Federal State Autonomous Educational Institution of Higher Education "Moscow Institute of Physics and Technology (National Research University)"

APPROVED
Head of the Phystech School of Applied Mathematics and Informatics
A.M. Raygorodskiy

Work program of the course (training module)

course:	Matrix Algebra Methods and Applications to Computer Science and Engineering/Методы матричной алгебры и приложения к информатике и инженерии
major:	Applied Mathematics and Informatics
specialization:	Advanced Methods of Modern Combinatorics/Продвинутые методы современной комбинаторики
	Phystech School of Applied Mathematics and Informatics
	Chair of Discrete Mathematics
term:	1
qualification:	Master

Semester, form of interim assessment: 2 (spring) - Exam

Academic hours: 60 AH in total, including:
lectures: 15 AH.
seminars: 45 AH.
laboratory practical: 0 AH.

Independent work: 45 AH.

Exam preparation: 30 AH.

In total: 135 AH, credits in total: 3

Author of the program: A.M. Elishev, candidate of physics and mathematical sciences, associate professor

The program was discussed at the Chair of Discrete Mathematics 05.03.2022

Annotation

This course is designed to serve as a natural development of the first term linear algebra program, building on the matrix algebra framework. The course is essentially a survey of notable and important applications of methods of linear algebra (in particular, matrix decompositions, orthogonalization, spectral theorem, and singular value decomposition) to problems in computer science and engineering.

1. Study objective

Purpose of the course

The purpose of teaching the discipline is to form students' deep theoretical knowledge and practical skills in the study and numerical solution of extreme problems with limitations arising in scientific res- изучение основных понятий и методов решения задач выпуклого программирования.

Tasks of the course

- study of the basic concepts and methods of solving convex programming problems.
- application of numerical methods of convex optimization for solving extreme problems

2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

Code and the name of the competence	Competency indicators
UC-1 Use a systematic approach to critically analyze a problem, and develop an action plan	UC-1.2 Search for solutions by using available sources
Gen.Pro.C-2 Iimprove upon and implement new mathematical methods in applied problem solving	Gen.Pro.C-2.1 Assess the current state of mathematical research within professional settings
Pro.C-2 Understands and is able to apply modern mathematical apparatus and algorithms, the basic laws of natural science, modern programming languages and software; operating systems and networking technologies in research and applied activities	Pro.C-2.1 Demonstrate expert knowledge of research basics in the field of ICTs, philosophy and methodology of science, scientific research methods, and apply skills to use them

3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

concepts, representations and statements of algebra

be able to:

proofs of basic linear algebra theorems

master:

the main methods of calculations and methods of solving algebraic problems

4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

4.1. The sections of the course (training module) and the complexity of the types of training sessions

№	Topic (section) of the course	Types of training sessions, including independent work			
		Lectures	Seminars	Laboratory practical	Independent work
1	Recap and review	2	4		4

2	Algorithmic viewpoint on gaussian elimination	2	4		4
3	Orthogonality	1	4		4
4	Symmetric matrices	2	4		4
5	Geometric interpretation	2	4		4
6	Matrix spaces	1	4		4
7	Vector norms	1	4		4
8	Basic applications of matrix algebra to physics	1	4		4
9	Markov matrices	1	4		4
10	Neural nets and the learning function	1	4		4
11	Linear programming	1	5		5
AH in total		15	45		45
Exam preparation		30 AH.			
Total complexity		135 AH., credits in total 3			

4.2. Content of the course (training module), structured by topics (sections)

Semester: 2 (Spring)

1. Recap and review

Systems of linear equations. Solutions. Homogeneous systems. Matrixalgebra. Column-vectors and row-vectors. The transpose of a matrix. Block matrices. Coordinates. Dimension. Rank. Solving Ax = b. The four fundamental subspaces. Gaussian elimination. LU factorization. Linear transformations and their matrices. Change of basis.

2. Algorithmic viewpoint on gaussian elimination

Computational efficiency. Applications:
computing determinants, finding the inverse of a matrix, ranks and bases.

3. Orthogonality

Orthogonal bases. Gram – Schmidt. QR factorization. Projection matrices. Determinants. Cofactors. Eigenvalues and eigenvectors. The characteristic polynomial. Algebraic and geometric multiplicities. Diagonalization. Powers of a square matrix. The exponent of a matrix.

4. Symmetric matrices

Positive definiteness and semidefiniteness. Minima.Diagonalization of symmetric transformations. Polar decomposition. Similar matrices. The Jordan form. Normal matrices. The spectral theorem.

5. Geometric interpretation

Singular value decomposition (SVD). Geometric interpretation. First applications: the pseudoinverse, solving homogeneous linear equations. Existence. Relation to the spectral theorem. Matrix norms.

