Federal State Autonomous Educational Institution of Higher Education "Moscow Institute of Physics and Technology (National Research University)"

APPROVED
Head of the Phystech School of Applied Mathematics and Informatics
A.M. Raygorodskiy

Work program of the course (training module)

course:	Algorithmic Game Theory/Алгоритмическая теория игр
major:	Applied Mathematics and Informatics
specialization:	Contemporary Combinatorics/Современная комбинаторика
	"Pusk" Online and Supplementary Education Centre
	Chair of Discrete Mathematics
term:	2
qualification:	Master

Semester, form of interim assessment: 3 (fall) - Grading test

Academic hours: 45 AH in total, including:
lectures: 15 AH.
seminars: 30 AH.
laboratory practical: 0 AH.

Independent work: 45 AH.

In total: 90 AH, credits in total: 2

Author of the program: D.G. Ilinskiy, candidate of economic sciences, associate professor

The program was discussed at the Chair of Discrete Mathematics 05.03.2021

Annotation

The course is a set of different subjects that lie at the intersection of game theory and computer science. The plots are to a small extent tied to each other, so the content can vary widely depending on the existing knowledge and interests of the listeners. The leitmotif of the course: if we assume that the game-theoretic model describes reality to some extent, then we must be able to at least calculate its outcome.

1. Study objective

Purpose of the course

To acquaint students with the basic concepts and results of non-cooperative and cooperative game theory. Central to the course is the concept of Nash equilibrium, sequential equilibrium, and the concept of the core in co-op games with side payments.

Tasks of the course

- mastering by students of basic knowledge (concepts, concepts, methods and models) in game theory;
- acquisition of theoretical knowledge and practical skills in game theory;
- providing advice and assistance to students in their own theoretical research in game theory.

2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

Code and the name of the competence	Competency indicators
UC-1 Use a systematic approach to critically analyze a problem, and develop an action plan	UC-1.2 Search for solutions by using available sources
Gen.Pro.C-1 Address current challenges in fundamental and applied mathematics	Gen.Pro.C-1.3 Understand interdisciplinary relations in applied mathematics and computer science and apply them in professional tasks
Gen.Pro.C-2 Iimprove upon and implement new mathematical methods in applied problem solving	Gen.Pro.C-2.3 Understand professional terminology used in modern scientific and technical literature and present scientific results in oral and written form
Pro.C-1 Become part of a professional community and conduct local research under scientific guidance using methods specific to a particular professional setting	Pro.C-1.2 Understand the verification process of software models used to solve related scientific problems
Pro.C-2 Understands and is able to apply modern mathematical apparatus and algorithms, the basic laws of natural science, modern programming languages and software; operating systems and networking technologies in research and applied activities	Pro.C-2.2 Demonstrate practical experience of applying methods and digital signal processing algorithms, using the Internet, abstracting, referencing, searching for bibliographic sources, and working with scientific sources

3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

- fundamental concepts, laws, game theory;
- modern problems of the relevant sections of game theory;
- concepts, axioms, methods of proofs and proofs of the main theorems in the sections included in the basic part of the game theory cycle;
- basic properties of the corresponding mathematical objects;
- analytical and numerical approaches and methods for solving typical applied problems of game theory.

be able to:

 understand the task at hand;
 use your knowledge to solve fundamental and applied problems;
 evaluate the correctness of the problem setting;
 strictly prove or disprove the statement;
 independently find algorithms for solving problems, including non-standard ones, and analyze them;
 independently see the consequences of the results obtained;
 Accurately represent mathematical knowledge in topology both orally and in writing.

master:

 skills of mastering a large amount of information and solving problems (including complex ones);
 skills of independent work and mastering new disciplines;
 culture of formulation, analysis and solution of mathematical and applied problems that require the use of mathematical approaches and methods for their solution;
 the subject language of topology and the skills of competently describing the solution of problems and presenting the results obtained.

