Federal State Autonomous Educational Institution of Higher Education "Moscow Institute of Physics and Technology (National Research University)"

APPROVED
Vice Rector for Academic Affairs

A.A. Voronov

Work program of the course (training module)

course:	Fourier Analysis/Фурье анализ
major:	Applied Mathematics and Informatics
specialization:	Computer Science/Информатика
	Phystech School of Applied Mathematics and Informatics
	Chair of Higher Mathematics
term:	3
qualification:	Bachelor

Semester, form of interim assessment: 5 (fall) - Exam

Academic hours: 60 AH in total, including:
lectures: 30 AH.
seminars: 30 AH.
laboratory practical: 0 AH.

Independent work: 90 AH.

Exam preparation: 30 AH.

In total: 180 AH, credits in total: 4

Authors of the program:

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The program was discussed at the Chair of Higher Mathematics 20.05.2020

Annotation

Refers to the basic part of the educational course. Topics are discussed such as Trigonometric Fourier series for absolutely integrable functions, summation of Fourier series by the method of arithmetic means, Metric and linear normed spaces, Infinite-dimensional Euclidean spaces, Trigonometric Fourier series for functions that are absolutely integrable with a square, Own integrals and improper Fourier integrals, The space of basic functions and the space of generalized functions, Fourier transform of generalized functions. Mastering the discipline is aimed at developing the ability to apply methods of mathematical analysis, modeling, optimization and statistics to solve problems arising in the course of professional activity.

1. Study objective

Purpose of the course

The formation of systematic knowledge about the methods of mathematical analysis, the expansion and deepening of concepts such as function and series.

Tasks of the course

- Formation of students' theoretical knowledge and practical skills in the theory of trigonometric Fourier series and the principles of functional analysis;
- preparing students for the study of related mathematical disciplines;
- acquisition of skills in the application of methods of mathematical analysis in physics and other natural science disciplines.

2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

<table>
<tr><th>Code and the name of the competence</th><th>Competency indicators</th></tr>
<tr><td rowspan="4">UC-1 Search and identify, critically evaluate and synthesize information, apply a systematic approach to problem-solving</td><td>UC-1.1 Analyze problems, highlight the stages of their solution, plan the actions required to solve them</td></tr>
<tr><td>UC-1.2 Find, critically assess, and select information required for the task in hand</td></tr>
<tr><td>UC-1.3 Consider various options for solving a problem, assess the advantages and disadvantages of each option</td></tr>
<tr><td>UC-1.4 Make competent judgments and estimates supported by logic and reasoning</td></tr>
<tr><td>UC-6 Use time-management skills, apply principles of self-development and lifelong learning</td><td>UC-6.2 Plan independent activities in professional problem-solving; critically analyze the work performed; find creative ways to use relevant experience for self-development</td></tr>
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3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

- Basic facts of the theory of trigonometric Fourier series of absolutely integrable functions: sufficient conditions for pointwise and uniform convergence;
- theorems on term-by-term integration and differentiation, order of decreasing coefficients, a theorem on the summation of Fourier series by the method of arithmetic means and its application;
- definition of convergence in metric and linear normed spaces, examples of complete and incomplete spaces;
- examples of complete systems in normed linear spaces;
- basic concepts of the theory of Fourier series in an orthonormal system in an infinite-dimensional Euclidean space;
- definition of proper and improper integrals depending on a parameter, their properties; theorems on continuity, differentiation and integration with respect to the parameter of improper integrals, their application to the calculation of integrals;
- a sufficient condition for the representation of a function by the Fourier integral;
- Fourier transform of an absolutely integrable function and its properties;
- basic concepts of the theory of generalized functions, the Fourier transform of generalized functions, its properties.

be able to:

- To expand functions in trigonometric Fourier series, to investigate it for uniform convergence, to determine the order of decreasing of Fourier coefficients;
- to investigate the completeness of systems in functional spaces;
- investigate the convergence and uniform convergence of improper integrals with a parameter, differentiate and integrate them with respect to the parameter;
- to represent functions by the Fourier integral; perform Fourier transforms;
- operate with generalized functions.

master:

- Thinking, methods of proof of mathematical statements;
- skills of working with Fourier series and integrals in various forms;
- the skills of applying the studied theory in mathematical and physical applications;
- the ability to use the necessary literature to solve problems.

