**Federal State Autonomous Educational Institution of Higher Education "Moscow Institute of Physics and Technology (National Research University)"**

**APPROVED**
**Head of the Phystech School of Applied Mathematics and Informatics**
**A.M. Raygorodskiy**

**Work program of the course (training module)**

| | |
|---|---|
| **course:** | Convex Optimization/Выпуклая оптимизация |
| **major:** | Applied Mathematics and Informatics |
| **specialization:** | Computer Science/Информатика |
| | Phystech School of Applied Mathematics and Informatics |
| | Chair of Discrete Mathematics |
| **term:** | 3 |
| **qualification:** | Bachelor |

Semester, form of interim assessment: 6 (spring) - Exam

Academic hours: 60 AH in total, including:
lectures: 30 AH.
seminars: 30 AH.
laboratory practical: 0 AH.

Independent work: 90 AH.

Exam preparation: 30 AH.

In total: 180 AH, credits in total: 4

Author of the program: I.M. Mitricheva, candidate of physics and mathematical sciences, associate professor

The program was discussed at the Chair of Discrete Mathematics 05.03.2020

**Annotation**

The course is devoted to the theoretical and applied aspects of solving convex optimization problems. Such problems arise in numerous applications, some of which are considered in the classroom. The course is divided into two parts. The first part of the course examines the basic concepts of convex analysis, optimality conditions, and gives an introduction to the theory of duality. The main objects of study are convex sets, convex functions and cones. The second part of the course is devoted to numerical methods for solving convex optimization problems, both unconditional and conditional. The analysis of the efficiency of the algorithms under consideration is carried out from the point of view of obtaining theoretical estimates of the convergence and the speed of solving specific classes of convex optimization problems. It also demonstrates the use of the convex optimization apparatus for calculating approximate solutions to discrete optimization problems.

## 1. Study objective

**Purpose of the course**

- mastering convex optimization.

**Tasks of the course**

- mastering by students of basic knowledge (concepts, concepts, methods and models) in the field of convex optimization;
- acquisition of theoretical knowledge and practical skills in the field of convex optimization;
- providing advice and assistance to students in carrying out their own theoretical studies in the field of convex optimization.

## 2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

| Code and the name of the competence | Competency indicators |
|---|---|
| Gen.Pro.C-5 Participate in fundamental and applied research and development activities; independently devise new theoretical research methods (including mathematical research methods) and work with cutting-edge scientific equipment (measuring, analytical, technological) | Gen.Pro.C-5.1 Perform tasks in the field of theoretical and experimental research and development activities |
| Pro.C-1 Assign, formalize, and solve tasks, develop and research mathematical models of studied phenomena and processes, systematically analyze scientific problems, obtain new scientific outcomes | Pro.C-1.2 Make hypotheses, build mathematical models of the studied phenomena and processes, evaluate the quality of the developed model |

## 3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

- fundamental concepts, laws, theory of convex optimization;
- modern problems of the corresponding sections of convex optimization;
- concepts, axioms, methods of proofs and proofs of the main theorems in the sections included in the basic part of the cycle;
- basic properties of the corresponding mathematical objects;
- analytical and numerical approaches and methods for solving typical applied problems of convex optimization.

be able to:

- understand the task at hand;
- use your knowledge to solve fundamental and applied problems of convex optimization;
- evaluate the correctness of the problem setting;
- strictly prove or disprove the statement;
- independently find algorithms for solving problems, including non-standard ones, and analyze them;
- independently see the consequences of the results obtained;
- accurately represent mathematical knowledge in complex calculations, orally and in writing.

master:

- skills of mastering a large amount of information and solving problems (including complex ones);
- skills of independent work and mastering new disciplines;
- culture of formulation, analysis and solution of mathematical and applied problems, requiring the use of mathematical approaches and methods of convex optimization for their solution;
- subject language of complex calculations and skills of competent description of problem solving and presentation of the results obtained.

