Federal State Autonomous Educational Institution of Higher Education "Moscow Institute of Physics and Technology (National Research University)"

APPROVED
Head of the Phystech School of Applied Mathematics and Informatics
A.M. Raygorodskiy

Work program of the course (training module)

course:	Additive Combinatorics/Аддитивная комбинаторика
major:	Applied Mathematics and Informatics
specialization:	Advanced Methods of Modern Combinatorics/Продвинутые методы современной комбинаторики
	Phystech School of Applied Mathematics and Informatics
	Chair of Discrete Mathematics
term:	2
qualification:	Master

Semester, form of interim assessment: 3 (fall) - Exam

Academic hours: 60 AH in total, including:
- lectures: 15 AH.
- seminars: 45 AH.
- laboratory practical: 0 AH.

Independent work: 45 AH.

Exam preparation: 30 AH.

In total: 135 AH, credits in total: 3

Author of the program: A.A. Glibichuk, candidate of physics and mathematical sciences, associate professor

The program was discussed at the Chair of Discrete Mathematics 05.03.2020

Annotation

The purpose of the course is to provide the foundations of an actively developing science, which is additive combinatorics. The main object of studying additive combinatorics is the sum of two sets. The course will cover the issues of extremal additive theory, which studies the structure of sets whose sum is small. The course covers the classical Freiman theorem, the Plünnecke-Ruzi inequalities, the Vosper theorem, the Kneser theorem, and other things mainly related to the extremal additive theory.

1. Study objective

Purpose of the course

mastering additive combinatorics.

Tasks of the course

- students mastering basic knowledge (concepts, concepts, methods and models) in the field of additive combinatorics;
- acquisition of theoretical knowledge and practical skills in the field of additive combinatorics;
- providing advice and assistance to students in conducting their own theoretical research in the field of additive combinatorics.

2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

Code and the name of the competence	Competency indicators
UC-1 Use a systematic approach to critically analyze a problem, and develop an action plan	UC-1.1 Systematically analyze the problem situation, identify its components and the relations between them
	UC-1.3 Develop a step-by-step strategy for achieving a goal, foresee the result of each step, evaluate the overall impact on the planned activity and its participants
UC-2 Able to manage a project through all stages of its life cycle	UC-2.1 Set an objective within a defined scientific problem; formulate the agenda, relevance, significance (scientific, practical, methodological or other depending on the project type), forecast the expected results and possible areas of their application
	UC-2.2 Forecast the project outcomes, plan necessary steps to achieve the outcomes, chart the project schedule and monitoring plan
	UC-2.3 Organize and coordinate the work of project stakeholders, provide the team with necessary resources
UC-6 Determine priorities and ways to improve performance through self-assessment	UC-6.1 Achieve personal growth and professional development, determine priorities and ways to improve performance
Gen.Pro.C-1 Address current challenges in fundamental and applied mathematics	Gen.Pro.C-1.2 Consolidate and critically assess professional experience and research findings
	Gen.Pro.C-1.1 Apply fundamental scientific knowledge, new scientific principles, and research methods in applied mathematics and computer science
	Gen.Pro.C-1.3 Understand interdisciplinary relations in applied mathematics and computer science and apply them in professional tasks
Gen.Pro.C-2 Iimprove upon and implement new mathematical methods in applied problem solving	Gen.Pro.C-2.2 Assess the relevance and practical importance of applied mathematical research in professional settings
Gen.Pro.C-4 Combine and adapt current information and communications technologies (ICTs) to meet professional challenges	Gen.Pro.C-4.1 Use ICTs to search and analyze professional information, highlight, structure, format, and present it in the form of analytical reviews with sound conclusions and recommendations

3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

- fundamental concepts, laws of additive combinatorics;
- modern problems of the corresponding sections of additive combinatorics;
- concepts, axioms, methods of proof and proof of the main theorems in the sections included in the basic part of the cycle;
- basic properties of the corresponding mathematical objects;
- analytical and numerical approaches and methods for solving typical applied problems of additive combinatorics.

be able to:

- understand the task;
- use your knowledge to solve fundamental and applied problems of additive combinatorics;
- evaluate the correctness of the problem statements;
- strictly prove or disprove the statement;
- independently find algorithms for solving problems, including non-standard ones, and conduct their analysis;
- independently see the consequences of the results;
- accurately represent mathematical knowledge in the field of complex computing in oral and written form.

master:

- skills of mastering a large amount of information and solving problems (including complex ones);
- skills of independent work and mastering new disciplines;
- the culture of the formulation, analysis and solution of mathematical and applied problems requiring the use of mathematical approaches and methods of additive combinatorics for their solution;
- the subject language of complex calculations and the skills of competent description of problem solving and presentation of the results.