4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

4.1. The sections of the course (training module) and the complexity of the types of training sessions

№	Topic (section) of the course	Types of training sessions, including independent work			
		Lectures	Seminars	Laboratory practical	Independent work
1	Summation of Fourier series by the method of arithmetic means.	6	6		18
2	Metric and linear normed spaces.	2	2		6
3	Infinite-dimensional Euclidean spaces.	4	4		12
4	Trigonometric Fourier series for functions absolutely square integrable.	4	4		12
5	Proper integrals and improper integrals.	3	3		8
6	Fourier integral.	5	5		16
7	The space of basic functions and the space of generalized functions.	2	2		6
8	Fourier transform of generalized functions.	2	2		6
9	Trigonometric Fourier series for absolutely integrable functions.	2	2		6
AH in total		30	30		90
Exam preparation		30 AH.			

Total complexity	180 AH., credits in total 4

4.2. Content of the course (training module), structured by topics (sections)

Semester: 5 (Fall)

1. Summation of Fourier series by the method of arithmetic means.

Riemann's lemma. Trigonometric Fourier series for absolutely integrable functions, the tendency of their coefficients to zero. Representation of the partial sum of the Fourier series by an integral in terms of the Dirichlet kernel. Localization principle. Dini and Lipschitz tests for convergence of Fourier series, consequences of the Lipschitz test. Uniform convergence of Fourier series. Term-by-term integration and differentiation of Fourier series. Decreasing order of Fourier coefficients. Fourier series in complex form.

2. Metric and linear normed spaces.

Summation of Fourier series by the method of arithmetic means. Weierstrass' theorems on the approximation of continuous functions by trigonometric and algebraic polynomials.

3. Infinite-dimensional Euclidean spaces.

Metric and linear normed spaces. Convergence in metric spaces. Complete metric spaces, complete normed linear (Banach) spaces. Completeness of space Incompleteness of the space of continuous functions on an interval with integral norms. Comparison of norms: comparison of uniform convergence, convergence in mean and mean square. Complete systems in normed linear spaces.

4. Trigonometric Fourier series for functions absolutely square integrable.

Infinite-dimensional Euclidean spaces. Fourier series in the orthonormal system. Minimal property of Fourier coefficients, Bessel inequality. Parseval's equality. Orthonormal basis in infinite-dimensional Euclidean space. Hilbert spaces. A necessary and sufficient condition for a sequence of numbers to be a sequence of Fourier coefficients of an element of a Hilbert space with a fixed orthonormal basis. Relationship between the concepts of completeness and closedness of an orthonormal system.

5. Proper integrals and improper integrals.

Trigonometric Fourier series for functions that are absolutely square integrable. Completeness of the trigonometric system, Parseval's equality. Completeness of the system of Legendre polynomials.

6. Fourier integral.

Eigen integrals depending on a parameter and their properties. Improper integrals depending on a parameter; uniform convergence. Cauchy criterion for uniform convergence, Weierstrass test. Dirichlet test. Continuity, differentiation and integration with respect to the parameter of improper integrals. Application of the theory of integrals depending on a parameter to the calculation of definite integrals. Dirichlet and Laplace integrals. Euler's integrals - gamma and beta functions.

7. The space of basic functions and the space of generalized functions.

Fourier integral. Representation of a function by the Fourier integral. Fourier transform of an absolutely integrable function and its properties: continuity, tending to zero at infinity. Conversion formulas. The Fourier transform of the derivative and the derivative of the Fourier transform.

8. Fourier transform of generalized functions.

Space of basic functions and space of generalized functions. Regular and singular generalized functions. Delta function. Multiplication of generalized by infinitely differentiable. Convergence in the space of generalized functions. Differentiation of generalized functions.

