Federal State Autonomous Educational Institution of Higher Education "Moscow Institute of Physics and Technology (National Research University)"

APPROVED
Head of the Phystech School of Applied Mathematics and Informatics
A.M. Raygorodskiy

Work program of the course (training module)

course:	Applied Discrete Optimization/Прикладная дискретная оптимизация
major:	Applied Mathematics and Informatics
specialization:	Advanced Methods of Modern Combinatorics/Продвинутые методы современной комбинаторики
	Phystech School of Applied Mathematics and Informatics
	Chair of Discrete Mathematics
term:	2
qualification:	Master

Semester, form of interim assessment: 3 (fall) - Exam

Academic hours: 60 AH in total, including:
lectures: 30 AH.
seminars: 30 AH.
laboratory practical: 0 AH.

Independent work: 45 AH.

Exam preparation: 30 AH.

In total: 135 AH, credits in total: 3

Author of the program: A.B. Daynyak, candidate of physics and mathematical sciences, associate professor

The program was discussed at the Chair of Discrete Mathematics 01.03.2021

Annotation

The course is devoted to the study of classical and modern optimization methods. Examples of their use in applied problems of physics, mathematics and computer science are considered.

1. Study objective

Purpose of the course

study of classical and modern optimization methods. Consideration of examples of their use in applied problems of physics, mathematics and computer science.

Tasks of the course

study of the mathematical foundations of modern combinatorics;

acquisition of theoretical knowledge in the field of combinatorial analysis of problems arising in practice;

mastering the analytical and algebraic apparatus of discrete mathematics and obtaining skills in working with basic discrete structures.

2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

Code and the name of the competence	Competency indicators
UC-1 Use a systematic approach to critically analyze a problem, and develop an action plan	UC-1.1 Systematically analyze the problem situation, identify its components and the relations between them
	UC-1.3 Develop a step-by-step strategy for achieving a goal, foresee the result of each step, evaluate the overall impact on the planned activity and its participants
UC-2 Able to manage a project through all stages of its life cycle	UC-2.1 Set an objective within a defined scientific problem; formulate the agenda, relevance, significance (scientific, practical, methodological or other depending on the project type), forecast the expected results and possible areas of their application
	UC-2.2 Forecast the project outcomes, plan necessary steps to achieve the outcomes, chart the project schedule and monitoring plan
	UC-2.3 Organize and coordinate the work of project stakeholders, provide the team with necessary resources
UC-6 Determine priorities and ways to improve performance through self-assessment	UC-6.1 Achieve personal growth and professional development, determine priorities and ways to improve performance
Gen.Pro.C-1 Address current challenges in fundamental and applied mathematics	Gen.Pro.C-1.2 Consolidate and critically assess professional experience and research findings
	Gen.Pro.C-1.1 Apply fundamental scientific knowledge, new scientific principles, and research methods in applied mathematics and computer science
	Gen.Pro.C-1.3 Understand interdisciplinary relations in applied mathematics and computer science and apply them in professional tasks
Gen.Pro.C-2 Iimprove upon and implement new mathematical methods in applied problem solving	Gen.Pro.C-2.2 Assess the relevance and practical importance of applied mathematical research in professional settings
Gen.Pro.C-4 Combine and adapt current information and communications technologies (ICTs) to meet professional challenges	Gen.Pro.C-4.1 Use ICTs to search and analyze professional information, highlight, structure, format, and present it in the form of analytical reviews with sound conclusions and recommendations

3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

fundamental concepts, laws, theories of a part of discrete mathematics;
modern problems of the corresponding sections of discrete mathematics;
concepts, axioms, methods of proofs and proofs of the main theorems in the sections included in the basic part of the cycle;
basic properties of the corresponding mathematical objects;
analytical and numerical approaches and methods for solving typical applied problems of discrete mathematics.

be able to:

understand the task at hand;
use your knowledge to solve fundamental and applied problems;
evaluate the correctness of the problem setting;
strictly prove or disprove the statement;
independently find algorithms for solving problems, including non-standard ones, and analyze them;
independently see the consequences of the results obtained;
Accurately present mathematical knowledge in the field orally and in writing.

master:

skills of mastering a large amount of information and solving problems (including complex ones);
skills of independent work and mastering new disciplines;
culture of setting, analyzing and solving mathematical and applied problems that require the use of mathematical approaches and methods for their solution;
the subject language of discrete mathematics and the skills of competently describing the solution of problems and presenting the results obtained.

