Federal State Autonomous Educational Institution of Higher Education "Moscow Institute of Physics and Technology (National Research University)"

APPROVED
Head of the Phystech School of Applied Mathematics and Informatics
A.M. Raygorodskiy

Work program of the course (training module)

course:	Discrete Mathematics/Дискретная математика
major:	Applied Mathematics and Informatics
specialization:	Computer Science/Информатика
	Phystech School of Applied Mathematics and Informatics
	Chair of Discrete Mathematics
term:	1
qualification:	Bachelor

Semester, form of interim assessment: 2 (spring) - Exam

Academic hours: 60 AH in total, including:
lectures: 30 AH.
seminars: 30 AH.
laboratory practical: 0 AH.

Independent work: 90 AH.

Exam preparation: 30 AH.

In total: 180 AH, credits in total: 4

Number of course papers, tasks: 2

Authors of the program:

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A.G. Zdor, candidate of physics and mathematical sciences

The program was discussed at the Chair of Discrete Mathematics 21.05.2020

Annotation

This course provides an introduction to the important sections of Discrete Mathematics.

Operations on sets, the algebra of sets, and its basic identities are considered.

For finite sets, based on the theorems of addition and multiplication, the properties of the main types of samples are considered, such as: permutations, placement, combination, placement and combination with repetitions, permutations with repetitions. A polynomial theorem is proved. The formula of inclusion and exclusion is proved in the form generalized for counting the number of elements with exactly r properties.

The definition of a Boolean function is introduced, a tabular way of setting is discussed, and the concept of essential and dummy variables is introduced. Representation of Boolean functions by formulas, equivalence of formulas, basic identities of binary Boolean algebra are considered. A theorem on the (disjunctive) expansion of a Boolean function in the first m variables is proved. Using the concept of a dual Boolean function on the basis of the duality principle, a theorem on the (conjunctive) expansion of a Boolean function in the first m variables is proved. We consider SDNF and SKNF of a Boolean function. The Zhegalkin polynomials are introduced, the existence and uniqueness of the representation of an arbitrary Boolean function by the canonical Zhegalkin polynomial is proved. Definitions of closed and complete systems of Boolean functions are introduced, five Post classes are considered, their closedness and the Post completeness theorem are proved. The main approaches to the analysis and synthesis of switching circuits are considered, as well as the questions of minimizing Boolean functions in the DNF class.

Statements and operations on them, functions, formulas and basic identities of the algebra of logic are considered.

The concept of a graph is introduced, methods of its assignment are discussed. The concepts of directed and undirected graphs, isomorphism of graphs, subgraphs, paths, circuits, cycles, graph connectivity, Euler, Hamiltonian, planar graphs are defined. For Euler graphs, an appropriate criterion and an algorithm for constructing an Euler cycle are formulated. We consider weighted undirected and directed graphs, algorithms of the "wave front" and Dijkstra for finding the shortest paths from the selected vertex of the graph to the rest. The concepts of a tree and a spanning tree of a graph are introduced, and a "greedy" algorithm for constructing a minimal spanning tree of a weighted undirected graph is considered.

For the introduced concept of a transport network, the full and maximum flows are determined, an algorithm for constructing the complete and a method for constructing the maximum (based on the graph of increments) flows are discussed. The concept of a cut of a transport network is introduced, a theorem on a minimal cut is formulated.

Code definitions, alphabetic coding are introduced. The prefix code is considered, its one-to-one is proved. The Kraft - Macmillan inequality is proved. For Fano and Huffman codes, algorithms for their construction are discussed. The Hamming code and its method of using it for coding that can be corrected for at most one error are considered. A geometric interpretation of self-correcting codes on the set of vertices of an n-dimensional cube is considered.

1. Study objective

Purpose of the course

The purpose of the discipline "Discrete Mathematics" is to form:

- world outlook in thematic areas of natural science, associated with the study of the properties of finite or infinite structures with discontinuous processes or the separability of their constituent elements;
- basic knowledge for further use in other areas of mathematics and disciplines of natural science content;
- mathematical culture, research skills and the ability to understand, improve and apply in practice modern mathematical apparatus.

Tasks of the course

- acquainting students with the main thematic areas of discrete mathematics and the formulation of typical mathematical problems;
- the formation of students' basic knowledge and skills in the application of basic methods for solving typical mathematical problems of discrete mathematics;
- the formation of a general mathematical culture, the ability to think logically, to prove the main statements, to establish logical connections and analogies between concepts;
- the formation of skills and abilities to apply the acquired knowledge for independent problem solving and analysis of the results.

