

**Federal State Autonomous Educational Institution of Higher Education "Moscow  
Institute of Physics and Technology  
(National Research University)"**

**APPROVED**  
**Vice Rector for Academic Affairs**

**A.A. Voronov**

**Work program of the course (training module)**

**course:** Geometry/Геометрия  
**major:** Biotechnology  
**specialization:** Biomedical Engineering/Биомедицинская инженерия  
Phystech School of Biological and Medical Physics  
Chair of Higher Mathematics  
**term:** 1  
**qualification:** Bachelor

Semester, form of interim assessment: 1 (fall) - Exam

Academic hours: 60 AH in total, including:

lectures: 30 AH.

seminars: 30 AH.

laboratory practical: 0 AH.

Independent work: 90 AH.

Exam preparation: 30 AH.

In total: 180 AH, credits in total: 4

Authors of the program:

O.G. Podlipskaya, candidate of physics and mathematical sciences, assistant

A.V. Zukhba, candidate of physics and mathematical sciences, assistant

The program was discussed at the Chair of Higher Mathematics 21.05.2020

## Annotation

The discipline belongs to the basic part of the educational program. The development of the discipline is aimed at developing the ability to acquire new scientific and professional knowledge using modern educational and information technologies. Topics such as Initial geometric data, Triangles, Polygons, Circles, stereometry Elements, Matrices, Vector space, Basis, Cartesian coordinate systems, Scalar product, Vector product, Mixed product, Algebraic lines and surfaces, Straight lines and planes are considered.

### 1. Study objective

#### Purpose of the course

to provide students with foundations of analytic geometry that will help them to study advanced mathematical disciplines – differential equations, complex analysis, mathematical physics, functional analysis, analytical mechanics, theoretical physics, methods of optimal control, etc.

#### Tasks of the course

- to provide students with theoretical knowledge and practical skills in geometry;
- to motivate students towards treatment of related mathematical disciplines;
- to equip students with skills to apply techniques of analytic geometry in physics and other natural sciences.

### 2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

Code and the name of the competence	Competency indicators
UC-1 Search and identify, critically assess, and synthesize information, apply a systematic approach to problem-solving	UC-1.1 Analyze problems, highlight the stages of their solution, plan the actions required to solve them
	UC-1.2 Find, critically assess, and select information required for the task in hand
	UC-1.3 Consider various options for solving a problem, assess the advantages and disadvantages of each option
	UC-1.4 Make competent judgments and estimates supported by logic and reasoning
UC-6 Use time-management skills, apply principles of self-development and lifelong learning	UC-6.2 Plan independent activities in professional problem-solving; critically analyze the work performed; find creative ways to use relevant experience for self-development

### 3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

- methodical foundations of study and use of mathematical statements;
- foundations of the course.

be able to:

- develop, use and apply definitions and theorems;
- study and form systems of mathematical knowledge;
- prove main theorems of the course;
- solve standard problems on topics covered in the course.

master:

- the essential concept of the course.

### 4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

4.1. The sections of the course (training module) and the complexity of the types of training sessions

№	Topic (section) of the course	Types of training sessions, including independent work			
		Lectures	Seminars	Laboratory practical	Independent work
1	The initial geometric information	2	2		6
2	Triangle	4	4		8
3	Polygons	2	2		6
4	Circle	2	2		6
5	Elements of stereometry	2	2		6
6	Matrixes	2	2		6
7	Vector space	2	2		6
8	Basis	2	2		6
9	Cartesian coordinate system	2	2		6
10	Scalar product	2	2		6
11	Vector product	2	2		6
12	Mixed product	2	2		6
13	Algebraic lines and surfaces	2	2		8
14	Straight and planes	2	2		8
AH in total		30	30		90
Exam preparation		30 AH.			
Total complexity		180 AH., credits in total 4			

#### 4.2. Content of the course (training module), structured by topics (sections)

Semester: 1 (Fall)

##### 1. The initial geometric information

Points, lines, line segments. Ray. Angle. Comparison of segments and angles. Measurement of segments. Measurement of angles. Adjacent and vertical angles. Perpendicular lines. Parallel lines.