4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

4.1. The sections of the course (training module) and the complexity of the types of training sessions

№	Topic (section) of the course	Types of training sessions, including independent work			
		Lectures	Seminars	Laboratory practical	Independent work
1	Prim and Boruvka's Algorithms for Solving the MST Problem	5	5		7
2	Duality in linear programming	4	4		7
3	Discrete linear subset problem (DLS problem)	4	4		7
4	The problem of constructing a matching of maximum cardinality in an arbitrary graph	4	4		6
5	Branch and bound method	4	4		6
6	Dijkstra's Algorithm Modifications	5	4		6
7	Distinctive features of discrete optimization problems	4	5		6
AH in total		30	30		45
Exam preparation		30 AH.			
Total complexity		135 AH., credits in total 3			

4.2. Content of the course (training module), structured by topics (sections)

Semester: 3 (Fall)

1. Prim and Boruvka's Algorithms for Solving the MST Problem

Reminder of basic concepts from linear programming. Problem in standard and canonical forms. Transition from inequalities to equalities and vice versa. Geometry of the problem: a simplex algorithm as a local search along the vertices of a polyhedron.

2. Duality in linear programming

Statement of the TSP problem in terms of the CLP. Miller – Tucker – Zemlin conditions (polynomial number of inequalities in the TSP). Remark "on the non-catastrophic nature of the exponential number of constraints in LP problems".

3. Discrete linear subset problem (DLS problem)

TSP and MST problems as special cases of DLS minimization problems; transition to maximization. Hereditary systems. Bases and cycles. Rank and lower rank of a set, rank spread. Matroids: equivalent definitions, examples. Evaluation of the performance of a greedy algorithm on a hereditary system through its rank spread. Corollary on the correctness of the greedy algorithm for constructing the shortest spanning tree. Estimation of the rank spread through the limitation on the number of cycles. Submodularity of the rank function of the matroid. Enumeration of matroids. Estimation of the number of cycles for a hereditary system in terms of the number of matroids in the intersection.

4. The problem of constructing a matching of maximum cardinality in an arbitrary graph

Augmenting paths (the statement that matching is non-maximal ⇔ is an augmenting path). The problem with finding augmentation paths in the absence of dicotyledonous: flowers. Flower shrinkage statements. Edmonds algorithm.

5. Branch and bound method

Exhaustive enumeration of complex discrete objects. Reed's approach: ordered enumeration. Avis-Fukuda local search inversion method.

6. Dijkstra's Algorithm Modifications

Two algorithms: gradual minimization of the flow cost at a constant value; an increment in value due to the smallest possible increment in value.

7. Distinctive features of discrete optimization problems

An overview of the formulations of classical discrete optimization problems: set coverage, vertex coverage, shortest path, minimal spanning tree, matching problems, assignment problem, scheduling problems, packing problems (bin packing, knapsack), flow problems (largest flow, flow least cost, multi-product flows), transport problem (Hitchcock problem), traveling salesman problem.

5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)

Classroom equipped with a computer and multimedia equipment (projector, sound system).

6. List of the main and additional literature, that is necessary for the course (training module) mastering

Main literature

1. Математический анализ [Текст] : [в 3т.] : учебник для вузов : доп. М-вом высш. и сред. спец. образов. СССР. Т. 1. Начальный курс / В. А. Ильин, В. А. Садовничий, Бл. Х. Сендов ; под ред. А. Н. Тихонова .— / 2-е изд., перераб. — М. : МГУ, 1985 .— 660 с.

2. Введение в теорию алгоритмов и структур данных [Текст] : [учеб. пособие для вузов] / М. А. Бабенко, М. В. Левин .— М. : МЦНМО ; ФМОП, 2012 .— 144 с.

3. Комбинаторная оптимизация. Теория и алгоритмы [Текст] = Combinatorial Optimization. Theory and Algorithms : [учеб. пособие для вузов] / Б. Корте, Й. Фиген ; пер. с англ. М. А. Бабенко .— М. : МЦНМО, 2015 .— 720 с.

Additional literature

1. Линейная алгебра [Текст] : учебник для вузов / В. А. Ильин, Э. Г. Позняк .— 3 - е изд., доп. — М. : Наука, 1984 .— 295 с.

