Federal State Autonomous Educational Institution of Higher Education "Moscow Institute of Physics and Technology (National Research University)"

APPROVED
Head of the Phystech School of Applied Mathematics and Informatics
A.M. Raygorodskiy

Work program of the course (training module)

course:	Extremal Combinatorics/Экстремальная комбинаторика
major:	Applied Mathematics and Informatics
specialization:	Advanced Methods of Modern Combinatorics/Продвинутые методы современной комбинаторики
	Phystech School of Applied Mathematics and Informatics
	Chair of Discrete Mathematics
term:	1
qualification:	Master

Semester, form of interim assessment: 2 (spring) - Exam

Academic hours: 45 AH in total, including:
- lectures: 15 AH.
- seminars: 30 AH.
- laboratory practical: 0 AH.

Independent work: 60 AH.

Exam preparation: 30 AH.

In total: 135 AH, credits in total: 3

Number of course papers, tasks: 2

Author of the program: V.L. Dolnikov, doctor of physics and mathematical sciences, full professor, professor

The program was discussed at the Chair of Discrete Mathematics 04.03.2022

Annotation

One of the classic statements of extremal combinatorics is the 1961 Erdös-Ko-Rado theorem, which establishes the size of the largest collection of pairwise intersecting k-element subsets of an n-element set. In the talk, we will tell about the history of the problematic that arose in connection with this theorem. We will show that this problem is at the very center of modern combinatorial analysis, we will demonstrate its connections with coding theory, combinatorial geometry, and algebraic topology. We will pay special attention to the recent probabilistic interpretation of the problem - in terms of the theory of random graphs.

1. Study objective

Purpose of the course

mastering the elective concepts of extreme combinatorics.

Tasks of the course

- students mastering basic knowledge (concepts, concepts, methods and models) in the field of extreme combinatorics;
- acquisition of theoretical knowledge and practical skills in the field of extreme combinatorics;
- providing advice and assistance to students in conducting their own theoretical research in the field of extreme combinatorics.

2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

Code and the name of the competence	Competency indicators
UC-3 Able to organise and lead a team, developing a team strategy to achieve a goal	UC-3.2 Consider the interests, specific behavior, and diversity of opinions of team members/colleagues/counterparties
Gen.Pro.C-2 Iimprove upon and implement new mathematical methods in applied problem solving	Gen.Pro.C-2.1 Assess the current state of mathematical research within professional settings
Pro.C-3 Participate in scholarly discussions, make speeches and presentations (oral, written, and online) on scientific topics, present research materials, proofread, edit, reference scientific works	Pro.C-3.2 Hold an appropriate discussion of ICTs and information systems, ask and answer questions related to a particular scientific subject

3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

fundamental concepts, laws, theories of extreme combinatorics;
current problems of the relevant sections of extreme combinatorics;
concepts, axioms, methods of proof and proof of the main theorems in the sections included in the basic part of the cycle;
basic properties of the corresponding mathematical objects;
analytical and numerical approaches and methods for solving typical applied problems of extreme combinatorics.

be able to:

understand the task;
use your knowledge to solve fundamental and applied problems of extreme combinatorics;
evaluate the correctness of the problem statements;
strictly prove or disprove the statement;
independently find algorithms for solving problems, including non-standard ones, and conduct their analysis;
independently see the consequences of the results;
accurately represent mathematical knowledge in the field of complex computing in oral and written form.

master:

skills of mastering a large amount of information and solving problems (including complex ones);
skills of independent work and mastering new disciplines;
the culture of the formulation, analysis and solution of mathematical and applied problems requiring the use of mathematical approaches and methods of extreme combinatorics for their solution;
the subject language of complex calculations and the skills of competent description of problem solving and presentation of the results.

4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

4.1. The sections of the course (training module) and the complexity of the types of training sessions

№	Topic (section) of the course	Types of training sessions, including independent work			
		Lectures	Seminars	Laboratory practical	Independent work
1	Canonical bundle over Grassmann space	3	6		12
2	Basic concepts and definitions of convex geometry	3	6		12
3	Polynomial division of one measure in the spirit of Gut – Katz and its properties	3	6		12
4	Applications of Helly's Theorem	3	6		12
5	The Borsuk – Ulam theorem in the simplest case	3	6		12
AH in total		15	30		60
Exam preparation		30 AH.			
Total complexity		135 AH., credits in total 3			

4.2. Content of the course (training module), structured by topics (sections)

Semester: 2 (Spring)

1. Canonical bundle over Grassmann space

Connection of points on a plane by a graph with a small number of intersections with any line, Chazel – Weltzl theorem

2. Basic concepts and definitions of convex geometry

Caratheodory's theorem and Helly's theorem.