4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

4.1. The sections of the course (training module) and the complexity of the types of training sessions

№	Topic (section) of the course	Types of training sessions, including independent work			
		Lectures	Seminars	Laboratory practical	Independent work
1	Polynomial Growth Groups	3	9		9
2	Groups generated by automata	3	9		9
3	Classification of automaton groups with two states and the alphabet $\{0, 1\}$	3	9		9
4	Nielsen Method	3	9		9
5	Plynneke Inequality	3	9		9
AH in total		15	45		45
Exam preparation		30 AH.			
Total complexity		135 AH., credits in total 3			

4.2. Content of the course (training module), structured by topics (sections)

Semester: 3 (Fall)

1. Polynomial Growth Groups

The increase in complexity of the group.

2. Groups generated by automata

Actions on Root Trees

3. Classification of automaton groups with two states and the alphabet {0, 1}

Balog-Semeredi-Gowers Theorem. Higher energies, structural theorems.

4. Nielsen Method

Its geometric interpretation

5. Plynneke Inequality

The simplest relations between the sizes of the sums of sets.

5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)

A standard classroom.

6. List of the main and additional literature, that is necessary for the course (training module) mastering

Main literature

1. Основы комбинаторики и теории чисел [Текст] : сборник задач : учеб. пособие для вузов / А. А. Глибичук [и др.] .— Долгопрудный : Изд. Дом "Интеллект", 2015 .— 104 с. - Предм. указ.: с. 101-102. - Библиогр.: с.103. - 500 экз. - ISBN 978-5-91559-201-7 .— Полный текст (Режим доступа : доступ из сети МФТИ).
2.Комбинаторика и информация [Текст]. Ч. 2. Информационные модели : учеб. пособие для вузов / В. К. Леонтьев ; М-во образования и науки РФ, Моск. физ.-техн. ин-т (гос. ун-т) .— М. : МФТИ, 2016 .— 112 с. + pdf-версия. - Библиогр.: с. 111. - 250 экз. - ISBN 978-5-74170586-5 .— Полный текст (Доступ из сети МФТИ).
3.Комбинаторика и теория вероятностей [Текст] : [учеб. пособие для вузов] / А. М. Райгородский .— Долгопрудный : Интеллект, 2013 .— 104 с. - Библиогр.: с. 99. - 3000 экз. - ISBN 978-5-91559-147-8 .— Полный текст (Режим доступа : доступ из сети МФТИ).

Additional literature

1. Линейно-алгебраический метод в комбинаторике [Текст] : [учеб. пособие для вузов] / А. М. Райгородский .— М. : МЦНМО, 2007 .— 136 с.

7. List of web resources that are necessary for the course (training module) mastering

http://dm.fizteh.ru/
http://web.stanford.edu/class/ee364b/lectures.html

8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)

Multimedia technologies can be employed during lectures and practical lessons, including presentations.

9. Guidelines for students to master the course

1. It is recommended to successfully pass the test papers, as this simplifies the final certification in the subject.
2. To prepare for the final certification in the subject, it is best to use the lecture materials.

SUPPLEMENT

Assessment funds for course (training module)

major:	Applied Mathematics and Informatics
specialization:	Advanced Methods of Modern Combinatorics/Продвинутые методы современной комбинаторики
	Phystech School of Applied Mathematics and Informatics
	Chair of Discrete Mathematics
term:	2
qualification:	Master

Semester, form of interim assessment: 3 (fall) - Exam

Author: A.A. Glibichuk, candidate of physics and mathematical sciences, associate professor

1. Competencies formed during the process of studying the course

Code and the name of the competence	Competency indicators
UC-1 Use a systematic approach to critically analyze a problem, and develop an action plan	UC-1.1 Systematically analyze the problem situation, identify its components and the relations between them
	UC-1.3 Develop a step-by-step strategy for achieving a goal, foresee the result of each step, evaluate the overall impact on the planned activity and its participants
UC-2 Able to manage a project through all stages of its life cycle	UC-2.1 Set an objective within a defined scientific problem; formulate the agenda, relevance, significance (scientific, practical, methodological or other depending on the project type), forecast the expected results and possible areas of their application
	UC-2.2 Forecast the project outcomes, plan necessary steps to achieve the outcomes, chart the project schedule and monitoring plan
	UC-2.3 Organize and coordinate the work of project stakeholders, provide the team with necessary resources
UC-6 Determine priorities and ways to improve performance through self-assessment	UC-6.1 Achieve personal growth and professional development, determine priorities and ways to improve performance
Gen.Pro.C-1 Address current challenges in fundamental and applied mathematics	Gen.Pro.C-1.2 Consolidate and critically assess professional experience and research findings
	Gen.Pro.C-1.1 Apply fundamental scientific knowledge, new scientific principles, and research methods in applied mathematics and computer science
	Gen.Pro.C-1.3 Understand interdisciplinary relations in applied mathematics and computer science and apply them in professional tasks
Gen.Pro.C-2 Iimprove upon and implement new mathematical methods in applied problem solving	Gen.Pro.C-2.2 Assess the relevance and practical importance of applied mathematical research in professional settings
Gen.Pro.C-4 Combine and adapt current information and communications technologies (ICTs) to meet professional challenges	Gen.Pro.C-4.1 Use ICTs to search and analyze professional information, highlight, structure, format, and present it in the form of analytical reviews with sound conclusions and recommendations