##### 2. Triangle

Signs of equality of triangles. Medians, bisectors and heights of a triangle. Properties of an isosceles triangle. Sum of the angles of a triangle. Equilateral triangles. Area of a triangle. Pythagorean theorem. Similar triangles. Law of sines. Law of cosines.

##### 3. Polygons

Convexpolygon. Quadrilateral. Parallelogram. Characteristics of a parallelogram. Trapezoid. Rectangle. Rhombus. Square.

##### 4. Circle

Tangent to a circle. Degree measure of an arc of a circle. Inscribed angle theorem. Inscribed circle. Circumscribed circle. Length of a circle and area of a circle.

##### 5. Elements of stereometry

Polyhedron. Parallelepiped. Prism. Cylinder. Cone. Sphere and ball. Body volume.

## 6. Matrixes

Operations of addition of matrices and multiplication of matrices by numbers. Matrix multiplication and inversion. Determinants of square matrices of 2-nd and 3-rd orders. Solving systems of linear equations by the Cramer method.

## 7. Vector space

Linear spaces and their basic properties. Directed segments and actions on them. Operations of adding directed segments and multiplying them by numbers. Their property. Commutativity, associativity and distributivity of vector operations.

## 8. Basis

Linearly dependent and linearly independent systems of vectors. Basis, coordinates of vectors in the basis. Coordinate representation of vectors. Operations with vectors in coordinate representation. Changing the coordinates of the vector when replacing the basis. Necessary and sufficient condition for linear dependence of vectors in coordinate form.

## 9. Cartesian coordinate system

Linearly dependent and linearly independent systems of vectors. Basis, coordinates of vectors in the basis. Coordinate representation of vectors. Operations with vectors in coordinate representation. Changing the coordinates of the vector when replacing the basis. Necessary and sufficient condition for linear dependence of vectors in coordinate form.

## 10. Scalar product

Orthogonal projections of vectors and their properties. Scalar product: properties, coordinate expression. Formulas for the distance between two points and the angle between two directions.

## 11. Vector product

Oriented set of vectors. Vector product, its properties, expression in orthonormal basis. Geometric meaning of the vector product. Expression of a vector product in an arbitrary basis.

## 12. Mixed product

Triple product of vectors, its properties, expression in arbitrary and orthonormal bases. The geometric meaning of the triple product. Conditions of collinearity and coplanarity of vectors. The formula of a double vector product. Derivation of the double vector product formula.

## 13. Algebraic lines and surfaces

Coordinate equation of lines on the plane, surfaces in space. Invariance of the order of algebraic lines on the plane when replacing the Cartesian coordinate system. Coordinate equation of a lines in space. Invariance of the order of algebraic lines and surfaces in space when replacing the Cartesian coordinate system. Coordinate equations of figures on the plane and bodies in space.

## 14. Straight and planes

A line on a plane. Vector and coordinate equations of a line. Positional and metric problems on lines on a plane. Translation of one form of description of lines on a plane into other form.

## **5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)**

A classroom equipped with multimedia projector, screen, and microphone.

## **6. List of the main and additional literature, that is necessary for the course (training module) mastering**

### Main literature

Strang, G. Linear algebra and its applications / G. Strange ; Massachusetts Institute of Technology. - 4th edition. - USA : Brooks/Cole : Cengage Learning, 2006. - 488 p. - Index: p. 482-487. - ISBN 978-0-03-010567-8

### Additional literature

Ефимов, Н. В.

Краткий курс аналитической геометрии [Текст] : учебник для вузов / Н. В. Ефимов .— 10-е изд., стереотип. — М. : Наука, 1969 .— 272 с. - 77 500 экз.

