Federal State Autonomous Educational Institution of Higher Education "Moscow Institute of Physics and Technology (National Research University)"

APPROVED
Head of the Phystech School of Applied Mathematics and Informatics
A.M. Raygorodskiy

Work program of the course (training module)

course:	Introduction to Graph Theory/Введение в теорию графов
major:	Applied Mathematics and Informatics
specialization:	Contemporary Combinatorics/Современная комбинаторика
	“Pusk” Online and Supplementary Education Centre
	Chair of Discrete Mathematics
term:	1
qualification:	Master

Semester, form of interim assessment: 1 (fall) - Grading test

Academic hours: 60 AH in total, including:
lectures: 30 AH.
seminars: 30 AH.
laboratory practical: 0 AH.

Independent work: 30 AH.

In total: 90 AH, credits in total: 2

Author of the program: A.B. Daynyak, candidate of physics and mathematical sciences, associate professor, associate professor

The program was discussed at the Chair of Discrete Mathematics 05.03.2020

Annotation

The course is intended for masters of mathematics interested in modern discrete mathematics and big data representations using graphs. In addition to the main objects of graph theory, the course includes some of the most important chapters of graph theory - criteria for Hamiltonianness, Turan's theorem, Ramsey's problem.

1. Study objective

Purpose of the course

mastering the main modern methods of graph theory

Tasks of the course

- students mastering basic knowledge (concepts, concepts, methods and models) in graph theory;
- acquisition of theoretical knowledge and practical skills in graph theory;
- providing advice and assistance to students in conducting their own theoretical research in graph theory.

2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

Code and the name of the competence	Competency indicators
UC-6 Determine priorities and ways to improve performance through self-assessment	UC-6.1 Achieve personal growth and professional development, determine priorities and ways to improve performance
Gen.Pro.C-1 Address current challenges in fundamental and applied mathematics	Gen.Pro.C-1.1 Apply fundamental scientific knowledge, new scientific principles, and research methods in applied mathematics and computer science
Pro.C-2 Understands and is able to apply modern mathematical apparatus and algorithms, the basic laws of natural science, modern programming languages and software; operating systems and networking technologies in research and applied activities	Pro.C-2.1 Demonstrate expert knowledge of research basics in the field of ICTs, philosophy and methodology of science, scientific research methods, and apply skills to use them

3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

- fundamental concepts, laws, graph theory;
- modern problems of the relevant sections of graph theory;
- concepts, axioms, methods of proof and proof of the main theorems in the sections included in the basic part of the graph theory cycle;
- basic properties of the corresponding mathematical objects;
- analytical and numerical approaches and methods for solving typical applied problems of graph theory.

be able to:

- understand the task;
- use your knowledge to solve fundamental and applied problems;
- evaluate the correctness of the problem statements;
- strictly prove or disprove the statement;
- independently find algorithms for solving problems, including non-standard ones, and conduct their analysis;
- independently see the consequences of the results;
- accurately represent mathematical knowledge in topology orally and in writing.

master:

 skills of mastering a large amount of information and solving problems (including complex ones);
 skills of independent work and mastering new disciplines;
 the culture of the formulation, analysis and solution of mathematical and applied problems requiring the use of mathematical approaches and methods for their solution;
 the subject language of topology and the skills of competent description of problem solving and presentation of the results.

4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

4.1. The sections of the course (training module) and the complexity of the types of training sessions

№	Topic (section) of the course	Types of training sessions, including independent work			
		Lectures	Seminars	Laboratory practical	Independent work
1	Definitions of basic graphs-theoretic notions	4	4		4
2	Counting objects up to equivalence. Example	4	4		4
3	Planar graphs	4	4		4
4	Graph colorings. Definitions and example applications	4	4		4
5	Ford—Fulkerson's "Max-Flow-Min-Cut" theorem on flows in networks	4	4		4
6	More on connectivity of graphs. Menger's theorem. Proof of Menger's theorem via Ford—Fulkerson's	4	4		4
7	BFS: Lovasz' proof of Brooks' theorem via greedy coloring; fundamental cycles in graphs	6	6		6
AH in total		30	30		30
Exam preparation		0 AH.			
Total complexity		90 AH., credits in total 2			

