Federal State Autonomous Educational Institution of Higher Education "Moscow Institute of Physics and Technology (National Research University)"

APPROVED
Head of the Phystech School of Applied Mathematics and Informatics
A.M. Raygorodskiy

Work program of the course (training module)

course:	Advanced Graph Theory/Современная теория графов
major:	Applied Mathematics and Informatics
specialization:	Contemporary Combinatorics/Современная комбинаторика
	“Pusk” Online and Supplementary Education Centre
	Chair of Discrete Mathematics
term:	1
qualification:	Master

Semester, form of interim assessment: 2 (spring) - Exam

Academic hours: 60 AH in total, including:
lectures: 30 AH.
seminars: 30 AH.
laboratory practical: 0 AH.

Independent work: 45 AH.

Exam preparation: 30 AH.

In total: 135 AH, credits in total: 3

Author of the program: A.B. Daynyak, candidate of physics and mathematical sciences, associate professor, associate professor

The program was discussed at the Chair of Discrete Mathematics 05.03.2020

Annotation

While studying the above mentioned topics we'll see that many graph invariants that seem to measure absolutely different properties of graphs turn to be subtly connected, and thus we'll see the beautiful integrity of Graph theory as a mature discipline, that might seem to the beginner as just some collection of random results.

1. Study objective

Purpose of the course

1. Familiarity with standard proof techniques in graph theory.
2. Familiarity with classical graph invariants and standard research questions.
3. Familiarity with classical and modern results in graph theory.

Tasks of the course

- students mastering basic knowledge (concepts, concepts, methods and models) in graph theory;
- acquisition of theoretical knowledge and practical skills in graph theory;
- In an open way and helping students conduct theoretical research in graph theory.

2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

Code and the name of the competence	Competency indicators
UC-4 Use modern communication tools in the academic and professional field, including those in a foreign language	UC-4.4 Use modern ICT tools for academic and professional collaboration
Gen.Pro.C-2 Iimprove upon and implement new mathematical methods in applied problem solving	Gen.Pro.C-2.1 Assess the current state of mathematical research within professional settings
Gen.Pro.C-3 Develop mathematical models and conduct their analysis in the processes of professional problem-solving	Gen.Pro.C-3.2 Employ research methods to solve new problems, and apply knowledge from various science and technology fields
Pro.C-2 Understands and is able to apply modern mathematical apparatus and algorithms, the basic laws of natural science, modern programming languages and software; operating systems and networking technologies in research and applied activities	Pro.C-2.1 Demonstrate expert knowledge of research basics in the field of ICTs, philosophy and methodology of science, scientific research methods, and apply skills to use them

3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

- fundamental concepts, laws, graph theory;
- modern problems of the relevant sections of graph theory;
- concepts, axioms, methods of proof and proof of the main theorems in the sections included in the basic part of the graph theory cycle;
- basic properties of the corresponding mathematical objects;
- analytical and numerical approaches and methods for solving typical applied problems of graph theory.

be able to:

- understand the task;
- use your knowledge to solve fundamental and applied problems;
- evaluate the correctness of the problem statements;
- strictly prove or disprove the statement;
- independently find algorithms for solving problems, including non-standard ones, and conduct their analysis;
- independently see the consequences of the results;
- accurately represent mathematical knowledge in topology orally and in writing.

master:

- skills of mastering a large amount of information and solving problems (including complex ones);
- skills of independent work and mastering new disciplines;
- the culture of the formulation, analysis and solution of mathematical and applied problems requiring the use of mathematical approaches and methods for their solution;
- the subject language of topology and the skills of competent description of problem solving and presentation of the results.

4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

4.1. The sections of the course (training module) and the complexity of the types of training sessions

№	Topic (section) of the course	Types of training sessions, including independent work			
		Lectures	Seminars	Laboratory practical	Independent work
1	Recall of basic notation and facts	4	4		6
2	Graph Factoring	4	4		6
3	Connectivity	4	4		6
4	Coloring	4	4		6
5	Embedding	6	6		6
6	Extremal problems	4	4		7
7	Traversals	4	4		8
AH in total		30	30		45
Exam preparation		30 AH.			
Total complexity		135 AH., credits in total 3			

4.2. Content of the course (training module), structured by topics (sections)

Semester: 2 (Spring)

1. Recall of basic notation and facts

We recall some elementary properties of graphs, many of which were introduced in the discrete structures courser the previous semester. We also introduce some convenient notation for local graph modifications
which will be extensively used throughout the course.