2. Competency assessment indicators

As a result of studying the course the student should:

know:

- fundamental concepts, laws of additive combinatorics;
- modern problems of the corresponding sections of additive combinatorics;
- concepts, axioms, methods of proof and proof of the main theorems in the sections included in the basic part of the cycle;
- basic properties of the corresponding mathematical objects;
- analytical and numerical approaches and methods for solving typical applied problems of additive combinatorics.

be able to:

understand the task;
use your knowledge to solve fundamental and applied problems of additive combinatorics;
evaluate the correctness of the problem statements;
strictly prove or disprove the statement;
independently find algorithms for solving problems, including non-standard ones, and conduct their analysis;
independently see the consequences of the results;
accurately represent mathematical knowledge in the field of complex computing in oral and written form.

master:

skills of mastering a large amount of information and solving problems (including complex ones);
skills of independent work and mastering new disciplines;
the culture of the formulation, analysis and solution of mathematical and applied problems requiring the use of mathematical approaches and methods of additive combinatorics for their solution;
the subject language of complex calculations and the skills of competent description of problem solving and presentation of the results.

3. List of typical control tasks used to evaluate knowledge and skills

Current control

Question 1. Rusza triangle inequality.

Question 2. Additive energy. Inequalities on additive energy. Proof of the statement that any set with small sumset has large additive energy. For any nonempty subset A of an abelian group G we denote H (A) = {g G: g + A = A} which is a group of it's symmetry.

Problem 1. Given an arbitrary abelian group G. For any subgroup H G and any subset S denote S / H: = {s + H: s S} G / H. Suppose that A and B are arbitrary nonempty subsets of the group G and H = H (A + B). Prove that either | A + B |> | A | + | B |, or | (A + B) / H | = | A / H | + | B / H | 1.

Problem 2. Deduce from Knesser's theorem the following corollary: Let m> 2 any natural number and A, B be nonempty subsets of Z / mZ. If 0 B and (b, m) = 1 for any element b B \ {0}, then | A + B |> min {m, | A | + | B | 1}.

Problem 3. Suppose we proved that in any abelian group G and for any two nite nonempty subsets A and B we have | A + B |> | A | + | B | | H (A + B) |. Deduce from this statement Knesser's theorem.

Problem 4. Prove that for any two nite subsets A and B of an abelian group G equivalent following 7 statements: 1) | A + B | = | A || B |. 2) | A B | = | A || B |. 3) | {(a1, a2, b1, b2) ACHAHBCHB: a1 + b1 = a2 + b2} | = | A || B |. 4) | {(a1, a2, b1, b2) ACHAHBCHB: a1 b1 = a2 b2} | = | A || B |. 5) | A (x B) | = 1 for any x A + B. 6) | A (B + y) | = 1 for any y A B. 7) (A A) (B B) = {0}.

Problem 5. Consider any nite eld F and any subeld P F. Prove that for any nonzero elements c, d F = F \ {0} the set A = c + dP satisfy inequality | A + A | = | A | and | A · A | = | A | simultaneously if and only if c dP.

Problem 6. Consider any prime number p> 3. Prove that for any nontrivial multiplicative subgroup H of the eld Fp and any element b Fp, b / H the cardinality of the set X = H + bH: = {h1 + bh2: h1 , h2 H} either multiple of | H | or equal to 1 modulo | H |.

Problem 7. Let p be any prime number and (ab, p) = 1. Given a function f (x, y) = ax2 + by2. Using CauchyDavenport theorem, prove that the congruence f (x, y) n (mod p) have solutions for any n.

Problem 8. Let p be any prime number and c1, c2, ..., ck any nonzero coecients from Zp. Consider a function f (x1, x2, ..., xk) = c1xk 1 + c2x2 2 + ... + ckxk k. Prove that the congruence f (x1, x2, ..., xk) n (mod p) have solutions for any n.

.

