Federal State Autonomous Educational Institution of Higher Education "Moscow Institute of Physics and Technology (National Research University)"

APPROVED
Head of the Phystech School of Applied Mathematics and Informatics
A.M. Raygorodskiy

Work program of the course (training module)

course:	Introduction to Combinatorics/Введение в комбинаторику
major:	Applied Mathematics and Informatics
specialization:	Contemporary Combinatorics/Современная комбинаторика
	"Pusk" Online and Supplementary Education Centre
	Chair of Discrete Mathematics
term:	1
qualification:	Master

Semester, form of interim assessment: 1 (fall) - Exam

Academic hours: 60 AH in total, including:
lectures: 30 AH.
seminars: 30 AH.
laboratory practical: 0 AH.

Independent work: 45 AH.

Exam preparation: 30 AH.

In total: 135 AH, credits in total: 3

Author of the program: D.G. Ilinskiy, candidate of economic sciences, associate professor

The program was discussed at the Chair of Discrete Mathematics 05.03.2020

Annotation

The course is devoted to the main modern methods of extreme combinatorics (EC): probabilistic method, linear-algebraic method, topological method.

First of all, the basic probabilistic method is introduced, as well as the method of the first moment and the method of alternation. The Erdos problem and the Ramsey number are considered.

We consider randomized algorithms, the method of the second moment and the general method of moments (the Beck – Spencer theorem, connectedness of random graphs, Azuma's inequality, martingales of edge and vertex types, Bollobash's theorem on the chromatic number of a random graph).

We discuss the properties of the quantity m (n, k, t) (the largest number of edges in a k-uniform hypergraph on n vertices for which no two edges intersect in t elements), the chromatic numbers of the spaces, and the Borsuk problem.

Then we consider the Erd –s – Ko – Rado theorem (the maximum number of edges in a 1-intersecting hypergraph) and the chromatic numbers of different types of graphs.

Finally, we discuss "trivial" lower and upper bounds, lower bounds using generalized systems of common representatives.

1. Study objective

Purpose of the course

studying the mathematical foundations of modern combinatorics, as well as preparing students for further independent work in the field of combinatorial problems of applied mathematics, physics and economics.

Tasks of the course

- study of the mathematical foundations of modern combinatorics;
- acquisition by students of theoretical knowledge in the field of combinatorial analysis of problems arising in practice;
- mastering the analytical and algebraic apparatus of discrete mathematics and gaining skills in working with basic discrete structures.

2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

Code and the name of the competence	Competency indicators
UC-6 Determine priorities and ways to improve performance through self-assessment	UC-6.1 Achieve personal growth and professional development, determine priorities and ways to improve performance
Gen.Pro.C-1 Address current challenges in fundamental and applied mathematics	Gen.Pro.C-1.1 Apply fundamental scientific knowledge, new scientific principles, and research methods in applied mathematics and computer science
Pro.C-2 Understands and is able to apply modern mathematical apparatus and algorithms, the basic laws of natural science, modern programming languages and software; operating systems and networking technologies in research and applied activities	Pro.C-2.1 Demonstrate expert knowledge of research basics in the field of ICTs, philosophy and methodology of science, scientific research methods, and apply skills to use them

3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

- fundamental concepts, laws, theories of combinatorial geometry;
- modern problems of the corresponding sections of combinatorial geometry;
- concepts, axioms, methods of proof and proof of the main theorems in the sections included in the basic part of the cycle;
- basic properties of the corresponding mathematical objects;
- analytical and numerical approaches and methods for solving typical applied problems of combinatorial geometry.

be able to:

- understand the task;
- use your knowledge to solve fundamental and applied problems of combinatorial geometry;
- evaluate the correctness of the problem statements;
- strictly prove or disprove the statement;
- independently find algorithms for solving problems, including non-standard ones, and conduct their analysis;
- independently see the consequences of the results;
- accurately represent mathematical knowledge in the field of complex computing in oral and written form.

master:

- skills of mastering a large amount of information and solving problems (including complex ones);
- skills of independent work and mastering new disciplines;
- the culture of the formulation, analysis and solution of mathematical and applied problems requiring the use of mathematical approaches and methods of combinatorial geometry for their solution;
- the subject language of complex calculations and the skills of competent description of problem solving and presentation of the results.