6. Matrix spaces

Rank one. Small world graphs. Graphs, networks, incidence matrices. Directed graphs and paths. Adjacency matrices. Eigenvalues. Search engines and PageRank.

7. Vector norms

Linear regression. Least squares. Approaches to the least squares problem.Generalizations. Functions evaluated on matrices. Polynomials. Power series. The exponent and the logarithm. Solving basic ODEs. The trace and the determinant.

8. Basic applications of matrix algebra to physics

Springs and masses. Oscillations. Tensionforces. Truss bridges. Basic electrical engineering. Kirchhoff's current law.Counting parameters in SVD, LU, QR decompositions. Saddle points. Lagrange multipliers. Max-min principle. Gradient descent. Generalizations.

9. Markov matrices

Fourier series. Complex matrices. Fast Fourier transform (FFT). Applications to signal processing. Spectral analysis.Principal component analysis. The Eckart – Young theorem. Data sets. Predictive models.Covariance matrices. Relation to SVD.

10. Neural nets and the learning function

Deep learning. Convolutional neural networks. ImageNet. The convolution rule. Distance matrices. Procrustes problem.The problem of finding clusters in graphs.

11. Linear programming

Kantorovich's problem. Slackness. Duality. Existence of optimalsolutions. Dantzig's simplex algorithm.

5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)

A classroom equipped with a computer and multimedia equipment (projector, sound system).

6. List of the main and additional literature, that is necessary for the course (training module) mastering

Main literature

1. Численные методы оптимизации [Текст] : [учеб.пособие для вузов] / А.Ф.Измайлов, М.В.Солодов .— М. : Физматлит, 2003, 2005 .— 304 с.
2. Элементы линейной алгебры и линейного програмирования [Текст] / Ф. И. Карпелевич, Л. Е. Садовский - М.Физматгиз,1963

Additional literature

1. Численные методы [Текст] : в 2 кн. : учебник для вузов / Н. Н. Калиткин, Е. А. Альшина .— М. : Академия, 2013 .— (Университетский учебник. Прикладная математика и информатика) .— Кн. 1 : Численный анализ. - 2013. - 304 с.

7. List of web resources that are necessary for the course (training module) mastering

http://dm.fizteh.ru/

8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)

Multimedia technologies are used in lecture classes, including demonstration of presentations.
In the process of independent work of students, it is possible to use such software tools as Mathcad, MATLAB, Maple, etc.

9. Guidelines for students to master the course

1. It is recommended to successfully pass the test papers, as this simplifies the final certification of the subject.
2. To prepare for the final certification on the subject, it is best to use lecture materials.

SUPPLEMENT

Assessment funds for course (training module)

major:	Applied Mathematics and Informatics
specialization:	Advanced Methods of Modern Combinatorics/Продвинутые методы современной комбинаторики Phystech School of Applied Mathematics and Informatics Chair of Discrete Mathematics
term:	1
qualification:	Master

Semester, form of interim assessment: 2 (spring) - Exam

Author: A.M. Elishev, candidate of physics and mathematical sciences, associate professor

1. Competencies formed during the process of studying the course

Code and the name of the competence	Competency indicators
UC-1 Use a systematic approach to critically analyze a problem, and develop an action plan	UC-1.2 Search for solutions by using available sources
Gen.Pro.C-2 Iimprove upon and implement new mathematical methods in applied problem solving	Gen.Pro.C-2.1 Assess the current state of mathematical research within professional settings
Pro.C-2 Understands and is able to apply modern mathematical apparatus and algorithms, the basic laws of natural science, modern programming languages and software; operating systems and networking technologies in research and applied activities	Pro.C-2.1 Demonstrate expert knowledge of research basics in the field of ICTs, philosophy and methodology of science, scientific research methods, and apply skills to use them

2. Competency assessment indicators

As a result of studying the course the student should:

know:

concepts, representations and statements of algebra

be able to:

proofs of basic linear algebra theorems

master:

the main methods of calculations and methods of solving algebraic problems

3. List of typical control tasks used to evaluate knowledge and skills

1. Summing up and review.
2. Systems of linear equations.
3. Homogeneous systems.
4. Matrixal-gebra.5. Column vectors and row vectors.
6. Matrix transposition.
7. Block matrices.
8. Coordinates.
9. Four fundamental subspaces.
10. Gauss exclusion.
11.LU factorization.
12. Linear transformations and their matrices.
13. Changing the basis.
14. Algorithmic point of view on Gaussian elimination.
15.Computational efficiency.
16. Applications:calculation of determinants, finding the inverse matrix, ranks and bases.
17. Review the second part.
18. Orthogonality.
19. Orthogonal bases.
20. Gram – Schmidt.
21. QR factorization.
22. Projection matrices.
23. Determinants.
24. Cofactors.
25.Eigenvalues and eigenvectors.
26. Characteristic polynomial.
27. Algebraic and geometric multiplicities.