9. Trigonometric Fourier series for absolutely integrable functions.

Fourier transform of generalized functions. The Fourier transform of the derivative and the derivative of the Fourier transform.

5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)

Classroom equipped with a multimedia projector, screen and microphone.

6. List of the main and additional literature, that is necessary for the course (training module) mastering

Main literature

1. Mathematical analysis II /V. A. Zorich. Berlin, Springer, 2016

Additional literature

1. Функциональный анализ [Текст] : учебник для вузов / В. А. Треногин .— 4-е изд., испр. — М. : Физматлит, 2007 .— 488 с.

7. List of web resources that are necessary for the course (training module) mastering

http://mathnet.ru – all-Russian mathematical portal.
http://www.edu.ru – Federal portal "Russian education".
http://benran.ru –library on natural Sciences of the Russian Academy of Sciences.
http://www.i-exam.ru – single portal of online testing in education.

8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)

The lectures use multimedia technologies, including the demonstration of presentations.

9. Guidelines for students to master the course

Successful mastering of the course requires intense independent work of the student. The course program provides the minimum required time for a student to work on each topic.
Independent work includes:
- study of lectures and recommended literature,
- study of educational material (based on lecture notes and educational literature),
- preparation of answers to questions intended for self-study,
- proof of individual statements, properties;
- solving problems offered to students in lectures and practical classes,
- preparation for practical exercises, colloquia, exam.

Guidance and control over the student's independent work is carried out in the form of individual consultations. An indicator of mastery of the material is the ability to solve practical and theoretical problems. To form the ability to apply theoretical knowledge in practice, the student needs to solve as many problems as possible. When solving problems, each action must be argued, referring to the known theoretical information. In preparation for practical exercises, it is necessary to repeat the previously studied basic definitions, theorem formulations. At the beginning of the lesson, as a rule, a short (10-15 minutes) survey is conducted on the material of the past lessons in oral or written form. Usually they adhere to the following scheme: study of the material of the lecture on the synopsis on the same day when the lecture was listened to (10-15 minutes); repetition of the material on the eve of the next lecture (10-15 minutes), study of educational material based on lecture notes, educational and scientific literature, preparation of answers to questions intended for self-study (1 hour a week), preparation for a practical lesson, problem solving (1 hour) ... It is important to achieve an understanding of the material being studied, not its mechanical memorization. If you find it difficult to study certain topics, issues, you should seek advice from a lecturer or teacher leading practical classes. A prerequisite is the performance of household chores, which are drawn up in a notebook specially designated for this and are systematically submitted for verification. Intermediate control of knowledge is carried out in the form of intermediate mini-control works, in which the student is invited to answer in writing a theoretical question and solve two problems on the topic being handed over.

SUPPLEMENT

Assessment funds for course (training module)

major:	Applied Mathematics and Informatics
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	Phystech School of Applied Mathematics and Informatics
	Chair of Higher Mathematics
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Authors:

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1. Competencies formed during the process of studying the course

Code and the name of the competence	Competency indicators
UC-1 Search and identify, critically evaluate and synthesize information, apply a systematic approach to problem-solving	UC-1.1 Analyze problems, highlight the stages of their solution, plan the actions required to solve them
	UC-1.2 Find, critically assess, and select information required for the task in hand
	UC-1.3 Consider various options for solving a problem, assess the advantages and disadvantages of each option
	UC-1.4 Make competent judgments and estimates supported by logic and reasoning
UC-6 Use time-management skills, apply principles of self-development and lifelong learning	UC-6.2 Plan independent activities in professional problem-solving; critically analyze the work performed; find creative ways to use relevant experience for self-development

2. Competency assessment indicators

As a result of studying the course the student should:

know:

- Basic facts of the theory of trigonometric Fourier series of absolutely integrable functions: sufficient conditions for pointwise and uniform convergence;
- theorems on term-by-term integration and differentiation, order of decreasing coefficients, a theorem on the summation of Fourier series by the method of arithmetic means and its application;
- definition of convergence in metric and linear normed spaces, examples of complete and incomplete spaces;
- examples of complete systems in normed linear spaces;
- basic concepts of the theory of Fourier series in an orthonormal system in an infinite-dimensional Euclidean space;
- definition of proper and improper integrals depending on a parameter, their properties; theorems on continuity, differentiation and integration with respect to the parameter of improper integrals, their application to the calculation of integrals;
- a sufficient condition for the representation of a function by the Fourier integral;
- Fourier transform of an absolutely integrable function and its properties;
- basic concepts of the theory of generalized functions, the Fourier transform of generalized functions, its properties.

be able to:

- To expand functions in trigonometric Fourier series, to investigate it for uniform convergence, to determine the order of decreasing of Fourier coefficients;
- to investigate the completeness of systems in functional spaces;
- investigate the convergence and uniform convergence of improper integrals with a parameter, differentiate and integrate them with respect to the parameter;
- to represent functions by the Fourier integral; perform Fourier transforms;
- operate with generalized functions.

master:

- Thinking, methods of proof of mathematical statements;
- skills of working with Fourier series and integrals in various forms;
- the skills of applying the studied theory in mathematical and physical applications;
- the ability to use the necessary literature to solve problems.

3. List of typical control tasks used to evaluate knowledge and skills

Current control is carried out on the basis of a point-rating system (BRS) for evaluating knowledge in the discipline being studied. The BRS takes into account the students ' performance of a set of homework assignments and tests in accordance with the curriculum. Data on attendance and current academic performance are entered by teachers in special journals and recorded in the BRS.

Current control on the basis of homework is carried out during the academic semester in the terms set by the Educational Department, in accordance with the curriculum.
To pass the task, the student must provide a solution to the homework problem in writing, answer the questions of the teacher and write a test paper on the task, which checks the knowledge of concepts and statements on the topics of the task and the ability to solve problems.
You can't use other people's help, computers, or mobile phones during the test.

* A BAR is attached to the subject being studied.

4. Evaluation criteria

Certification in the discipline "Fourier Analysis/Фурье анализ" is carried out in the form of an exam.
The exam is conducted taking into account the control tasks previously completed by the students.

Control tasks:

1. Riemann's lemma. Trigonometric Fourier series for absolutely integrable functions, the tendency of their coefficients to zero.
2. Representation of the partial sum of the Fourier series by an integral in terms of the Dirichlet kernel. Localization principle.
3. Dini and Lipschitz tests for convergence of Fourier series, consequences of the Lipschitz test. Uniform convergence of Fourier series.
4. Term-by-term integration and differentiation of Fourier series. Decreasing order of Fourier coefficients. Fourier series in complex form.
5. Summation of Fourier series by the method of arithmetic means.
6. Weierstrass theorems on the approximation of continuous functions by trigonometric and algebraic polynomials.
7. Metric and linear normed spaces. Convergence in metric spaces.
8. Complete metric spaces, complete normed linear (Banach) spaces. Completeness of space Incompleteness of the space of continuous functions on an interval with integral norms.
9. Comparison of norms: comparison of uniform convergence, convergence in mean and mean square. Complete systems in normed linear spaces.
10. Infinite-dimensional Euclidean spaces. Fourier series in the orthonormal system.
11. The minimal property of the Fourier coefficients, the Bessel inequality. Parseval's equality. Orthonormal basis in infinite-dimensional Euclidean space. Hilbert spaces.
12. Necessary and sufficient conditions for a sequence of numbers to be a sequence of Fourier coefficients of an element of a Hilbert space with a fixed orthonormal basis. Relationship between the concepts of completeness and closedness of an orthonormal system.
13. Trigonometric Fourier series for functions absolutely square integrable. Completeness of the trigonometric system, Parseval's equality. Completeness of the system of Legendre polynomials.
14. Proper integrals depending on a parameter and their properties.
15. Improper integrals depending on a parameter; uniform convergence. Cauchy criterion for uniform convergence, Weierstrass test.
16. The Dirichlet test. Continuity, differentiation and integration with respect to the parameter of improper integrals.
17. Application of the theory of integrals depending on a parameter to the calculation of definite integrals. Dirichlet and Laplace integrals. Euler's integrals - gamma and beta functions.
18. Fourier integral. Representation of a function by the Fourier integral. Fourier transform of an absolutely integrable function and its properties: continuity, tending to zero at infinity. Conversion formulas. The Fourier transform of the derivative and the derivative of the Fourier transform.
19. Space of basic functions and space of generalized functions. Regular and singular generalized functions. Delta function. Multiplication of generalized by infinitely differentiable. Convergence in the space of generalized functions. Differentiation of generalized functions.
20. Fourier transform of generalized functions. The Fourier transform of the derivative and the derivative of the Fourier transform.