4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

4.1. The sections of the course (training module) and the complexity of the types of training sessions

№	Topic (section) of the course	Types of training sessions, including independent work			
		Lectures	Seminars	Laboratory practical	Independent work
1	Introducing combinatorics as the art of counting (enumeration)	4	4		6
2	Pigeonhole principle, double counting, averaging techniques	4	4		6
3	Estimates in combinatorics. Combinatorial sums	4	4		6
4	Recurrence relations in combinatorics.	4	4		6
5	More on recurrence relations in combinatorics and graph theory	4	4		6
6	Formal power series and generating functions	4	4		6
7	Combinatorics of permutations. Cyclic structure of permutations	6	6		9
AH in total		30	30		45
Exam preparation		30 AH.			
Total complexity		135 AH., credits in total 3			

4.2. Content of the course (training module), structured by topics (sections)

Semester: 1 (Fall)

1. Introducing combinatorics as the art of counting (enumeration)

Why counting? Introducing combinatorics as the art of counting (enumeration). Rules of summation and multiplication. Counting tuples and subsets. Structuring the counting with decision trees.

2. Pigeonhole principle, double counting, averaging techniques

Pigeonhole principle, double counting, averaging techniques. Using counting to prove existence

3. Estimates in combinatorics. Combinatorial sums

Binomial theorem, binomial coefficients. Big-O notation, Stirling's formula and asymptotic estimates.

4. Recurrence relations in combinatorics.

The number of independent sets in a path, Fibonacci numbers. Binet's formula. Fast computation of Fibonacci numbers via matrix multiplication. A glimpse into general linear recurrences.

5. More on recurrence relations in combinatorics and graph theory

Integer partitions, counting graph colorings (chromatic polynomial).

6. Formal power series and generating functions

Generating functions for linear recurrences. Derivation of a formula for Catalan numbers. Generating functions and integer partitions.

7. Combinatorics of permutations. Cyclic structure of permutations

Permutation groups. Other examples of groups. Cayley's theorem. Counting w.r.t. group actions. Counting w.r.t. group actions (continued). Cauchy—Frobenius—Burnside lemma and Redfield—Polya counting framework.

5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)

A classroom equipped with a computer and multimedia equipment (projector, sound system).

6. List of the main and additional literature, that is necessary for the course (training module) mastering

Main literature

1. Введение в комбинаторные методы дискретной математики [Текст] : [учеб. пособие для вузов] / В. Н. Сачков .— М. : МЦНМО, 2004 .— 424 с.
2. Комбинаторика [Текст], [учеб. пособие для вузов] /Н. Я. Виленкин, А. Н. Виленкин, П. А. Виленкин. -М., ФИМА : МЦНМО, 2015

Additional literature

1. Комбинаторика и теория вероятностей [Текст] / А. М. Райгородский - М.МФТИ,2012

7. List of web resources that are necessary for the course (training module) mastering

http://dm.fizteh.ru

8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)

Multimedia technologies can be employed during lectures and practical lessons, including presentations.

9. Guidelines for students to master the course

1. It is recommended to successfully pass the test papers, as this simplifies the final certification in the subject.
2. To prepare for the final certification in the subject, it is best to use the lecture materials.

SUPPLEMENT

Assessment funds for course (training module)

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Author: D.G. Ilinskiy, candidate of economic sciences, associate professor

1. Competencies formed during the process of studying the course

Code and the name of the competence	Competency indicators
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Pro.C-2 Understands and is able to apply modern mathematical apparatus and algorithms, the basic laws of natural science, modern programming languages and software; operating systems and networking technologies in research and applied activities	Pro.C-2.1 Demonstrate expert knowledge of research basics in the field of ICTs, philosophy and methodology of science, scientific research methods, and apply skills to use them

2. Competency assessment indicators

As a result of studying the course the student should:

know:

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3. List of typical control tasks used to evaluate knowledge and skills

In order to control the mastering of educational material by students, an oral survey is conducted at the beginning of the lesson on the topic of the previous lesson.

4. Evaluation criteria

The list of questions for the exam:

1. Basic concepts and definitions of convex geometry.
2. The Caratheodory theorem and Helly's theorem.
3. Applications of Helly's theorem: Young's inequality, center point theorem.

4. Technique of minimization and its application. Carathéodory's color theorem and Helly's color theorem. Tverberg theorem.

5. The degree of display and some of its applications. A topological lemma on the mapping of a simplex into itself. Division of a measure into convex parts of a given size. The Knaster – Kuratovsky – Mazurkevich theorem and the Brauer fixed point theorem. Strengthenings of the Carathéodory color theorem.

6. The Borsuk – Ulam theorem in the simplest case.

7. The sandwich theorem. Curve of moments and its generalization, polynomial version of the sandwich theorem.

8. Polynomial division of one measure in the spirit of Gut – Katz and its properties.

9. Cemeredi – Trotter theorem on the incidence number of points and lines. Estimates for the set of sums and the set of products of real numbers.