7. List of web resources that are necessary for the course (training module) mastering

Not used

8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)

Not used

9. Guidelines for students to master the course

1. It is recommended to successfully pass test papers, as this simplifies the final certification in the subject.
2. To prepare for the final certification in the subject, it is best to use the lecture materials.

SUPPLEMENT

Assessment funds for course (training module)

major:	Applied Mathematics and Informatics
specialization:	Advanced Methods of Modern Combinatorics/Продвинутые методы современной комбинаторики
	Phystech School of Applied Mathematics and Informatics
	Chair of Discrete Mathematics
term:	2
qualification:	Master

Semester, form of interim assessment: 3 (fall) - Exam

Author:	A.B. Daynyak, candidate of physics and mathematical sciences, associate professor

1. Competencies formed during the process of studying the course

Code and the name of the competence	Competency indicators
UC-1 Use a systematic approach to critically analyze a problem, and develop an action plan	UC-1.1 Systematically analyze the problem situation, identify its components and the relations between them
	UC-1.3 Develop a step-by-step strategy for achieving a goal, foresee the result of each step, evaluate the overall impact on the planned activity and its participants
UC-2 Able to manage a project through all stages of its life cycle	UC-2.1 Set an objective within a defined scientific problem; formulate the agenda, relevance, significance (scientific, practical, methodological or other depending on the project type), forecast the expected results and possible areas of their application
	UC-2.2 Forecast the project outcomes, plan necessary steps to achieve the outcomes, chart the project schedule and monitoring plan
	UC-2.3 Organize and coordinate the work of project stakeholders, provide the team with necessary resources
UC-6 Determine priorities and ways to improve performance through self-assessment	UC-6.1 Achieve personal growth and professional development, determine priorities and ways to improve performance
Gen.Pro.C-1 Address current challenges in fundamental and applied mathematics	Gen.Pro.C-1.2 Consolidate and critically assess professional experience and research findings
	Gen.Pro.C-1.1 Apply fundamental scientific knowledge, new scientific principles, and research methods in applied mathematics and computer science
	Gen.Pro.C-1.3 Understand interdisciplinary relations in applied mathematics and computer science and apply them in professional tasks
Gen.Pro.C-2 Iimprove upon and implement new mathematical methods in applied problem solving	Gen.Pro.C-2.2 Assess the relevance and practical importance of applied mathematical research in professional settings
Gen.Pro.C-4 Combine and adapt current information and communications technologies (ICTs) to meet professional challenges	Gen.Pro.C-4.1 Use ICTs to search and analyze professional information, highlight, structure, format, and present it in the form of analytical reviews with sound conclusions and recommendations

2. Competency assessment indicators

As a result of studying the course the student should:

know:

fundamental concepts, laws, theories of a part of discrete mathematics;
modern problems of the corresponding sections of discrete mathematics;
concepts, axioms, methods of proofs and proofs of the main theorems in the sections included in the basic part of the cycle;
basic properties of the corresponding mathematical objects;
analytical and numerical approaches and methods for solving typical applied problems of discrete mathematics.

be able to:

understand the task at hand;
use your knowledge to solve fundamental and applied problems;
evaluate the correctness of the problem setting;
strictly prove or disprove the statement;
independently find algorithms for solving problems, including non-standard ones, and analyze them;
independently see the consequences of the results obtained;
Accurately present mathematical knowledge in the field orally and in writing.

master:

skills of mastering a large amount of information and solving problems (including complex ones);
skills of independent work and mastering new disciplines;
culture of setting, analyzing and solving mathematical and applied problems that require the use of mathematical approaches and methods for their solution;
the subject language of discrete mathematics and the skills of competently describing the solution of problems and presenting the results obtained.

3. List of typical control tasks used to evaluate knowledge and skills

current control takes place in the form of a survey.