Examples of exam tickets:

Ticket 1

1. Representation of the partial sum of the Fourier series by an integral in terms of the Dirichlet kernel. Localization principle.
2. Eigen integrals depending on a parameter and their properties. Fourier series in complex form.

Ticket 2

1. Summation of Fourier series by the method of arithmetic means. Cauchy criterion for uniform convergence, Weierstrass test.
2. Fourier transform of generalized functions. The Fourier transform of the derivative and the derivative of the Fourier transform.

Grade "excellent (10)" is given to a student who has exhibited extensive and deep knowledge of the course and ability to apply skills when solving specific tasks;

Grade "excellent (9)" is given to a student who has exhibited extensive and deep knowledge of the course and ability to apply skills when solving specific tasks, but he has made minor errors that were independently found and corrected;

Grade "excellent (8)" is given to a student who has exhibited extensive and deep knowledge of the course and ability to apply skills when solving specific tasks, but he has made minor errors that were independently corrected after the instructions of an examiner;

Grade "good (7)" is given to a student who has a good command of the course and is able to apply skills when solving specific tasks, but has made minor mistakes when answering questions or solving problems;

Grade "good (6)" is given to a student who has a good command of the course and is able to apply skills when solving specific tasks, but has made rare mistakes when answering questions or solving problems;

Grade "good (5)" is given to a student who has a good command of the course and is able to apply skills when solving specific tasks, but has made mistakes when answering questions or solving problems;

Grade "satisfactory (4)" is given to a student who has exhibited fragmented knowledge, has made inaccurate formulation of the basic concepts, but understands the subject well, is able to apply the knowledge in standard situations and possesses skills necessary for the future study;

Grade "satisfactory (3)" is given to a student who has exhibited fragmented knowledge, has made inaccurate formulation of the basic concepts, has inconsistencies in understanding the course, but is able to apply the knowledge in standard situations and possesses skills necessary for the future study;

Grade "unsatisfactory (2)" is given to a student who does not possess knowledge of the essential concept of the course, has made gross mistakes in formulations of basic concepts and cannot use the knowledge in solving typical tasks;

Grade "unsatisfactory (1)" is given to a student who has exhibited total lack of knowledge of the course.

5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience

During the oral examination, the student is given one (astronomical) hour to prepare an answer to the question posed. The discussion schedule between the examiner and the student is limited to two hours.

During the exam, students can use the discipline program.

Scoring for Exams without Written Part
Department of Higher Mathematics

NN	Types of Control	Points
1	Control test 1	0—9
2	Control test 2	0—9
3	Homework 1	0—3
4	Homework 2	0—3
5	Theory checking	0—3
6	Class attendance and activity during seminars	0—3
7	Oral exam	0—70
	Total Score	0—100

The score for the oral exam is computed by the formula $7 \cdot N$, where $N \geq 3$ is a grade gained by a student during the exam. If $N = 1$ or 2, then the student grade is equal to N.

Conversion Scale between the total score and the student grade

Total Score	Student Grade	
93—100	10	Excellent
86—92	9	
79—85	8	
72—78	7	Good
65—71	6	
58—64	5	
51—57	4	Satisfactory
44—50	3	
30—43	2	Unsatisfactory
0—29	1	