4.2. Content of the course (training module), structured by topics (sections)

Semester: 1 (Fall)

1. Definitions of basic graphs-theoretic notions

Graphs/digraphs/multigraphs, adjacency and incidence, degrees and neighbourhoods, connectivity, paths, cycles. Trees and their basic properties. Examples of graph-theoretic models in the real world

2. Counting objects up to equivalence. Example

The number of cycles in graphs. Graph isomorphism. Estimating the number of classes of isomorphism of trees. Cayley's formula for the number of trees; a proof by constructing a bijection with integer tuples, one or two other proofs as time permits. Handshake lemma and variations, two-coloring a uniform hypergraph, finding a large bipartite subgraph in an arbitrary graph

3. Planar graphs

Triangulations, the maximal number of edges in a planar graph. Euler's formula. Better bound for the number of edges in a triangle-free graph. Proving non-planarity of certain graphs. Estimating the number of crossings in a drawing of a complete graph. A forbidden minor/subgraph approach to characterizing planar graphs (Wagner and Kuratowski criteria).

4. Graph colorings. Definitions and example applications

Graph colorings. Definitions and example applications. Greedy coloring. Chromatic number. Simple bounds for the chromatic number. Coloring planar graphs. Digraphs. Some similarities and differences between directed and undirected graphs. Tournaments. Kernels.

5. Ford—Fulkerson's "Max-Flow-Min-Cut" theorem on flows in networks

Cuts and flows as an example of "combinatorial duality". Combinatorial application of integer flows: Hall's theorem on matchings in bipartite graphs.

6. More on connectivity of graphs. Menger's theorem. Proof of Menger's theorem via Ford—Fulkerson's

Biconnectivity. Bridges, cut-vertices and blocks. Importance of blocks in planarity testing, coloring, shortest paths. Two notions of k-connectivity, Graph traversals. Breadth-first search (BFS) and depth-first search (DFS).

7. BFS: Lovasz' proof of Brooks' theorem via greedy coloring; fundamental cycles in graphs

Eulerian and Hamiltonian circuits. The TSP problem. Euler's criterion: inductive and algorithmic proofs. Dirac's sufficient conditions for the existence of a hamiltonian circuit. An approximate solution to the TSP problem on graphs with triangle inequality using any Eulerian walk or DFS.

5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)

A standard classroom.

6. List of the main and additional literature, that is necessary for the course (training module) mastering

Main literature

1. Теория графов [Текст] = Graph Theory : [учеб. пособие для вузов] / Ф. Харари ; пер. с англ. В. П. Козырева ; под ред. Г. П. Гаврилова .— М. : Мир, 1973 .— 300 с.

2. Дискретная математика [Текст] : графы, матроиды, алгоритмы : учеб. пособие для вузов / М. О. Асанов, В. А. Баранский, В. В. Расин .— Ижевск : НИЦ Регулярная и хаотическая динамика, 2001 .— 288 с.

Additional literature

Графы. Алгоритмы на языке С [Текст] : учеб. пособие для студентов 1 курса МФТИ / В. В. Прут ; М-во образования и науки РФ, Моск. физ.-техн. ин-т (гос. ун-т) .— М. : МФТИ, 2017 .— 213 с. + pdf-версия. - Библиогр.: с. 207-209. - 200 экз. - ISBN 978-5-7417-0633-6 .— Полный текст (Доступ из сети МФТИ).

7. List of web resources that are necessary for the course (training module) mastering

http://dm.fizteh.ru

8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)

Multimedia technologies can be employed during lectures and practical lessons, including presentations.

9. Guidelines for students to master the course

1. It is recommended to successfully pass the test papers, as this simplifies the final certification in the subject.
2. To prepare for the final certification in the subject, it is best to use the lecture materials.