## **7. List of web resources that are necessary for the course (training module) mastering**

<http://www.math.mipt.ru>

<http://lib.mipt.ru>

<https://arxiv.org/pdf/1111.6521.pdf>

## **8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)**

The lectures use multimedia technologies, including presentations.

## **9. Guidelines for students to master the course**

Given in the annually developed homework.

**Assessment funds for course (training module)**

**major:** Biotechnology  
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Chair of Higher Mathematics  
**term:** 1  
**qualification:** Bachelor

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**Authors:**

O.G. Podlipskaya, candidate of physics and mathematical sciences, assistant  
A.V. Zukhba, candidate of physics and mathematical sciences, assistant

## 1. Competencies formed during the process of studying the course

Code and the name of the competence	Competency indicators
UC-1 Search and identify, critically assess, and synthesize information, apply a systematic approach to problem-solving	UC-1.1 Analyze problems, highlight the stages of their solution, plan the actions required to solve them
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UC-6 Use time-management skills, apply principles of self-development and lifelong learning	UC-6.2 Plan independent activities in professional problem-solving; critically analyze the work performed; find creative ways to use relevant experience for self-development

## 2. Competency assessment indicators

As a result of studying the course the student should:

### know:

- methodical foundations of study and use of mathematical statements;
- foundations of the course.

### be able to:

- develop, use and apply definitions and theorems;
- study and form systems of mathematical knowledge;
- prove main theorems of the course;
- solve standard problems on topics covered in the course.

### master:

- the essential concept of the course.

## 3. List of typical control tasks used to evaluate knowledge and skills

Current control is carried out on the basis of a point-rating system (BRS) for evaluating knowledge in the discipline being studied. The BRS takes into account the students' performance of a set of homework assignments and tests in accordance with the curriculum. Data on attendance and current academic performance are entered by teachers in special journals and recorded in the BRS.

Current control on the basis of homework is carried out during the academic semester in the terms set by the Educational Department, in accordance with the curriculum.

To pass the task, the student must provide a solution to the homework problem in writing, answer the questions of the teacher and write a test paper on the task, which checks the knowledge of concepts and statements on the topics of the task and the ability to solve problems.

You can't use other people's help, computers, or mobile phones during the test.

\* A BRS is attached to the subject being studied.

## 4. Evaluation criteria

Certification in the discipline "Geometry/Геометрия" is carried out in the form of an exam.

The examination is conducted in accordance with the previously performed by the students in the control tasks.

Control tasks:

1. Find adjacent angles if the difference is known. Find the vertical angles, if given their sum.

2. Prove that the bisectors of vertical angles lie on the same line.
3. Formulate and prove a theorem expressing the first (second, third) sign of equality of triangles.
4. Prove that the midpoints of the sides of an isosceles triangle are the vertices of another isosceles triangle.
5. Prove that at the intersection of two parallel lines of the cross-cutting lying angles are equal.
6. Prove that the doubled sum of the distances from any point inside the triangle to its vertices is greater than the perimeter of the triangle.
7. In the parallelogram whose adjacent angles are not equal, bisectors of angles are carried out. Prove that at their intersection a rectangle is formed.
8. The diagonals of the isosceles trapezoid are mutually perpendicular, and the sum of its bases is  $2a$ . Find the area of the trapezoid.
9. Prove that the vertices of the triangle are equidistant from the line containing its middle line.
10. Prove that if the lines containing the bases of the trapezoid touch the circle, the line passing through the middle of the sides of the trapezoid passes through the center of the circle.
11. Prove that the diagonal of the parallelepiped is less than the sum of three edges having a common vertex.
12. Find the product of matrices.
13. Find the inverse matrix.
14. Solve a system of linear equations with 3 unknowns.
15. Two different coordinate systems and the coordinates of some point in the first system are given. Find its coordinates in the second system.
16. Find the distance from the point to the plane.
17. Explicitly Express the relationship between the parameters of two different forms of the equation of the line.

