**Federal State Autonomous Educational Institution of Higher Education "Moscow Institute of Physics and Technology (National Research University)"**

**APPROVED**
**Head of the Phystech School of Applied Mathematics and Informatics**
**A.M. Raygorodskiy**

**Work program of the course (training module)**

| | |
|---|---|
| **course:** | Applied Mathematical Modeling/Прикладное математическое моделирование |
| **major:** | Applied Mathematics and Informatics |
| **specialization:** | Contemporary Combinatorics/Современная комбинаторика |
| | “Pusk” Online and Supplementary Education Centre |
| | Chair of Discrete Mathematics |
| **term:** | 1 |
| **qualification:** | Master |

Semester, form of interim assessment: 2 (spring) - Grading test

Academic hours: 90 AH in total, including:
lectures: 45 AH.
seminars: 45 AH.
laboratory practical: 0 AH.

Independent work: 45 AH.

In total: 135 AH, credits in total: 3

Author of the program: A.M. Raygorodskiy, doctor of physics and mathematical sciences, head of chair

The program was discussed at the Chair of Discrete Mathematics 05.03.2021

**Annotation**

This course examines basic equilibrium models in various fields of application, such as economics and telecommunications systems. These include equilibrium models under conditions of perfect and imperfect competition, with various types of information use, including exchange models, Arrow-Debreu, Wald, Cournot and Bertrand.

## 1. Study objective

### Purpose of the course

Equilibrium-based mathematical modeling is one of the main ways to study complex systems.This course examines basic equilibrium models in various fields of application, such as economics and telecommunications systems. These include equilibrium models under conditions of perfect and imperfect competition, with various types of information use, including exchange models, Arrow-Debreu, Wald, Cournot and Bertrand. The dynamic models of Leontief and von Neumann are presented. In addition, models of spatial economic equilibrium, transport equilibrium, and models of population migration processes are considered. Variational inequality is used as a basic equilibrium model in complex systems. The elements of the theory and basic methods for solving variational inequalities are considered, as well as their connection with other general problems of nonlinear analysis.

### Tasks of the course

- mastering by students of basic knowledge (concepts, concepts, methods and models) in applied mathematical modeling;
- acquisition of theoretical knowledge and practical skills in applied mathematical modeling;
- providing advice and assistance to students in conducting their own theoretical research in applied mathematical modeling.

## 2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

| Code and the name of the competence | Competency indicators |
|---|---|
| UC-2 Able to manage a project through all stages of its life cycle | UC-2.4 Publicly present the project results (or results of its stages) via reports, articles, presentations at scientific conferences, seminars, and similar events |
| Gen.Pro.C-1 Address current challenges in fundamental and applied mathematics | Gen.Pro.C-1.1 Apply fundamental scientific knowledge, new scientific principles, and research methods in applied mathematics and computer science |
| Gen.Pro.C-4 Combine and adapt current information and communications technologies (ICTs) to meet professional challenges | Gen.Pro.C-4.3 Create original algorithms and use software tools and modern smart technologies for professional problem-solving |
| Pro.C-1 Become part of a professional community and conduct local research under scientific guidance using methods specific to a particular professional setting | Pro.C-1.1 Apply principles of scientific work, methods of data collection and analysis, ways of argumentation; prepare scientific reviews, publications, abstracts, and bibliographies on research topics in Russian and English |
| | Pro.C-1.2 Understand the verification process of software models used to solve related scientific problems |