4. Evaluation criteria

1. Distinctive features of discrete optimization problems. An overview of the formulations of classical discrete optimization problems: set coverage, vertex coverage, shortest path, minimal spanning tree, matching problems, assignment problem, scheduling problems, packing problems (bin packing, knapsack), flow problems (largest flow, flow least cost, multi-product flows), transport problem (Hitchcock problem), traveling salesman problem.
2. Local search as a broad general approach to solving discrete optimization problems. Neighborhood systems. Example of a Neighborhood System in TSP: A tradeoff between neighborhood strength and size. An example in which the 2-neighborhood does not allow reaching the global optimum. Kernighan-Lin heuristic: local variable depth search. Local search add-ons: simulated annealing and taboo search.
3. Non-existence of a polynomially observable exact system of neighborhoods in the TSP problem (under the assumption $P \neq NP$). Local greedy heuristics in the TSP problem that do not explicitly fit into the local search paradigm (do not go from loop to loop, but build a loop from scratch). Performance indicators of heuristic (approximate) algorithms: appproximation ratio and domination number. Nearest neighbor algorithm: idea, theorem, that the approximation ratio is estimated from above O (log # of vertices).
4. Discrete linear subset problem (DLS problem). TSP and MST problems as special cases of DLS minimization problems; transition to maximization. Hereditary systems. Bases and cycles. Rank and lower rank of a set, rank spread. Matroids: equivalent definitions, examples. Evaluation of the performance of a greedy algorithm on a hereditary system through its rank spread. Corollary on the correctness of the greedy algorithm for constructing the shortest spanning tree. Estimation of the rank spread through the limitation on the number of cycles. Submodularity of the rank function of the matroid. Enumeration of matroids. Estimation of the number of cycles for a hereditary system in terms of the number of matroids in the intersection. Probability of uniqueness of a solution to the DLS problem with a random choice of weights: the isolation lemma.
5. Algorithms of Prim and Boruvka for solving the MST problem: examples, implementation (without using heaps). Prim's algorithm using Fibonacci heaps.
6. Problems of the distribution of a discrete homogeneous resource: the problem of discrete maximin, maximization of the sum of concave functions. Optimality criteria (Germeier's principle of equalization, Gross criterion). Algorithm "double binary search". Optimizing the product for a fixed amount.
7. Reminder of basic concepts from linear programming. Problem in standard and canonical forms. Transition from inequalities to equalities and vice versa. Geometry of the problem: a simplex algorithm as a local search along the vertices of a polyhedron.
8. An example of a polytope on which, under certain conditions, a simplex algorithm may require exponentially many steps: the Klee – Minty theorem. An upper bound on the number of steps in the "lucky simplex method": the Kalai – Kleitman theorem on the diameter of a polytope graph.
9. Concept of smoothed analysis: the average between analysis on random inputs and analysis of the worst case. The Spielman-Teng theorem on the simplex method.
10. Duality in linear programming: solution of a dual problem as a certificate of optimality of a solution to a direct problem. Fourier-Motzkin elimination. Farkas' lemma: the existence of a certificate of undecidability of a system of linear inequalities. Derivation of the strong duality theorem from the Farkas lemma.

11. Statement of the TSP problem in terms of the CLP. Miller – Tucker – Zemlin conditions (polynomial number of inequalities in the TSP). Remark "on the non-catastrophic nature of the exponential number of constraints in LP problems".

12. Simple "combinatorial" algorithm for the vertex coverage (VP) problem with approximation ratio = 2. Statement of the weighted vertex coverage (VP) problem in terms of linear integer programming. Algorithm for solving the GDP problem of the form "we solve the LP problem → round off"; the statement that approximation ratio = 2 is achieved. Formulation of the dual problem for the GDP problem: potentials on the edges. "Combinatorial" (without using LP) algorithm for solving the GDP problem based on duality; proof that approximation ratio = 2 for this algorithm.

13. General covering problem (equivalent to the covering problem for sets). Formulation in terms of matrices. Statement in the form of an ILP, formulation of a dual problem. The theorem that the size / weight of a greedy cover is at most ln k times the size / weight of the optimal (where k is the maximum number of ones in a line). Achievability (in order) of this estimate. Estimation of the weight of a greedy cover in terms of the weight of the optimal cover with limited weights of individual rows.

14. The problem of constructing a matching of maximum cardinality in an arbitrary graph. Augmenting paths (the statement that matching is non-maximal ⇔ is an augmenting path). The problem with finding augmentation paths in the absence of dicotyledonous: flowers. Flower shrinkage statements. Edmonds algorithm.

15. Modifications of Dijkstra's algorithm for a quick practical solution of the shortest path problem: "two-way" Dijkstra's algorithm, the use of landmarks (in the case of a triangle inequality).