Examples examination tickets:

Ticket 1

1. Find adjacent angles if the difference is known. Find the vertical angles, if given their sum.
2. Prove that the bisectors of vertical angles lie on the same line.

Ticket 2

1. The diagonals of the isosceles trapezoid are mutually perpendicular, and the sum of its bases is  $2a$ . Find the area of the trapezoid.
2. Prove that the vertices of the triangle are equidistant from the line containing its middle line.

Grade "excellent (10)" is given to a student who has exhibited extensive and deep knowledge of the course and ability to apply skills when solving specific tasks;

Grade "excellent (9)" is given to a student who has exhibited extensive and deep knowledge of the course and ability to apply skills when solving specific tasks, but he has made minor errors that were independently found and corrected;

Grade "excellent (8)" is given to a student who has exhibited extensive and deep knowledge of the course and ability to apply skills when solving specific tasks, but he has made minor errors that were independently corrected after the instructions of an examiner;

Grade "good (7)" is given to a student who has a good command of the course and is able to apply skills when solving specific tasks, but has made minor mistakes when answering questions or solving problems;

Grade "good (6)" is given to a student who has a good command of the course and is able to apply skills when solving specific tasks, but has made rare mistakes when answering questions or solving problems;

Grade "good (5)" is given to a student who has a good command of the course and is able to apply skills when solving specific tasks, but has made mistakes when answering questions or solving problems;

Grade "satisfactory (4)" is given to a student who has exhibited fragmented knowledge, has made inaccurate formulation of the basic concepts, but understands the subject well, is able to apply the knowledge in standard situations and possesses skills necessary for the future study;

Grade "satisfactory (3)" is given to a student who has exhibited fragmented knowledge, has made inaccurate formulation of the basic concepts, has inconsistencies in understanding the course, but is able to apply the knowledge in standard situations and possesses skills necessary for the future study;

Grade "unsatisfactory (2)" is given to a student who does not possess knowledge of the essential concept of the course, has made gross mistakes in formulations of basic concepts and cannot use the knowledge in solving typical tasks;

Grade "unsatisfactory (1)" is given to a student who has exhibited total lack of knowledge of the course.

## **5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience**

During the oral exam, a student is given one (astronomical) hour to prepare the answer to the question. The schedule of discussion between an examiner and a student is limited by two hours.

During the exam, students can use the discipline program.

**Балльно-рейтинговая система оценки знаний студентов**

Дисциплина: **Geometry/Геометрия**  
**1 курс, 1 семестр, экзамен**

Кафедра: **высшей математики**

№	Вид занятий	Сумма баллов
1.	Посещение лекций	0–3
2.	Проверка теоретических знаний	0–3
3.	Контрольная работы, проводимые в классе	0–18
4.	Домашняя работа	0–6
5.	Итоговый контроль Экзамен (устный ответ)	0–70
	<b>ИТОГО</b>	<b>0–100</b>

\*Если при учете этого вида работы итоговая сумма за работу в семестре превосходит 30 баллов, то считать ее равной 30 баллам.

Сумма баллов за устный ответ начисляется по формуле  $N * 7$ , где  $N \geq 3$  — предварительная оценка за устный ответ по десятибалльной шкале. Если  $N = 1, 2$ , то итоговая оценка совпадает с  $N$ .

Соответствие оценок итоговой академической успеваемости балльно-рейтинговой системы.

Баллы БРС	Оценки	
93–100	10	отлично
86–92	9	
79–85	8	
72–78	7	хорошо
65–71	6	
58–64	5	
51–57	4	удовлетворительно
44–50	3	
30–43	2	неудовлетворительно
0–29	1	

Регламент принятия домашних заданий и проведения экзамена определяется «Положением о текущем контроле успеваемости и промежуточной аттестации студентов на кафедре высшей математики».

Зав.кафедрой

\_\_\_\_\_ Г. Е. Иванов