**4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions**

4.1. The sections of the course (training module) and the complexity of the types of training sessions

| № | Topic (section) of the course | Types of training sessions, including independent work | | | |
|---|---|---|---|---|---|
| | | Lectures | Seminars | Laboratory practical | Independent work |
| 1 | Convergence analysis | 6 | 6 | | 18 |
| 2 | Convex sets. | 6 | 6 | | 18 |
| 3 | Localization methods. | 6 | 6 | | 18 |
| 4 | Proximal operator | 6 | 6 | | 18 |
| 5 | Subgradient. | 6 | 6 | | 18 |
| AH in total | | 30 | 30 | | 90 |
| Exam preparation | | 30 AH. | | | |
| Total complexity | | 180 AH., credits in total 4 | | | |

4.2. Content of the course (training module), structured by topics (sections)

Semester: 6 (Spring)

1. Convergence analysis

Newton's method in a problem with equality constraints.

2. Convex sets.

Affine sets. Convex functions

3. Localization methods.

Метод отсекающих гиперплоскостей

4. Proximal operator

Proximal algorithms: minimization, gradient method, accelerated gradient method, directional multiplier method

5. Subgradient.

Subgradient methods

**5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)**

Standard classroom.

**6. List of the main and additional literature, that is necessary for the course (training module) mastering**


Main literature

1. Введение в выпуклую оптимизацию [Текст], [монография]/Ю. Е. Нестеров , -М., МЦНМО, 2010
2. Основы методов оптимизации [Электронный ресурс] : учеб. пособие / В. В. Лесин, Ю. П. Лисовец .— 4-е изд., стереотип. — СПб. : Лань, 2016 .— (Учебники для вузов. Специальная литература) .— Электрон. версия печ. публикации .— Полный текст (Доступ из сети МФТИ / Удаленный доступ).
3. Методы оптимизации [Текст]. Ч. 1. Введение в выпуклый анализ и теорию оптимизации : учеб. пособие для вузов / В. Г. Жадан ; М-во образования и науки РФ, Моск. физ.-техн. ин-т (гос. ун-т) .— М. : МФТИ, 2014 .— 271 с. + pdf-версия. - Библиогр.: с. 267-270. - 300 экз. - ISBN 978-5-7417-0514-8. — Полный текст (Доступ из сети МФТИ).

Additional literature

1. Методы оптимизации. Численные алгоритмы поиска экстремума [Текст] / Т. Н. Данильченко, Ю. П. Иванилов - М.МФТИ,1991


**7. List of web resources that are necessary for the course (training module) mastering**

http://dm.fizteh.ru/
http://web.stanford.edu/class/ee364b/lectures.html

**8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)**

The class uses multimedia technologies, including the presentation of presentations.
In the process of independent work of students, it is possible to use software such as Mathcad, MATLAB, Maple, etc.

**9. Guidelines for students to master the course**

1. It is recommended to successfully pass test papers, as this simplifies the final certification in the subject.
2. To prepare for the final certification in the subject, it is best to use the lecture materials.

**SUPPLEMENT**

**Assessment funds for course (training module)**

| | |
|---|---|
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Semester, form of interim assessment: 6 (spring) - Exam

**Author:** I.M. Mitricheva, candidate of physics and mathematical sciences, associate professor

## 1. Competencies formed during the process of studying the course

| Code and the name of the competence | Competency indicators |
|---|---|
| Gen.Pro.C-5 Participate in fundamental and applied research and development activities; independently devise new theoretical research methods (including mathematical research methods) and work with cutting-edge scientific equipment (measuring, analytical, technological) | Gen.Pro.C-5.1 Perform tasks in the field of theoretical and experimental research and development activities |
| Pro.C-1 Assign, formalize, and solve tasks, develop and research mathematical models of studied phenomena and processes, systematically analyze scientific problems, obtain new scientific outcomes | Pro.C-1.2 Make hypotheses, build mathematical models of the studied phenomena and processes, evaluate the quality of the developed model |

## 2. Competency assessment indicators

As a result of studying the course the student should:

**know:**

- fundamental concepts, laws, theory of convex optimization;
- modern problems of the corresponding sections of convex optimization;
- concepts, axioms, methods of proofs and proofs of the main theorems in the sections included in the basic part of the cycle;
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**be able to:**

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**master:**

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- culture of formulation, analysis and solution of mathematical and applied problems, requiring the use of mathematical approaches and methods of convex optimization for their solution;
- subject language of complex calculations and skills of competent description of problem solving and presentation of the results obtained.

