical hours.

During the exam, students can use only the program of disciplines.

Scoring for Exams with Written Part
Department of Higher Mathematics

NN	Types of Control	Points
1	Control test 1	0—6
2	Control test 2	0—6
3	Control test 3	0—6
4	Homework 1	0—2
5	Homework 2	0—2
6	Homework 3	0—2
7	Theory checking	0—3
8	Class attendance and activity during seminars	0—3
9	Written exam	0—30
10	Oral exam	0—60
	Total Score	0—120

The score for the oral exam is computed by the formula $6 \cdot N$, where $N \geq 3$ is a grade gained by a student during the exam. If $N = 1$ or 2, then the student grade is equal to N.

Conversion Scale between the total score and the student grade

Total Score	Student Grade	
112—120	10	Excellent
103—111	9	
94—102	8	
85—93	7	Good
76—84	6	
67—75	5	
54—66	4	Satisfactory
41—53	3	
27—40	2	Unsatisfactory
0—26	1	