## 3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

theoretical knowledge of the basic properties of equilibrium models

be able to:

understand the main approaches to building equilibrium models in complex systems and their applications

master:

skills of formulating and solving the simplest equilibrium models

**4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions**

4.1. The sections of the course (training module) and the complexity of the types of training sessions

| № | Topic (section) of the course | Types of training sessions, including independent work | | | |
|---|---|---|---|---|---|
| | | Lectures | Seminars | Laboratory practical | Independent work |
| 1 | Types of equilibria in the market of a homogeneous product | 5 | 5 | | 5 |
| 2 | Equilibrium for the study of complex systems | 4 | 4 | | 4 |
| 3 | Perfect competition models | 4 | 4 | | 4 |
| 4 | Equilibrium game models | 4 | 4 | | 4 |
| 5 | Dynamic models of the economy | 4 | 4 | | 4 |
| 6 | Equilibrium models based on duality theory | 4 | 4 | | 4 |
| 7 | General models of economic equilibrium | 4 | 4 | | 4 |
| 8 | Equilibrium models in distributed information systems | 4 | 4 | | 4 |
| 9 | Population migration patterns | 4 | 4 | | 4 |
| 10 | Variational inequalities | 4 | 4 | | 4 |
| 11 | Methods for solving variational inequalities | 4 | 4 | | 4 |
| AH in total | | 45 | 45 | | 45 |
| Exam preparation | | 0 AH. | | | |
| Total complexity | | 135 AH., credits in total 3 | | | |

4.2. Content of the course (training module), structured by topics (sections)

Semester: 2 (Spring)

1. Types of equilibria in the market of a homogeneous product

Aggregated equilibrium models of interaction of economic agents. Types of equilibria in the market of a homogeneous product, static and dynamic market models.

2. Equilibrium for the study of complex systems

Construction of models of applied problems using the concept of equilibrium. Investigation of the properties of equilibrium problems.

3. Perfect competition models

Microeconomic models of interaction between economic agents and information exchange. Exchange model. Individual demand and balance. Arrow-Debreu model. Equilibrium price establishment processes.

4. Equilibrium game models

Oligopolistic markets according to Cournot and Bertrand, strategies of participants' behavior.

5. Dynamic models of the economy

Generalized dynamic Leontief model.

6. Equilibrium models based on duality theory

Duality in linear programming. Economic interpretation of the simplex method. Generalized optimality conditions.

7. General models of economic equilibrium

Equilibrium models of Kassel-Wald and Scarfe.

8. Equilibrium models in distributed information systems

Transport equilibrium models.

9. Population migration patterns

Modeling population migration processes

10. Variational inequalities

Properties of existence and uniqueness of solutions. Variational inequalities and other problems of nonlinear analysis.

11. Methods for solving variational inequalities

Newton's method. Projective method.

**5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)**

Classroom equipped with a computer and multimedia equipment (projector, sound system).

**6. List of the main and additional literature, that is necessary for the course (training module) mastering**

Main literature

1. Физические модели и методы теории равновесия в программировании и экономике [Текст]/Б. С. Разумихин, -М., Наука, 1975

Additional literature

1. Сборник задач по математическому анализу [Текст] : в 3 т. Т. 3 : Функции нескольких переменных : учеб. пособие для вузов / Л. Д. Кудрявцев ; под ред. А. Д. Кудрявцева .— 2-е изд., перераб. и доп. — М. : Физматлит, 2003, 2012 .— 472 с.

**7. List of web resources that are necessary for the course (training module) mastering**

http://dm.fizteh.ru/
http://www.mccme.ru/~anromash/courses/expanders-notes-2014.pdf

**8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)**

The lectures use multimedia technologies, including the demonstration of presentations.
In the process of independent work of students, it is possible to use lecture materials

**9. Guidelines for students to master the course**

1. It is recommended to successfully pass test papers, as this simplifies the final certification in the subject.
2. To prepare for the final certification in the subject, it is best to use the lecture materials.

**SUPPLEMENT**

**Assessment funds for course (training module)**

| | |
|---|---|
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Semester, form of interim assessment: 2 (spring) - Grading test

**Author:** A.M. Raygorodskiy, doctor of physics and mathematical sciences, head of chair

**1. Competencies formed during the process of studying the course**

| Code and the name of the competence | Competency indicators |
|---|---|
| UC-2 Able to manage a project through all stages of its life cycle | UC-2.4 Publicly present the project results (or results of its stages) via reports, articles, presentations at scientific conferences, seminars, and similar events |
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| | Pro.C-1.2 Understand the verification process of software models used to solve related scientific problems |