4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

4.1. The sections of the course (training module) and the complexity of the types of training sessions

№	Topic (section) of the course	Types of training sessions, including independent work			
		Lectures	Seminars	Laboratory practical	Independent work
1	Brower's theorem. Sperner's lemma. Kakutani's theorem	3	6		
2	Dominated strategies	3	6		
3	Definition of a game in normal form: strategy, player, utility	3	6		
4	Defining a mixed strategy	3	6		
5	Expanded form of the game	3	6		45
AH in total		15	30		45
Exam preparation		0 AH.			
Total complexity		90 AH., credits in total 2			

4.2. Content of the course (training module), structured by topics (sections)

Semester: 3 (Fall)

1. Brower's theorem. Sperner's lemma. Kakutani's theorem

Nash's theorem on the existence of equilibrium in mixed strategies.

2. Dominated strategies

Consistent elimination of heavily dominated strategies. Minimax and maximin. Zero-sum games. Saddle point

3. Definition of a game in normal form: strategy, player, utility

Nash equilibrium in pure
strategies. Examples. The Prisoner's Dilemma. Game "rock-paper-scissors".

4. Defining a mixed strategy

Nash Equilibrium in Mixed Strategies

5. Expanded form of the game

Equivalence with normal form. Equilibria perfect in subplay. Examples

5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)

Standard classroom.

6. List of the main and additional literature, that is necessary for the course (training module) mastering

Main literature

1. Лекции по теории игр и экономическому моделированию [Текст] : [учеб. пособие для вузов] / И. С. Меньшиков .— 2-е изд., испр. и доп. — М. : Контакт Плюс, 2010 .— 336 с.
2. Основы теории игр [Текст] : учеб. пособие для вузов / Л. В. Колобашкина .— М. : БИНОМ. Лаб. знаний, 2012 .— 164 с.

Additional literature

1. Теория игр [Текст] : учеб. пособие для вузов : рек. М-вом общ. и проф. образования РФ / Л. А. Петросян, Н. А. Зенкевич, Е. А. Семина .— М. : Высшая школа, 1998 .— 304 с.

7. List of web resources that are necessary for the course (training module) mastering

http://dm.fizteh.ru/

8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)

The class uses multimedia technologies, including the presentation of presentations.
In the process of independent work of students, it is possible to use software such as Mathcad, MATLAB, etc.

9. Guidelines for students to master the course

It is recommended to successfully pass test papers, as this simplifies the final certification in the subject.

SUPPLEMENT

Assessment funds for course (training module)

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Semester, form of interim assessment: 3 (fall) - Grading test

Author: D.G. Ilinskiy, candidate of economic sciences, associate professor

1. Competencies formed during the process of studying the course

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Pro.C-1 Become part of a professional community and conduct local research under scientific guidance using methods specific to a particular professional setting	Pro.C-1.2 Understand the verification process of software models used to solve related scientific problems
Pro.C-2 Understands and is able to apply modern mathematical apparatus and algorithms, the basic laws of natural science, modern programming languages and software; operating systems and networking technologies in research and applied activities	Pro.C-2.2 Demonstrate practical experience of applying methods and digital signal processing algorithms, using the Internet, abstracting, referencing, searching for bibliographic sources, and working with scientific sources

2. Competency assessment indicators

As a result of studying the course the student should:

know:

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- culture of formulation, analysis and solution of mathematical and applied problems that require the use of mathematical approaches and methods for their solution;
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3. List of typical control tasks used to evaluate knowledge and skills

Examples of tasks from homework

1.Use the Lemke-Howson algorithm to find the Nash equilibrium in mixed strategies in a 3 × 3 game.

2. Find all correlated equilibria in a 2 × 2 game. Draw the outcomes in the payment space of both players.

3. Find the balance and optimal flow in the transport network. Calculate the cost of anarchy.

Examples of tasks from the first part of the test

1. Give an example in which the total utility of two players in a correlated equilibrium is greater than in any mixed Nash equilibrium.