2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

Code and the name of the competence	Competency indicators
UC-1 Search and identify, critically evaluate and synthesize information, apply a systematic approach to problem-solving	UC-1.1 Analyze problems, highlight the stages of their solution, plan the actions required to solve them
	UC-1.2 Find, critically assess, and select information required for the task in hand
	UC-1.3 Consider various options for solving a problem, assess the advantages and disadvantages of each option
	UC-1.4 Make competent judgments and estimates supported by logic and reasoning
UC-6 Use time-management skills, apply principles of self-development and lifelong learning	UC-6.2 Plan independent activities in professional problem-solving; critically analyze the work performed; find creative ways to use relevant experience for self-development

3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

- operations on sets, basic identities of the algebra of sets;
- addition and multiplication theorems for finite sets;
- the main types of finite samples (permutations, placement, combination, placement and combination with repetitions, permutation with repetitions) and expressions for counting their quantities;
- generalization of the formula for inclusion and exclusion for counting the number of elements with exactly r properties;
- definition of a Boolean function, ways of specifying Boolean functions, elementary Boolean functions of one and two variables;
- canonical types of a Boolean function (SDNF, SKNF, Zhegalkin polynomial), the duality principle;
- definitions of closed and complete systems of Boolean functions, Post's completeness theorem;
- a way to implement a Boolean function in the form of a switch circuit conductance function;
- operations on statements, basic identities of the algebra of statements;
- definitions of the main types of graphs (graph, multigraph, directed and undirected graphs), ways to define them using matrices, definition of isomorphism and connectivity;
- the main types of subgraphs (paths, chains, cycles);
- definitions of Euler, Hamiltonian, semi-Hamiltonian, planar graphs;
- criteria for the Euler and planarity of graphs, an algorithm for constructing an Euler cycle;
- determination of a weighted graph, algorithms of the "wave front" and Dijkstra for finding the shortest paths from the selected vertex of the graph to the rest;
- definitions of a “tree”, “forest”, “spanning tree” of a graph, a “greedy” algorithm for constructing a minimal “spanning tree” of a weighted undirected graph;
- definition of the transport network, full and maximum flows, algorithms for their construction, the theorem on the minimum cut;
- definitions of the code, alphabetical code, properties of one-to-one code;
- definition of the prefix code and the theorem on its one-to-one;
- Kraft - Macmillan inequality;
- algorithms for constructing Fano and Huffman codes;
- determination of a self-correcting code, its geometric interpretation on a unit n-dimensional cube, estimates for the Gil lower bound and the Hamming upper bound;
- definition and properties of the Hamming code.

be able to:

- perform identical transformations according to the rules of set algebra;
- to use the main types of finite samples when solving the simplest combinatorial problems;
- apply the theorems of addition and multiplication for finite sets, generalization of the formula for inclusion and exclusion;
- to reduce the Boolean function to canonical forms (SDNF, SKNF, Zhegalkin polynomial) using the table and the method of algebraic transformations;
- to study the closedness and completeness of the systems of Boolean functions;
- to analyze and synthesize switching circuits, to minimize their conductance function in the DNF class;
- perform identical transformations according to the rules of propositional algebra, establish the truth of complex propositions;
- to specify the main types of graphs using matrices, to investigate the isomorphism of pairs of graphs;
- to apply the criteria of the Euler and planarity of the graphs, to construct the Euler cycle;
- investigate the graph for Hamiltonian and semi-Hamiltonian;
- find the shortest paths from the selected vertex of the weighted graph to the rest;
- find the minimum "spanning tree" of a weighted undirected graph;
- find the full flow in the transport network;
- compile a graph of increments for a flow in a transport network and find the maximum flow;
- find the minimum section of the transport network;
- apply the Craft - Macmillan inequality, build a "tree" of the prefix code;
- build "trees" for Fano and Huffman codes;
- using the Hamming code, encrypt, search for an error and correct it for information messages of arbitrary length.

master:

- methods for solving combinatorial problems;
- methods for solving problems of graph theory, in particular:
- an algorithm for constructing an Euler cycle;
- algorithms of the "wave front" and Dijkstra of finding the shortest paths from the selected vertex of the graph to the rest;
- a “greedy” algorithm for constructing a minimal “spanning tree” of a weighted undirected graph;
- the method of constructing a complete flow in the transport network;
- by the method of constructing the maximum flow in the transport network using the increment graph;
- methods for solving problems of coding theory, in particular:
- algorithms for constructing Fano and Huffman codes;
- the method of applying self-correcting codes.