16. Floyd-Warshall's algorithm for finding the shortest paths and cycles of negative weight.

17. The problem of the flow of the minimum cost (and a given value). Two algorithms: gradual minimization of the flow cost at a constant value; an increment in value due to the smallest possible increment in value.

18. "Biological" metaheuristics: genetic algorithms, ant colony algorithms. Illustration on tasks shortest path and TSP.

19. The method of branches and bounds.

20. Problems of exhaustive enumeration of complex discrete objects. Reed's approach: ordered enumeration. Avis-Fukuda local search inversion method.

21. Time Optimization: Online Optimization. Buy / rent task. "The task of the secretary." Analysis of the LRU (Least Recently Used) caching algorithm.

22. About the concept of reoptimization. Reoptimization in the metric traveling salesman problem with increasing weight of one edge: NP-hardness of the exact solution, an approximate algorithm with an approximation exponent of 7/5 (which is better than the Christofides algorithm).

Ticket 1:

1. Local search as a broad general approach to solving discrete optimization problems. Neighborhood systems. Example of a Neighborhood System in TSP: A tradeoff between neighborhood strength and size.An example in which the 2-neighborhood does not allow reaching the global optimum. Kernighan-Lin heuristic: local variable depth search. Local search add-ons: simulated annealing and taboo search.

2. The method of branches and bounds.

Ticket 2:

1. Discrete linear subset problem (DLS problem). TSP and MST problems as special cases of DLS minimization problems; transition to maximization. Hereditary systems. Bases and cycles. Rank and lower rank of a set, rank spread. Matroids: equivalent definitions, examples. Evaluation of the performance of a greedy algorithm on a hereditary system through its rank spread. Corollary on the correctness of the greedy algorithm for constructing the shortest spanning tree.Estimation of the rank spread through the limitation on the number of cycles. Submodularity of the rank function of the matroid. Enumeration of matroids. Estimation of the number of cycles for a hereditary system in terms of the number of matroids in the intersection. Probability of uniqueness of a solution to the DLS problem with a random choice of weights: the isolation lemma.

2. Statement of the TSP problem in terms of the CLP. Miller – Tucker – Zemlin conditions (polynomial number of inequalities in the TSP). Remark "on the non-catastrophic nature of the exponential number of constraints in LP problems".

- the mark "excellent (10)" is given to a student who has shown comprehensive, systematized, in-depth knowledge of the curriculum of the discipline and the ability to confidently apply them in practice when solving specific problems, free and correct justification of the decisions made;

- the mark "excellent (9)" is given to a student who has shown comprehensive, systematized, in-depth knowledge of the curriculum of the discipline and the ability to apply them in practice in solving specific problems, free and correct justification of the decisions made;
- the mark "excellent (8)" is given to a student who has shown a comprehensive, systematized, deep knowledge of the curriculum of the discipline and the ability to apply them in practice in solving specific problems, and the correct justification of the decisions made;
- the mark "good (7)" is given to a student if he knows the material well, expresses it competently and in essence, knows how to apply the knowledge gained in practice, but makes some inaccuracies in the answer or in solving problems;
-the mark "good (6)" is given to a student if he knows the material, presents it competently and to the point, knows how to apply the knowledge gained in practice, but makes some inaccuracies in the answer or in solving problems;
- the mark "good (5)" is given to a student if he knows the material and essentially expounds it, knows how to apply the knowledge gained in practice, but makes some inaccuracies in the answer or in solving problems;
- the mark "satisfactory (4)" is given to a student who has shown a fragmented, scattered nature of knowledge, insufficiently correct formulations of basic concepts, violation of the logical sequence in the presentation of the program material, but at the same time he owns the main sections of the curriculum necessary for further education and can apply the knowledge gained according to the sample in a standard situation;
- the mark "satisfactory (3)" is given to a student who has shown a fragmented, scattered nature of knowledge, insufficiently correct formulations of basic concepts, a violation of the logical sequence in the presentation of program material, but at the same time he knows fragmentarily the main sections of the curriculum necessary for further education and can apply the obtained knowledge by model in a standard situation;
- the mark "unsatisfactory (2)" is given to a student who does not know most of the main content of the curriculum of the discipline, makes gross errors in the formulation of the basic concepts of the discipline and is unable to use the knowledge gained in solving typical practical problems;
- the mark "unsatisfactory (1)" is given to a student who does not know the formulations of the basic concepts of the discipline.

5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience

During the exam, students can use the discipline program.