**2. Competency assessment indicators**

As a result of studying the course the student should:

**know:**

theoretical knowledge of the basic properties of equilibrium models

**be able to:**

understand the main approaches to building equilibrium models in complex systems and their applications

**master:**

skills of formulating and solving the simplest equilibrium models

**3. List of typical control tasks used to evaluate knowledge and skills**

1. The concept of equilibrium and its generalizations for the study of complex systems.
2. Types of equilibrium in the market for a homogeneous product, static and dynamic market models.
3. The exchange model.
4. Individual demand and balance.
5. The Arrow-Debreu model.
6. Processes of equilibrium prices.
7. General model of Cournot oligopoly.
8. Behavior of economic agents on the example of duopoly.
9. General model of oligopoly according to Bertrand.
10. Generalized dynamic Leontief model.
11. The von Neumann model of a developing economy.

**4. Evaluation criteria**

Test questions:

1 Duality theory in linear programming.
2. Economic interpretation of the simplex method.
3. Generalized optimality conditions.
4. The method of decomposition and its application to the management of complex systems.
5. Model of the Kassel-Wald equilibrium.
6. The Scarfe Equilibrium Model.
7. Models of spatial economic equilibrium.

8. Models of transport equilibrium.
9. Modeling the processes of population migration.
10. Equilibrium model of migration and its properties.
11. Variational inequalities.
12. Properties of existence and uniqueness of solutions.
13. Variational inequalities and other problems of nonlinear analysis.
14. Newton's method.
15. Projective method.
16. Methods of regularization and proximal point.

Examples of exam tickets:
1 Newton's method.
2 Solve the Cournot duopoly problem with linear functions.

1 Regularization method.
2 Solve the oligopoly problem with linear functions.

1 Equilibrium in the exchange model.
2 Perform the steps of the projective method for the linear complementarity problem.

- the mark "excellent (10)" is given to a student who has shown comprehensive, systematized, in-depth knowledge of the curriculum of the discipline and the ability to confidently apply them in practice in solving specific problems, free and correct justification of the decisions made
- the mark "excellent (9)" is given to a student who has shown comprehensive, systematized, in-depth knowledge of the curriculum of the discipline and the ability to apply them in practice in solving specific problems, free and correct justification of the decisions made
- the mark "excellent (8)" is given to a student who has shown comprehensive, systematized, deep knowledge of the curriculum of the discipline and the ability to apply them in practice in solving specific problems, and the correct justification of the decisions made
- the mark "good (7)" is given to a student if he knows the material well, expresses it competently and in essence, knows how to apply the acquired knowledge in practice, but makes some inaccuracies in the answer or in solving problems;
- the mark "good (6)" is given to a student if he knows the material, presents it competently and in essence, knows how to apply the knowledge gained in practice, but makes some inaccuracies in the answer or in solving problems;
- the mark "good (5)" is given to a student if he knows the material, and essentially expounds it, knows how to apply the knowledge gained in practice, but makes some inaccuracies in the answer or in solving problems;
- the mark "satisfactory (4)" is given to a student who has shown a fragmented, scattered nature of knowledge, insufficiently correct formulations of basic concepts, a violation of the logical sequence in the presentation of the program material, but at the same time he owns the main sections of the curriculum necessary for further education and can apply the received knowledge by model in a standard situation;
- the mark "satisfactory (3)" is given to a student who has shown a fragmented, scattered nature of knowledge, insufficiently correct formulations of basic concepts, violation of the logical sequence in the presentation of program material, but at the same time he has fragmentary knowledge of the main sections of the curriculum necessary for further education and can apply the knowledge gained by the model in a standard situation;
- the mark "unsatisfactory (2)" is given to a student who does not know most of the main content of the curriculum of the discipline, makes gross errors in the formulation of the basic concepts of the discipline and does not know how to use the knowledge gained in solving typical practical problems
- the mark "unsatisfactory (1)" is given to a student who does not know the formulations of the basic concepts of the dissertationциплины

**5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience**

During the test, students can use the discipline program.