4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

4.1. The sections of the course (training module) and the complexity of the types of training sessions

№	Topic (section) of the course	Types of training sessions, including independent work			
		Lectures	Seminars	Laboratory practical	Independent work
1	Algebra of propositions.	2	2		14
2	An introduction to boolean functions.	6	6		16
3	Elements of combinatorics.	6	6		14
4	Elements of graph theory.	7	7		16
5	Elements of coding theory.	7	7		14
6	Elements of set theory.	2	2		16
AH in total		30	30		90
Exam preparation		30 AH.			

Total complexity	180 AH., credits in total 4

4.2. Content of the course (training module), structured by topics (sections)

Semester: 2 (Spring)

1. Algebra of propositions.

Statements and operations on them. Functions, formulas and basic identities of the algebra of logic.

2. An introduction to boolean functions.

Boolean functions: definition, tabular way of assigning, lexicographic order of listing all sets of variables, elementary Boolean functions of one and two variables. Substantial and dummy variables. Representation of Boolean functions by formulas. Equivalence of formulas, basic identities of binary Boolean algebra. A theorem on the (disjunctive) expansion of a Boolean function in the first m variables. SDNF of a nonzero Boolean function. Dual boolean function. Duality principle. A theorem on the (conjunctive) expansion of a Boolean function in the first m variables. The SKNF is not identically equal to one of the Boolean function. Zhegalkin polynomials. Existence and uniqueness of the representation of an arbitrary Boolean function by the canonical Zhegalkin polynomial. Closed and complete systems of Boolean functions. Five classes of Fasting. Post's completeness theorem. Analysis and synthesis of switching circuits. Minimization of Boolean functions in the DNF class.

3. Elements of combinatorics.

Finite sets. Addition and multiplication theorems. Sampling, rearrangement, placement, combination. Placements and combinations with reps. Permutations with repetitions, a polynomial theorem. Inclusion and exclusion formula. Generalization of the inclusion and exclusion formula for counting the number of elements with exactly r properties.

4. Elements of graph theory.

The concept of a graph, methods of assignment. Directed and undirected graphs. Graph isomorphism. Subgraphs, paths, chains, cycles. Graph connectivity. Euler graphs: a criterion, an algorithm for constructing an Euler cycle. Hamiltonian and semi-Hamiltonian graphs. Planar graphs, planarity criterion. Weighted undirected and directed graphs. Algorithms of the "wave front" and Dijkstra for finding the shortest paths from the selected vertex of the graph to the rest. Trees. The spanning tree of the graph. A "greedy" algorithm for constructing the minimum spanning tree of a weighted undirected graph. Transport networks. Full and maximum flows. Algorithm for constructing a complete stream. Increment graph. Algorithm for constructing the maximum flow. Transport network sections. Minimum cut theorem.

5. Elements of coding theory.

The code. Alphabetic coding. Prefix code, its one-to-one. Craft-Macmillan inequality. Fano and Huffman codes, algorithms for their construction. Hamming code. Bug fix. Self-correcting codes. Partitioning the set of vertices of an n-dimensional cube into balls.

6. Elements of set theory.

Sets, operations on sets. Euler - Venn diagrams. Algebra of sets. Basic identities of the algebra of sets.

5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)

Classroom equipped with a blackboard, multimedia projector, screen and microphone.

6. List of the main and additional literature, that is necessary for the course (training module) mastering

Main literature

1. Основы комбинаторики и теории чисел [Текст] : сборник задач : учеб. пособие для вузов / А. А. Глибичук [и др.] .— Долгопрудный : Изд. Дом "Интеллект", 2015 .— 104 с.

2. Комбинаторика и теория вероятностей [Текст] : [учеб. пособие для вузов] / А. М. Райгородский .— Долгопрудный : Интеллект, 2013 .— 104 с. - Библиогр.: с. 99. - 3000 экз. - ISBN 978-5-91559-147-8 .— Полный текст (Режим доступа : доступ из сети МФТИ).