## 3. List of typical control tasks used to evaluate knowledge and skills

Homework examples:

1. Numerically compare the rate of convergence of gradient descent for a quadratic objective function with a positive definite matrix with a theoretical estimate
2. Give an example of a convex optimization problem for which the Slater condition is violated
3. Obtain an expression for the step size in the steepest gradient descent for the convex quadratic objective function.

Examples of tasks for the test:

1. What is a convex function?
2. Write the Karush-Kuhn-Tucker conditions for the convex optimization problem

3. What is a dual gap?
4. What is strong duality?
5. What is the rate of convergence of gradient descent for a strongly convex function?
6. What is the complexity of one iteration in the BFGS method?
7. What is the proximal gradient method and for what tasks is it advisable to use it?

**4. Evaluation criteria**

1. Convex sets. Convex-preserving operations.
2. Subgradient. Subgradient methods.
3. Directly - dual subgradient method.
4. Newton's method in a problem with equality constraints. Convergence analysis. Generalization of Newton's method for the case of starting from an invalid point.
5. Interior point methods. The method of barrier functions. Logarithmic barrier function.
6. Self-consistent functions. Analysis of the complexity of the method of barrier functions through self-consistent functions.
7. Directly dual methods of the interior point.
8. Method of cutting hyperplanes.
9. Method of analytical centers. Ellipsoid method.
10. Decomposition and parallel programming in convex optimization algorithms. Direct and dual decomposition.
11. Proximal operator. Examples of proximal algorithms.

Ticket 1
1. Subgradient. Subgradient methods.
2. Directly - dual subgradient method.

- the mark "excellent (10)" is given to a student who has shown comprehensive, systematized, in-depth knowledge of the curriculum of the discipline and the ability to confidently apply them in practice when solving specific problems, free and correct justification of the decisions made

- the mark "excellent (9)" is given to a student who has shown comprehensive, systematized, in-depth knowledge of the curriculum of the discipline and the ability to apply them in practice in solving specific problems, free and correct justification of the decisions

- the mark "excellent (8)" is given to a student who has shown comprehensive systematized, deep knowledge of the curriculum of the discipline and the ability to apply them in practice in solving specific problems, and the correct justification of the decisions

- the mark "good (7)" is given to a student if he firmly knows the material, expresses it competently and to the point, knows how to apply the acquired knowledge in practice, but makes some inaccuracies in the answer or in solving problems;

- the mark "good (6)" is given to the student if he knows the material, presents it competently and in essence, knows how to apply the knowledge gained in practice, but makes some inaccuracies in the answer or in solving problems;

- the mark "good (5)" is given to the student if he knows the material, and essentially expounds it, knows how to apply the knowledge gained in practice, but makes some inaccuracies in the answer or in solving problems;

- the mark "satisfactory (4)" is given to a student who has shown a fragmented, scattered nature of knowledge, insufficiently correct formulations of basic concepts, a violation of the logical sequence in the presentation of the program material, but at the same time he owns the main sections of the curriculum necessary for further education and can apply the obtained knowledge by model in a standard situation;

- the mark "satisfactory (3)" is given to a student who has shown a fragmentary, scattered nature of knowledge, insufficiently correct formulations of basic concepts, violation of the logical sequence in the presentation of program material, but at the same time he has fragmentary knowledge of the main sections of the curriculum necessary for further education and can apply the knowledge gained by the model in a standard situation;

- the mark "unsatisfactory (2)" is given to a student who does not know most of the main content of the curriculum of the discipline, makes gross mistakes in the formulation of the basic concepts of the discipline and does not know how to use the knowledge gained in solving typical practical problems

- the mark "unsatisfactory (1)" is given to a student who does not know the wording of the basic concepts of the discipline

**5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience**

During the exam of the student, use the program of disciplines.