2. Think of a shared traffic routing problem where the price of anarchy is exactly a.

3. Some situation of the public project is described. Formulate what the Vickrey-Clark-Groves mechanism would look like for her.

Examples of tasks from the second part of the test

1. Prove that for an Euler graph the problem of finding one partition of edges into spanning trees for another such partition lies in the PPA.

(2) Consider the following traffic routing problem: three roads lead from point A to point B, on one the transport costs are equal to 1, on the other x, on the third kx. You need to transmit traffic of size 2. Plot the dependence of the price of anarchy on the parameter k.

3. Calculate the optimal reserve price in the auction for the given distribution of estimates.

4. Evaluation criteria

Test questions:

1. Definition of the game in normal form: strategy, player, utility. Nash equilibrium in pure strategies. Examples. The Prisoner's Dilemma. Game "rock-paper-scissors".

2. Definition of a mixed strategy. Nash equilibrium in mixed strategies.

3. Brouwer's theorem. Sperner's lemma. Kakutani's theorem. Nash's theorem on the existence of equilibrium in mixed strategies.

4. Dominated strategies. Consistent elimination of heavily dominated strategies. Minimax and maximin. Zero-sum games. Saddle point.

5. Expanded form of the game. Equivalence with normal form. Equilibria perfect in subplay. Examples.

6. Weak and strong sequential Nash equilibria. The beliefs of the players.

7. Introduction to the theory of cooperative games. Balance concept. Kernel concept. Shapley vector.

8. Dynamic games. Endless and endless repeatable games. The endless prisoner's dilemma. The principle of single deflection. People's theorem. Duels. Simultaneous truelles.

- the mark "excellent (10)" is given to a student who has shown comprehensive, systematized, in-depth knowledge of the curriculum of the discipline and the ability to confidently apply them in practice when solving specific problems, free and correct justification of the decisions

- the mark "excellent (9)" is given to a student who has shown comprehensive, systematized, in-depth knowledge of the curriculum of the discipline and the ability to apply them in practice in solving specific problems, free and correct justification of the decisions

- the mark "excellent (8)" is given to a student who has shown comprehensive systematized, deep knowledge of the curriculum of the discipline and the ability to apply them in practice in solving specific problems, and the correct justification of the decisions made

- the mark "good (7)" is given to a student if he firmly knows the material, expresses it competently and to the point, knows how to apply the knowledge gained in practice, but makes some inaccuracies in the answer or in solving problems;

- the mark "good (6)" is given to a student if he knows the material, presents it competently and in essence, knows how to apply the knowledge gained in practice, but makes some inaccuracies in the answer or in solving problems;

- the mark "good (5)" is given to a student if he knows the material, and essentially expounds it, knows how to apply the knowledge gained in practice, but makes some inaccuracies in the answer or in solving problems;

- the mark "satisfactory (4)" is given to a student who has shown a fragmented, scattered nature of knowledge, insufficiently correct formulations of basic concepts, a violation of the logical sequence in the presentation of the program material, but at the same time he owns the main sections of the curriculum necessary for further education and can apply the obtained knowledge by model in a standard situation;

- the mark "satisfactory (3)" is given to a student who has shown a fragmented, scattered nature of knowledge, insufficiently correct formulations of basic concepts, violation of the logical sequence in the presentation of program material, but at the same time he knows fragmentarily the main sections of the curriculum necessary for further education and can apply the knowledge gained by the model in a standard situation;

- the mark "unsatisfactory (2)" is given to a student who does not know most of the main content of the curriculum of the discipline, makes gross mistakes in the formulation of the basic concepts of the discipline and does not know how to use the knowledge gained in solving typical practical problems

- grade "unsatisfactory (1)" is given to a student who does not know the formulations of the basic concepts of the discipline.

5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience

During the differentiated test, students can use the discipline program.