2. Graph Factoring

We study classical Hall’s theorem on the existence of perfect matchings in bipartite graphs. We prove this theorem using max-flow-min-cut theorem and also give a second proof using alternating chains. We then study graph coloring implications of Hall’s theorem and proceed to Tutte’s theorem on criterial conditions of
existence of perfect matchings in non-bipartite graphs. To finalise, we study different problems of splitting graph into complete bipartite graphs.

3. Connectivity

We introduce two notions of high connectivity, both of which generalise "standard" connectivity and prove the milestone Menger's theorem on the equivalence of these definitions, employing the max-flow-min-cut theorem. We then prove Mader's theorem on the existence of subgraph with high connectivity in a graph with high average degree. Here we illustrate graph saturation proof techniques. We then discuss properties of biconnected and triconnected graphs, recursive constructions. We also prove a beautiful result on cycles in graphs with high connectivity.

4. Coloring

We prove two classical theorems on graph coloring: Brooks' theorem on chromatic number (using Lovasz greedy coloring with special ordering of vertices) and Vizing's theorem on chromatic index. We also discuss properties of critical graphs, and in particular prove theorem on -constructible graphs. We prove some results on list coloring and connection of list chromatic number and ordinary chromatic number. In
the end we discuss a notion of perfect graphs and prove Weak perfect graph conjecture (proof due to Lovasz).

5. Embedding

We study planar graphs. We prove Kuratowski's and Wagner's planarity criteria. We also study properties of planar triangulations, in particular, prove that they are triconnected. We conclude with proving Tutte's barycentric embedding theorem and Lipton—Tarjan's theorem on planar graph separation.

6. Extremal problems

We prove the central result of extremal graph theory by Erdos, Stone and Simonovits on critical density of graphs with forbidden subgraphs.

7. Traversals

We prove several results (by Erdos, Chvatal et al.) on the existence of hamiltonian cycles in graphs.

5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)

A standard classroom.

6. List of the main and additional literature, that is necessary for the course (training module) mastering

Main literature

1. Дискретный анализ. Комбинаторика. Алгебра логики. Теория графов [Текст] : учеб. пособие для вузов / Ю. И. Журавлев, Ю. А. Флеров, О. С. Федько ; М-во образования и науки РФ, Моск. физ.-техн. ин-т (гос. ун-т .— М. : МФТИ, 2012 .— 248 с.
2. Теория графов [Текст] : [учеб. пособие для вузов] / О. Оре ; пер. с англ. И. Н. Врублевской ; под ред. Н. Н. Воробьева .— 2-е изд., стереотип. — М. : Наука, 1980 .— 336 с.

Additional literature

1. Сборник задач по дискретному анализу. Комбинаторика. Элементы алгебры логики. Теория графов [Текст] : учеб. пособие для вузов / Ю. И. Журавлев [и др.] ; М-во образования Рос. Федерации, Моск. физ.-техн. ин-т (гос. ун-т) .— 2-е изд. — М. : МФТИ, 2000, 2004 .— 100 с.

7. List of web resources that are necessary for the course (training module) mastering

http://dm.fizteh.ru/

http://www.mccme.ru/~anromash/courses/expanders-notes-2014.pdf

8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)

Multimedia technologies can be employed during lectures and practical lessons, including presentations.

9. Guidelines for students to master the course

1. It is recommended to successfully pass the test papers, as this simplifies the final certification in the subject.
2. To prepare for the final certification in the subject, it is best to use the lecture materials.

SUPPLEMENT

Assessment funds for course (training module)

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term:	~~1~~
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Author:	A.B. Daynyak, candidate of physics and mathematical sciences, associate professor, associate professor

1. Competencies formed during the process of studying the course

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Pro.C-2 Understands and is able to apply modern mathematical apparatus and algorithms, the basic laws of natural science, modern programming languages and software; operating systems and networking technologies in research and applied activities	Pro.C-2.1 Demonstrate expert knowledge of research basics in the field of ICTs, philosophy and methodology of science, scientific research methods, and apply skills to use them

2. Competency assessment indicators

As a result of studying the course the student should:

know:

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3. List of typical control tasks used to evaluate knowledge and skills

Examples of home tasks

1. What is the maximum number of edges in a graph with 12 vertices with 3 connected components?

2. How many pairwise non-isomorphic 2-regular bipartite graphs on 16 vertices exist?

3. How many different 2-regular graphs on 16 vertices with fixed parts {x1, ..., x8} and {y1, ..., y8} exist? 4. What is the maximum number of vertices in a graph in which there is neither an independent set on three vertices nor odd cycles?

5. Find the smallest possible independence number of a graph on 6 vertices without triangles.

4. Evaluation criteria

Exam Questions List:

Properties of the extension, the relationship between them. Definition of regular expanders, evidence of existence. Applications: improved success in algorithms.