28. Diagonalization.
29. Degrees of a square matrix.
30.The exponent of the matrix.
31. Review the third part.
32. Symmetric matrices.
33. Positive definiteness and semi-definiteness.
34. Diagonalization of symmetric transformations.
35. Polar decomposition.
36. Similar matrices.
37. The Jordanian form.
38. Normal matrices.
39. The spectral theorem.
40. Singular value Decomposition (SVD).
41. Geometric interpretation.
42.First applications: pseudo-inversion, solution of homogeneous linear equations.
43. Relation to the spectral theorem.
44. Matrix norms.
45. Matrix spaces.
46. Graphs of a small world.
47. Graphs, networks, incidence matrices.

4. Evaluation criteria

List of exam questions:
1. Vector norms.
2. Linear regression.
3. Least squares.
4. Approaches to the least squares problem.Generalizations.
5. Functions calculated on matrices. ,
6.Polynomials.
7. Power row.
8. Exponent and logarithm.
9. Solution of basic ODES.
10. Trace and determinant.
11. Basic applications of matrix algebra to physics.
12. Springs and weights.
13. Fluctuations.
14. Tension forces.
15. Truss bridges.
16. Fundamentals of electrical engineering.
17. The current Kirchhoff Law.
18. Calculation of parameters in SVD, LU, QR decompositions.
19. Saddle points.
20. Lagrange multipliers.
21. The principle of Max-min
. 22.Gradient descent.
23. Generalizations.
24. Markov matrices.
25.Fourier series.
26. Complex matrices.
27. Fast Fourier Transform (FFT).
28. Applications for signal processing.
29. Spectral analysis.
30. Analysis of the main components.

31The Eckart–Young theorem.
32. Data sets.
33. Predictive models.
34. Covariance matrices.
35.Attitude to the SVD.
36. Neural networks and the learning function.
37. Deep learning.
38. Convolutional neural networks.
39.ImageNet. Convolution rule.
40. Distance matrices.
41. The problem of Procrustes.
42. The problem of finding clusters in graphs.
43. Linear programming.
44 Kantorovich's problem.
45.Relaxation.
46. Duality.
47. Existence of optimal solutions.
48. Simplex Danzig algorithm.

Ticket examples:
Ticket 1:
1. Eigenvalues.
2.Search engines and PageRank.
Ticket 2:
1.Oriented graphs and paths.
2..Adjacency matrices.

- the grade "excellent (10)" is given to the student who has shown comprehensive, systematic, in-depth knowledge of the discipline's curriculum and the ability to confidently apply them in practice when solving specific tasks, free and correct justification of the decisions made
- the grade "excellent (9)" is given to the student who has shown comprehensive, systematic, in-depth knowledge of the discipline's curriculum and the ability to apply them in practice when solving specific tasks, free and correct justification of the decisions made
- the grade "excellent (8)" is given to the student who has shown comprehensive, systematic, in-depth knowledge of the discipline's curriculum and the ability to apply them in practice when solving specific tasks, and the correct justification of the decisions made
- the grade "good (7)" is given to the student if he firmly knows the material, competently and essentially expounds it, knows how to apply the knowledge gained in practice, but admits some inaccuracies in the answer or in solving problems;
- the grade "good (6)" is given to the student if he knows the material, competently and substantially expounds it, knows how to apply the knowledge gained in practice, but admits some inaccuracies in the answer or in solving problems;
- the grade "good (5)" is given to the student if he knows the material, and essentially expounds it, is able to apply the knowledge gained in practice, but admits some inaccuracies in the answer or in solving problems;
- the grade "satisfactory (4)" is given to a student who has shown a fragmentary, fragmented nature of knowledge, insufficiently correct formulations of basic concepts, violations of logical sequence in the presentation of program material, but at the same time he owns the main sections of the curriculum necessary for further study and can apply the knowledge gained on a model in a standard situation;
- the assessment "satisfactory (3)" is given to a student who has shown a fragmentary, fragmented nature of knowledge, insufficiently correct formulations of basic concepts, violations of logical sequence in the presentation of program material, but at the same time he owns fragmentary the main sections of the curriculum necessary for further study and can apply the knowledge obtained on a model in a standard situation;

- the grade "unsatisfactory (2)" is given to a student who does not know most of the main content of the discipline's curriculum, makes gross mistakes in the formulations of the basic concepts of the discipline and does not know how to use the knowledge gained in solving typical practical tasks
- the grade "unsatisfactory (1)" is given to a student who does not know the formulations of the basic concepts of the discipline

5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience

During the exam, students can use the discipline program.