10. Connection of points on a plane by a graph with a small number of intersections with any line, Chazel – Weltzl theorem.

11. Dolnikov's theorem on intersections with hyperplanes and the chromatic number of the Kneser graph. Generalizations of the Borsuk – Ulam theorem for the action of a group of two elements.

12. The canonical bundle over the Grassmann space, Dolnikov's transversal theorem and the central transversal theorem.

13. Generalizations of the Borsuk – Ulam theorem for the action of groups of simple order. Tverberg's topological theorem and division of measures into equal parts on a straight line.

14. Cech cohomology, cover nerve lemma, and Helly's topological theorem.

Ticket Examples:

Ticket 1:

1. Dolnikov's theorem on intersections with hyperplanes and the chromatic number of the Kneser graph. Generalizations of the Borsuk – Ulam theorem for the action of a group of two elements.

2. The Caratheodory theorem and Helly's theorem.

Ticket 2:

1. Polynomial division of one measure in the spirit of Gut – Katz and its properties.

2. Basic concepts and definitions of convex geometry.

Assessment "excellent (10)" is given to a student who has displayed comprehensive, systematic and deep knowledge of the educational program material, has independently performed all the tasks stipulated by the program, has deeply studied the basic and additional literature recommended by the program, has been actively working in the classroom, and understands the basic scientific concepts on studied discipline, who showed creativity and scientific approach in understanding and presenting educational program material, whose answer is characterized by using rich and adequate terms, and by the consistent and logical presentation of the material;

Assessment "excellent (9)" is given to a student who has displayed comprehensive, systematic knowledge of the educational program material, has independently performed all the tasks provided by the program, has deeply mastered the basic literature and is familiar with the additional literature recommended by the program, has been actively working in the classroom, has shown the systematic nature of knowledge on discipline sufficient for further study, as well as the ability to amplify it on one's own, whose answer is distinguished by the accuracy of the terms used, and the presentation of the material in it is consistent and logical;

Assessment "excellent (8)" is given to a student who has displayed complete knowledge of the educational program material, does not allow significant inaccuracies in his answer, has independently performed all the tasks stipulated by the program, studied the basic literature recommended by the program, worked actively in the classroom, showed systematic character of his knowledge of the discipline, which is sufficient for further study, as well as the ability to amplify it on his own;

Assessment "good (7)" is given to a student who has displayed a sufficiently complete knowledge of the educational program material, does not allow significant inaccuracies in the answer, has independently performed all the tasks provided by the program, studied the basic literature recommended by the program, worked actively in the classroom, showed systematic character of his knowledge of the discipline, which is sufficient for further study, as well as the ability to amplify it on his own;

Assessment “good (6)” is given to a student who has displayed a sufficiently complete knowledge of the educational program material, does not allow significant inaccuracies in his answer, has independently carried out the main tasks stipulated by the program, studied the basic literature recommended by the program, showed systematic character of his knowledge of the discipline, which is sufficient for further study;

Assessment “good (5)” is given to a student who has displayed knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, who while not being sufficiently active in the classroom, has nevertheless independently carried out the main tasks stipulated by the program, mastered the basic literature recommended by the program, made some errors in their implementation and in his answer during the test, but has the necessary knowledge for correcting these errors by himself;

Assessment “satisfactory (4)” is given to a student who has discovered knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, who while not being sufficiently active in the classroom, has nevertheless independently carried out the main tasks stipulated by the program, learned the main literature but allowed some errors in their implementation and in his answer during the test, but has the necessary knowledge for correcting these errors under the guidance of a teacher;

Assessment “satisfactory (3)” is given to a student who has displayed knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, not showed activity in the classroom, independently fulfilled the main tasks envisaged by the program, but allowed errors in their implementation and in the answer during the test, but possessing necessary knowledge for elimination under the guidance of the teacher of the most essential errors;

Assessment “unsatisfactory (2)” is given to a student who showed gaps in knowledge or lack of knowledge on a significant part of the basic educational program material, who has not performed independently the main tasks demanded by the program, made fundamental errors in the fulfillment of the tasks stipulated by the program, who is not able to continue his studies or start professional activities without additional training in the discipline in question;

Assessment “unsatisfactory (1)” is given to a student when there is no answer (refusal to answer), or when the submitted answer does not correspond at all to the essence of the questions contained in the task.

5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience

During examination the student are allowed to use the program of the discipline.