3. Polynomial division of one measure in the spirit of Gut – Katz and its properties

The sandwich theorem. Curve of moments and its generalization, polynomial version of the sandwich theorem

4. Applications of Helly's Theorem

Jung's inequality, central point theorem.

5. The Borsuk – Ulam theorem in the simplest case

Technique of minimization and its application. Carathéodory's color theorem and Helly's color theorem. Tverberg theorem.

5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)

A standard classroom.

6. List of the main and additional literature, that is necessary for the course (training module) mastering

Main literature

1. Дискретная математика [Текст] : графы, матроиды, алгоритмы : учеб. пособие для вузов / М. О. Асанов, В. А. Баранский, В. В. Расин .— Ижевск : НИЦ Регулярная и хаотическая динамика, 2001 .— 288 с.
2. Экстремальные задачи теории графов и Интернет [Текст] : [учеб. пособие для вузов] / А. М. Райгородский .— Долгопрудный : Интеллект, 2012 .— 104 с.
3. Комбинаторика [Текст], [учеб. пособие для вузов] /Н. Я. Виленкин, А. Н. Виленкин, П. А. Виленкин. -М., ФИМА : МЦНМО, 2015

Additional literature

1. Комбинаторика и теория вероятностей [Текст] : [учеб. пособие для вузов] / А. М. Райгородский .— Долгопрудный : Интеллект, 2013 .— 104 с. - Библиогр.: с. 99. - 3000 экз. - ISBN 978-5-91559-147-8 .— Полный текст (Режим доступа : доступ из сети МФТИ).

7. List of web resources that are necessary for the course (training module) mastering

http://dm.fizteh.ru/

8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)

Multimedia technologies can be employed during lectures and practical lessons, including presentations.

9. Guidelines for students to master the course

1. It is recommended to successfully pass the test papers, as this simplifies the final certification in the subject.
2. To prepare for the final certification in the subject, it is best to use the lecture materials.

SUPPLEMENT

Assessment funds for course (training module)

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	Chair of Discrete Mathematics
term:	1
qualification:	Master

Semester, form of interim assessment: 2 (spring) - Exam

Author:	V.L. Dolnikov, doctor of physics and mathematical sciences, full professor, professor

1. Competencies formed during the process of studying the course

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2. Competency assessment indicators

As a result of studying the course the student should:

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the subject language of complex calculations and the skills of competent description of problem solving and presentation of the results.

3. List of typical control tasks used to evaluate knowledge and skills

Homework example

Is it true that for any sets A and B the equality
$(A \setminus B) \cap ((A \cup B) \setminus (A \cap B)) = A \setminus B$?

Is it true that for any sets A, B, and C the equality
$((A \setminus B) \cup (A \setminus C)) \cap (A \setminus (B \cap C)) = A \setminus (B \cup C)$?

Is it true that for any sets A, B, and C the equality
$(A \cap B) \setminus C = (A \setminus C) \cap (B \setminus C)$?

Is it true that for any sets A and B the inclusion
$(A \cup B) \setminus (A \setminus B) \subseteq B$?

Let $P = [10,40]$; $Q = [20,30]$; it is known that the segment A satisfies the relation

$((x \in A) \to (x \in P)) \wedge ((x \in Q) \to (x \in A))$.
Find the segment A of the maximum possible length.
Find the segment A of the shortest possible length.
It is known about the sets A, B, X, Y that $A \cap X = B \cap X$, $A \cup Y = B \cup Y$. Is it true that then the equality $A \cup (Y \setminus X) = B \cup (Y \setminus X)$ holds?
Let $A_1 \supseteq A_2 \supseteq A_3 \supseteq \cdots \supseteq A_n \supseteq \cdots$ be a nonincreasing sequence of sets. It is known that $A_1 \setminus A_4 = A_6 \setminus A_9$. Prove that $A_2 \setminus A_7 = A_3 \setminus A_8$.
Let A, B, C, D be line segments such that $A \triangle B = C \triangle D$ (symmetric differences are equal). Is it true that the inclusion $A \cap B \subseteq C$ holds?