SUPPLEMENT

Assessment funds for course (training module)

major:	Applied Mathematics and Informatics
specialization:	Contemporary Combinatorics/Современная комбинаторика
	“Pusk” Online and Supplementary Education Centre
	Chair of Discrete Mathematics
term:	1
qualification:	Master

Semester, form of interim assessment: 1 (fall) - Grading test

Author: A.B. Daynyak, candidate of physics and mathematical sciences, associate professor, associate professor

1. Competencies formed during the process of studying the course

Code and the name of the competence	Competency indicators
UC-6 Determine priorities and ways to improve performance through self-assessment	UC-6.1 Achieve personal growth and professional development, determine priorities and ways to improve performance
Gen.Pro.C-1 Address current challenges in fundamental and applied mathematics	Gen.Pro.C-1.1 Apply fundamental scientific knowledge, new scientific principles, and research methods in applied mathematics and computer science
Pro.C-2 Understands and is able to apply modern mathematical apparatus and algorithms, the basic laws of natural science, modern programming languages and software; operating systems and networking technologies in research and applied activities	Pro.C-2.1 Demonstrate expert knowledge of research basics in the field of ICTs, philosophy and methodology of science, scientific research methods, and apply skills to use them

2. Competency assessment indicators

As a result of studying the course the student should:

know:

- fundamental concepts, laws, graph theory;
- modern problems of the relevant sections of graph theory;
- concepts, axioms, methods of proof and proof of the main theorems in the sections included in the basic part of the graph theory cycle;
- basic properties of the corresponding mathematical objects;
- analytical and numerical approaches and methods for solving typical applied problems of graph theory.

be able to:

- understand the task;
- use your knowledge to solve fundamental and applied problems;
- evaluate the correctness of the problem statements;
- strictly prove or disprove the statement;
- independently find algorithms for solving problems, including non-standard ones, and conduct their analysis;
- independently see the consequences of the results;
- accurately represent mathematical knowledge in topology orally and in writing.

master:

- skills of mastering a large amount of information and solving problems (including complex ones);
- skills of independent work and mastering new disciplines;
- the culture of the formulation, analysis and solution of mathematical and applied problems requiring the use of mathematical approaches and methods for their solution;
- the subject language of topology and the skills of competent description of problem solving and presentation of the results.

3. List of typical control tasks used to evaluate knowledge and skills

Examples of home tasks

1. What is the maximum number of edges in a graph with 12 vertices with 3 connected components?
2. How many pairwise non-isomorphic 2-regular bipartite graphs on 16 vertices exist?
3. How many different 2-regular graphs on 16 vertices with fixed parts {x1, ..., x8} and {y1, ..., y8} exist? 4. What is the maximum number of vertices in a graph in which there is neither an independent set on three vertices nor odd cycles?
5. Find the smallest possible independence number of a graph on 6 vertices without triangles.

4. Evaluation criteria

1. Definitions of basic graphs-theoretic notions, graphs/digraphs/multigraphs.
2. Adjacency and incidence, degrees and neighbourhoods, connectivity, paths, cycles. Trees and their basic properties.
3. Counting objects up to equivalence.
4. The number of cycles in graphs. Graph isomorphism.
5. Estimating the number of classes of isomorphism of trees.
6. Cayley's formula for the number of trees; a proof by constructing a bijection with integer tuples, one or two other proofs as time permits.
7. Handshake lemma and variations, two-coloring a uniform hypergraph, finding a large bipartite subgraph in an arbitrary graph.
8. Planar graphs. Triangulations, the maximal number of edges in a planar graph. Euler's formula.
9. Better bound for the number of edges in a triangle-free graph. Proving non-planarity of certain graphs. Estimating the number of crossings in a drawing of a complete graph. A forbidden minor/subgraph approach to characterizing planar graphs (Wagner and Kuratowski criteria).
10. Graph colorings. Definitions and example applications.
11. Greedy coloring. Chromatic number. Simple bounds for the chromatic number. Coloring planar graphs.
12. Digraphs. Some similarities and differences between directed and undirected graphs.
13. Tournaments. Kernels.
14. Ford—Fulkerson's "Max-Flow-Min-Cut" theorem on flows in networks.
15. More on connectivity of graphs. Biconnectivity. Bridges, cut-vertices and blocks.
16. Importance of blocks in planarity testing, coloring, shortest paths. Two notions of k-connectivity,
17. Menger's theorem. Proof of Menger's theorem via Ford—Fulkerson's.
18. Graph traversals. Breadth-first search (BFS) and depth-first search (DFS). Applications of.
19. Lovasz' proof of Brooks' theorem via greedy coloring; fundamental cycles in graphs.
20. Eulerian and Hamiltonian circuits.
21. The TSP problem. Euler's criterion: inductive and algorithmic proofs. Dirac's sufficient conditions for the existence of a hamiltonian circuit.
22. An approximate solution to the TSP problem on graphs with triangle inequality using any Eulerian walk or DFS.