4. Evaluation criteria

Questions for the exam

1. Introduction. The simplest relations between the sizes of the sums of sets. Plynneke's inequality. Universal sets.
2. The structure of sets with small doubling. Cover lemmas. Freiman's theorem in torsion groups.
3. Fourier analysis on abelian groups. Uniform sets of the first order. Roth's theorem.
4. The cemeredi regularity lemma. Rouge-Cemeredi theorem on triangles.
5. Large trigonometric sums.

6. Properties of Bohr sets.
7. Almost periodic convolution of characteristic functions. Arithmetic progressions in sums of sets.
8. Freiman's theorem, the polynomial Bogolyubov hypothesis - modern estimates.
9. Balog-Szemerédi-Gowers theorem. Higher energies, structural theorems.
10. The Berend construction of sets without solutions of affine equations. Top grades.
11. Gowers norms, uniform sets of higher orders.
12. Semeredi-Trotter theorem, convex sets. Sum of products: real case.

Exam ticket examples
Ticket number 1
1. The Berend construction of sets without solutions of affine equations. Top grades
2. The cemeredi regularity lemma. Rouge-Cemeredi theorem on triangles.
Ticket number 2
1. The structure of sets with small doubling. Cover lemmas.
2. Cemeredi-Trotter theorem, convex sets

Topics for term papers:
1. Sum of products: finite fields, uniform distribution of multiplicative subgroups.
2. The problem of Kakey.

Assessment “excellent (10)” is given to a student who has displayed comprehensive, systematic and deep knowledge of the educational program material, has independently performed all the tasks stipulated by the program, has deeply studied the basic and additional literature recommended by the program, has been actively working in the classroom, and understands the basic scientific concepts on studied discipline, who showed creativity and scientific approach in understanding and presenting educational program material, whose answer is characterized by using rich and adequate terms, and by the consistent and logical presentation of the material;
Assessment “excellent (9)” is given to a student who has displayed comprehensive, systematic knowledge of the educational program material, has independently performed all the tasks provided by the program, has deeply mastered the basic literature and is familiar with the additional literature recommended by the program, has been actively working in the classroom, has shown the systematic nature of knowledge on discipline sufficient for further study, as well as the ability to amplify it on one’s own, whose answer is distinguished by the accuracy of the terms used, and the presentation of the material in it is consistent and logical;
Assessment “excellent (8)” is given to a student who has displayed complete knowledge of the educational program material, does not allow significant inaccuracies in his answer, has independently performed all the tasks stipulated by the program, studied the basic literature recommended by the program, worked actively in the classroom, showed systematic character of his knowledge of the discipline, which is sufficient for further study, as well as the ability to amplify it on his own;
Assessment “good (7)” is given to a student who has displayed a sufficiently complete knowledge of the educational program material, does not allow significant inaccuracies in the answer, has independently performed all the tasks provided by the program, studied the basic literature recommended by the program, worked actively in the classroom, showed systematic character of his knowledge of the discipline, which is sufficient for further study, as well as the ability to amplify it on his own;
Assessment “good (6)” is given to a student who has displayed a sufficiently complete knowledge of the educational program material, does not allow significant inaccuracies in his answer, has independently carried out the main tasks stipulated by the program, studied the basic literature recommended by the program, showed systematic character of his knowledge of the discipline, which is sufficient for further study;
Assessment “good (5)” is given to a student who has displayed knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, who while not being sufficiently active in the classroom, has nevertheless independently carried out the main tasks stipulated by the program, mastered the basic literature recommended by the program, made some errors in their implementation and in his answer during the test, but has the necessary knowledge for correcting these errors by himself;

Assessment “satisfactory (4)” is given to a student who has discovered knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, who while not being sufficiently active in the classroom, has nevertheless independently carried out the main tasks stipulated by the program, learned the main literature but allowed some errors in their implementation and in his answer during the test, but has the necessary knowledge for correcting these errors under the guidance of a teacher;

Assessment “satisfactory (3)” is given to a student who has displayed knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, not showed activity in the classroom, independently fulfilled the main tasks envisaged by the program, but allowed errors in their implementation and in the answer during the test, but possessing necessary knowledge for elimination under the guidance of the teacher of the most essential errors;

Assessment “unsatisfactory (2)” is given to a student who showed gaps in knowledge or lack of knowledge on a significant part of the basic educational program material, who has not performed independently the main tasks demanded by the program, made fundamental errors in the fulfillment of the tasks stipulated by the program, who is not able to continue his studies or start professional activities without additional training in the discipline in question;

Assessment “unsatisfactory (1)” is given to a student when there is no answer (refusal to answer), or when the submitted answer does not correspond at all to the essence of the questions contained in the task.

5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience

During examination the student are allowed to use the program of the discipline.