4. Evaluation criteria

Exam Questions
(1) Basic concepts and definitions of convex geometry.
(2) Caratheodory's theorem and Helly's theorem.
(3) Applications of Helly's theorem: Young's inequality, center point theorem.
(4) Technique of minimization and its application. Carathéodory's color theorem and Helly's color theorem. Tverberg theorem.
(5) The degree of display and some of its applications. A topological lemma on the mapping of a simplex into itself. Division of a measure into convex parts of a given size. The Knaster – Kuratovsky – Mazurkevich theorem and the Brauer fixed point theorem. Strengthenings of the Carathéodory color theorem.
(6) The Borsuk – Ulam theorem in the simplest case.
(7) The sandwich theorem. Curve of moments and its generalization, polynomial version of the sandwich theorem.
(8) Polynomial division of one measure in the spirit of Gut – Katz and its properties.
(9) The Cemeredi – Trotter theorem on the incidence number of points and lines. Estimates for the set of sums and the set of products of real numbers.
(10) Connection of points on a plane by a graph with a small number of intersections with any line, Chazel – Weltzl theorem.
(11) Dolnikov's theorem on intersections with hyperplanes and the chromatic number of the Kneser graph. Generalizations of the Borsuk – Ulam theorem for the action of a group of two elements.
(12) The canonical bundle over the Grassmann space, Dolnikov’s transversal theorem and central transversal theorem.
(13) Generalizations of the Borsuk – Ulam theorem for the action of groups of simple order. Tverberg's topological theorem and division of measures into equal parts on a straight line.
(14) Cech cohomology, covering nerve lemma, and Helly's topological theorem.

Exam ticket example
Ticket number 1
1. Cech cohomology, covering nerve lemma, and Helly's topological theorem.
2. Polynomial division of one measure in the spirit of Gut – Katz and its properties.
Ticket number 2
1. The Borsuk – Ulam theorem in the simplest case
2. Basic concepts and definitions of convex geometry

Assessment “excellent (10)” is given to a student who has displayed comprehensive, systematic and deep knowledge of the educational program material, has independently performed all the tasks stipulated by the program, has deeply studied the basic and additional literature recommended by the program, has been actively working in the classroom, and understands the basic scientific concepts on studied discipline, who showed creativity and scientific approach in understanding and presenting educational program material, whose answer is characterized by using rich and adequate terms, and by the consistent and logical presentation of the material;

Assessment “excellent (9)” is given to a student who has displayed comprehensive, systematic knowledge of the educational program material, has independently performed all the tasks provided by the program, has deeply mastered the basic literature and is familiar with the additional literature recommended by the program, has been actively working in the classroom, has shown the systematic nature of knowledge on discipline sufficient for further study, as well as the ability to amplify it on one’s own, whose answer is distinguished by the accuracy of the terms used, and the presentation of the material in it is consistent and logical;

Assessment “excellent (8)” is given to a student who has displayed complete knowledge of the educational program material, does not allow significant inaccuracies in his answer, has independently performed all the tasks stipulated by the program, studied the basic literature recommended by the program, worked actively in the classroom, showed systematic character of his knowledge of the discipline, which is sufficient for further study, as well as the ability to amplify it on his own;

Assessment “good (7)” is given to a student who has displayed a sufficiently complete knowledge of the educational program material, does not allow significant inaccuracies in the answer, has independently performed all the tasks provided by the program, studied the basic literature recommended by the program, worked actively in the classroom, showed systematic character of his knowledge of the discipline, which is sufficient for further study, as well as the ability to amplify it on his own;

Assessment “good (6)” is given to a student who has displayed a sufficiently complete knowledge of the educational program material, does not allow significant inaccuracies in his answer, has independently carried out the main tasks stipulated by the program, studied the basic literature recommended by the program, showed systematic character of his knowledge of the discipline, which is sufficient for further study;

Assessment “good (5)” is given to a student who has displayed knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, who while not being sufficiently active in the classroom, has nevertheless independently carried out the main tasks stipulated by the program, mastered the basic literature recommended by the program, made some errors in their implementation and in his answer during the test, but has the necessary knowledge for correcting these errors by himself;

Assessment “satisfactory (4)” is given to a student who has discovered knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, who while not being sufficiently active in the classroom, has nevertheless independently carried out the main tasks stipulated by the program, learned the main literature but allowed some errors in their implementation and in his answer during the test, but has the necessary knowledge for correcting these errors under the guidance of a teacher;

Assessment “satisfactory (3)” is given to a student who has displayed knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, not showed activity in the classroom, independently fulfilled the main tasks envisaged by the program, but allowed errors in their implementation and in the answer during the test, but possessing necessary knowledge for elimination under the guidance of the teacher of the most essential errors;

Assessment “unsatisfactory (2)” is given to a student who showed gaps in knowledge or lack of knowledge on a significant part of the basic educational program material, who has not performed independently the main tasks demanded by the program, made fundamental errors in the fulfillment of the tasks stipulated by the program, who is not able to continue his studies or start professional activities without additional training in the discipline in question;

Assessment “unsatisfactory (1)” is given to a student when there is no answer (refusal to answer), or when the submitted answer does not correspond at all to the essence of the questions contained in the task.

5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience

During examination the student are allowed to use the program of the discipline.