Assessment "excellent (10)" is given to a student who has displayed comprehensive, systematic and deep knowledge of the educational program material, has independently performed all the tasks stipulated by the program, has deeply studied the basic and additional literature recommended by the program, has been actively working in the classroom, and understands the basic scientific concepts on studied discipline, who showed creativity and scientific approach in understanding and presenting educational program material, whose answer is characterized by using rich and adequate terms, and by the consistent and logical presentation of the material;

Assessment "excellent (9)" is given to a student who has displayed comprehensive, systematic knowledge of the educational program material, has independently performed all the tasks provided by the program, has deeply mastered the basic literature and is familiar with the additional literature recommended by the program, has been actively working in the classroom, has shown the systematic nature of knowledge on discipline sufficient for further study, as well as the ability to amplify it on one's own, whose answer is distinguished by the accuracy of the terms used, and the presentation of the material in it is consistent and logical;

Assessment "excellent (8)" is given to a student who has displayed complete knowledge of the educational program material, does not allow significant inaccuracies in his answer, has independently performed all the tasks stipulated by the program, studied the basic literature recommended by the program, worked actively in the classroom, showed systematic character of his knowledge of the discipline, which is sufficient for further study, as well as the ability to amplify it on his own;

Assessment "good (7)" is given to a student who has displayed a sufficiently complete knowledge of the educational program material, does not allow significant inaccuracies in the answer, has independently performed all the tasks provided by the program, studied the basic literature recommended by the program, worked actively in the classroom, showed systematic character of his knowledge of the discipline, which is sufficient for further study, as well as the ability to amplify it on his own;

Assessment “good (6)” is given to a student who has displayed a sufficiently complete knowledge of the educational program material, does not allow significant inaccuracies in his answer, has independently carried out the main tasks stipulated by the program, studied the basic literature recommended by the program, showed systematic character of his knowledge of the discipline, which is sufficient for further study;

Assessment “good (5)” is given to a student who has displayed knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, who while not being sufficiently active in the classroom, has nevertheless independently carried out the main tasks stipulated by the program, mastered the basic literature recommended by the program, made some errors in their implementation and in his answer during the test, but has the necessary knowledge for correcting these errors by himself;

Assessment “satisfactory (4)” is given to a student who has discovered knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, who while not being sufficiently active in the classroom, has nevertheless independently carried out the main tasks stipulated by the program, learned the main literature but allowed some errors in their implementation and in his answer during the test, but has the necessary knowledge for correcting these errors under the guidance of a teacher;

Assessment “satisfactory (3)” is given to a student who has displayed knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, not showed activity in the classroom, independently fulfilled the main tasks envisaged by the program, but allowed errors in their implementation and in the answer during the test, but possessing necessary knowledge for elimination under the guidance of the teacher of the most essential errors;

Assessment “unsatisfactory (2)” is given to a student who showed gaps in knowledge or lack of knowledge on a significant part of the basic educational program material, who has not performed independently the main tasks demanded by the program, made fundamental errors in the fulfillment of the tasks stipulated by the program, who is not able to continue his studies or start professional activities without additional training in the discipline in question;

Assessment “unsatisfactory (1)” is given to a student when there is no answer (refusal to answer), or when the submitted answer does not correspond at all to the essence of the questions contained in the task.

5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience

During examination the student are allowed to use the program of the discipline.