Dicotyledon expanders, existence. Applications: asymptotically good codes, data storage with ultrafast query.

Spectral theory of expanders. Algebraic expanders. Lower bound for a second eigenvalue. Existence theorems.

Mixing lemma. Edge extension theorem.

The relationship between combinatorial and algebraic expanders. Random walks on expanders. Applications

Wildcard product, tensor product, zigzag product of graphs. Spectral properties.

Recursive constructions of expanders. About the "explicit" assignment of graphs.

Counts of Ramanujan, design of Margulis.

Applications: Reinhold algorithm for checking graph connectivity.

Conductors of probability. Min entropy, its properties.

Zigzag construction for conductors.

Explicit construction of almost optimal dicotyledonous expanders: the use of spectral expanders and implicit constructions of size O (1).

Zemora codes. Encoding and decoding.

Codes on dicotyledonous expanders. Their coding and decoding.

Reliable data storage in untrusted cells. Reliable Boolean circuits.

Exam ticket examples.

Ticket number 1

1. Zigzag construction for conductors.
2. The relationship between combinatorial and algebraic expanders.

Ticket number 2

1. The mixing lemma. Edge extension theorem.
2. Recursive constructions of expanders. About the "explicit" assignment of graphs.

Assessment “excellent (10)” is given to a student who has displayed comprehensive, systematic and deep knowledge of the educational program material, has independently performed all the tasks stipulated by the program, has deeply studied the basic and additional literature recommended by the program, has been actively working in the classroom, and understands the basic scientific concepts on studied discipline, who showed creativity and scientific approach in understanding and presenting educational program material, whose answer is characterized by using rich and adequate terms, and by the consistent and logical presentation of the material;

Assessment “excellent (9)” is given to a student who has displayed comprehensive, systematic knowledge of the educational program material, has independently performed all the tasks provided by the program, has deeply mastered the basic literature and is familiar with the additional literature recommended by the program, has been actively working in the classroom, has shown the systematic nature of knowledge on discipline sufficient for further study, as well as the ability to amplify it on one’s own, whose answer is distinguished by the accuracy of the terms used, and the presentation of the material in it is consistent and logical;

Assessment “excellent (8)” is given to a student who has displayed complete knowledge of the educational program material, does not allow significant inaccuracies in his answer, has independently performed all the tasks stipulated by the program, studied the basic literature recommended by the program, worked actively in the classroom, showed systematic character of his knowledge of the discipline, which is sufficient for further study, as well as the ability to amplify it on his own;

Assessment “good (7)” is given to a student who has displayed a sufficiently complete knowledge of the educational program material, does not allow significant inaccuracies in the answer, has independently performed all the tasks provided by the program, studied the basic literature recommended by the program, worked actively in the classroom, showed systematic character of his knowledge of the discipline, which is sufficient for further study, as well as the ability to amplify it on his own;

Assessment “good (6)” is given to a student who has displayed a sufficiently complete knowledge of the educational program material, does not allow significant inaccuracies in his answer, has independently carried out the main tasks stipulated by the program, studied the basic literature recommended by the program, showed systematic character of his knowledge of the discipline, which is sufficient for further study;

Assessment “good (5)” is given to a student who has displayed knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, who while not being sufficiently active in the classroom, has nevertheless independently carried out the main tasks stipulated by the program, mastered the basic literature recommended by the program, made some errors in their implementation and in his answer during the test, but has the necessary knowledge for correcting these errors by himself;

Assessment “satisfactory (4)” is given to a student who has discovered knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, who while not being sufficiently active in the classroom, has nevertheless independently carried out the main tasks stipulated by the program, learned the main literature but allowed some errors in their implementation and in his answer during the test, but has the necessary knowledge for correcting these errors under the guidance of a teacher;

Assessment “satisfactory (3)” is given to a student who has displayed knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, not showed activity in the classroom, independently fulfilled the main tasks envisaged by the program, but allowed errors in their implementation and in the answer during the test, but possessing necessary knowledge for elimination under the guidance of the teacher of the most essential errors;

Assessment “unsatisfactory (2)” is given to a student who showed gaps in knowledge or lack of knowledge on a significant part of the basic educational program material, who has not performed independently the main tasks demanded by the program, made fundamental errors in the fulfillment of the tasks stipulated by the program, who is not able to continue his studies or start professional activities without additional training in the discipline in question;

Assessment “unsatisfactory (1)” is given to a student when there is no answer (refusal to answer), or when the submitted answer does not correspond at all to the essence of the questions contained in the task.

5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience

During examination the student are allowed to use the program of the discipline.