Additional literature

1. Комбинаторика и информатика [Текст]. Ч. 1. Комбинаторный анализ : учеб. пособие / В. К. Леонтьев; Моск. физ.- техн. ин-т (гос. ун-т) .— М : МФТИ , 2015 .— 174 с. + pdf-версия. - Библиогр.: с. 173. - 250 экз. - ISBN 978-5-7417-0545-2. — Полный текст (Доступ из сети МФТИ).

7. List of web resources that are necessary for the course (training module) mastering

1. http://lib.mipt.ru – электронная библиотека Физтеха.
2. http://www.exponenta.ru – образовательный математический сайт.
3. http://mathnet.ru – общероссийский математический портал.
4. http://www.edu.ru – федеральный портал «Российское образование».
5. http://benran.ru –библиотека по естественным наукам Российской академии наук.
6. http://techlibrary.ru/books.htm – техническая библиотека.
7. https://studizba.com/files/show/djvu/1717-1-gorbatov-v-a--fundamental-nye-osnovy.html
8. https://www.bookvoed.ru/files/3515/10/67/97.pdf

8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)

The lectures use multimedia technologies, including the demonstration of presentations.

To control and correct knowledge, students can use computer testing.

In the process of independent work of students, it is possible to use software such as Grin, Mathematica, Scilab, etc

9. Guidelines for students to master the course

A student studying a course in discrete mathematics must, on the one hand, master the general conceptual apparatus, and on the other hand, must learn to apply theoretical knowledge in practice.

Since in modern literature there are, as a rule, several variants of definitions of the concepts under consideration (for example, a graph), when studying theoretical material, the student is recommended to adhere to one main source - a course of lectures.

Successful mastering of the course requires independent student work. The course program contains the minimum required time for a student to work on a topic.

Independent work should include:

- reading and taking notes of the recommended literature;
- study of educational material (based on lecture notes, educational and scientific literature);
- preparation of answers to questions intended for self-study;
- proof of individual statements, properties;
- solving problems offered to students in lectures and practical classes;
- preparation for practical training, semester test work, exam.

The student's independent work is guided and monitored in the form of individual consultations.

An indicator of mastery of the material is the ability to solve problems. To form the ability to apply theoretical knowledge in practice, the student needs to solve as many problems as possible. When solving problems, each action must be argued, referring to the known theoretical information.

When mastering the course of discrete mathematics, it is advisable to adhere to the following scheme: study of the lecture material according to the synopsis on the same day when the lecture was listened to (approximately 10-15 minutes); repetition of the material on the eve of the next lecture (approximately 10-15 minutes), working out educational material based on lecture notes, educational and scientific literature, preparing answers to questions intended for self-study (approximately 1 hour a week), preparing for a practical lesson, solving problems (approximately 1 hour). In preparation for practical exercises, it is necessary to repeat the previously studied basic definitions, theorem formulations. At the beginning of the lesson, as a rule, a short (approximately 10-15 minutes) survey is conducted on the material of the past lessons in oral or written form.

It is important to achieve an understanding of the studied material, and not its mechanical memorization. If you find it difficult to study certain topics, issues, you should seek advice from a lecturer or teacher who conducts practical classes.

SUPPLEMENT

Assessment funds for course (training module)

major:	Applied Mathematics and Informatics
specialization:	Computer Science/Информатика
	Phystech School of Applied Mathematics and Informatics
	Chair of Discrete Mathematics
term:	1
qualification:	Bachelor

Semester, form of interim assessment: 2 (spring) - Exam

Authors:

L.P. Kuptsov, candidate of physics and mathematical sciences, associate professor
A.G. Zdor, candidate of physics and mathematical sciences

1. Competencies formed during the process of studying the course

Code and the name of the competence	Competency indicators
UC-1 Search and identify, critically evaluate and synthesize information, apply a systematic approach to problem-solving	UC-1.1 Analyze problems, highlight the stages of their solution, plan the actions required to solve them
	UC-1.2 Find, critically assess, and select information required for the task in hand
	UC-1.3 Consider various options for solving a problem, assess the advantages and disadvantages of each option
	UC-1.4 Make competent judgments and estimates supported by logic and reasoning
UC-6 Use time-management skills, apply principles of self-development and lifelong learning	UC-6.2 Plan independent activities in professional problem-solving; critically analyze the work performed; find creative ways to use relevant experience for self-development

2. Competency assessment indicators

As a result of studying the course the student should:

know:

- operations on sets, basic identities of the algebra of sets;
- addition and multiplication theorems for finite sets;
- the main types of finite samples (permutations, placement, combination, placement and combination with repetitions, permutation with repetitions) and expressions for counting their quantities;
- generalization of the formula for inclusion and exclusion for counting the number of elements with exactly r properties;
- definition of a Boolean function, ways of specifying Boolean functions, elementary Boolean functions of one and two variables;
- canonical types of a Boolean function (SDNF, SKNF, Zhegalkin polynomial), the duality principle;
- definitions of closed and complete systems of Boolean functions, Post's completeness theorem;
- a way to implement a Boolean function in the form of a switch circuit conductance function;
- operations on statements, basic identities of the algebra of statements;
- definitions of the main types of graphs (graph, multigraph, directed and undirected graphs), ways to define them using matrices, definition of isomorphism and connectivity;
- the main types of subgraphs (paths, chains, cycles);
- definitions of Euler, Hamiltonian, semi-Hamiltonian, planar graphs;
- criteria for the Euler and planarity of graphs, an algorithm for constructing an Euler cycle;
- determination of a weighted graph, algorithms of the "wave front" and Dijkstra for finding the shortest paths from the selected vertex of the graph to the rest;
- definitions of a “tree”, “forest”, “spanning tree” of a graph, a “greedy” algorithm for constructing a minimal “spanning tree” of a weighted undirected graph;
- definition of the transport network, full and maximum flows, algorithms for their construction, the theorem on the minimum cut;
- definitions of the code, alphabetical code, properties of one-to-one code;
- definition of the prefix code and the theorem on its one-to-one;
- Kraft - Macmillan inequality;
- algorithms for constructing Fano and Huffman codes;
- determination of a self-correcting code, its geometric interpretation on a unit n-dimensional cube, estimates for the Gil lower bound and the Hamming upper bound;
- definition and properties of the Hamming code.

be able to:

- perform identical transformations according to the rules of set algebra;
- to use the main types of finite samples when solving the simplest combinatorial problems;
- apply the theorems of addition and multiplication for finite sets, generalization of the formula for inclusion and exclusion;
- to reduce the Boolean function to canonical forms (SDNF, SKNF, Zhegalkin polynomial) using the table and the method of algebraic transformations;
- to study the closedness and completeness of the systems of Boolean functions;
- to analyze and synthesize switching circuits, to minimize their conductance function in the DNF class;
- perform identical transformations according to the rules of propositional algebra, establish the truth of complex propositions;
- to specify the main types of graphs using matrices, to investigate the isomorphism of pairs of graphs;
- to apply the criteria of the Euler and planarity of the graphs, to construct the Euler cycle;
- investigate the graph for Hamiltonian and semi-Hamiltonian;
- find the shortest paths from the selected vertex of the weighted graph to the rest;
- find the minimum "spanning tree" of a weighted undirected graph;
- find the full flow in the transport network;
- compile a graph of increments for a flow in a transport network and find the maximum flow;
- find the minimum section of the transport network;
- apply the Craft - Macmillan inequality, build a "tree" of the prefix code;
- build "trees" for Fano and Huffman codes;
- using the Hamming code, encrypt, search for an error and correct it for information messages of arbitrary length.

master:

- methods for solving combinatorial problems;
- methods for solving problems of graph theory, in particular:
- an algorithm for constructing an Euler cycle;
- algorithms of the "wave front" and Dijkstra of finding the shortest paths from the selected vertex of the graph to the rest;
- a “greedy” algorithm for constructing a minimal “spanning tree” of a weighted undirected graph;
- the method of constructing a complete flow in the transport network;
- by the method of constructing the maximum flow in the transport network using the increment graph;
- methods for solving problems of coding theory, in particular:
- algorithms for constructing Fano and Huffman codes;
- the method of applying self-correcting codes.

3. List of typical control tasks used to evaluate knowledge and skills

The current control is carried out on the basis of the fulfillment by students of a set of homework assignments and tests in accordance with the curriculum. Data on attendance and current performance are entered by teachers in special journals.

Current control based on homework is carried out during the academic semester within the timeframe established by the Educational Department, in accordance with the curriculum.

To pass the assignment, the student is obliged to provide a solution to the homework problem in writing, answer the teacher's questions and write a test on the assignment, which tests the knowledge of concepts and statements on the topics of the assigned assignment and the ability to solve problems.

During the execution of the test work, you cannot use the help of other persons, computers and mobile phones.

4. Evaluation criteria

Exam questions:

1. Give an example of a correspondence that has the following properties: a) surjective, not injective, not a mapping; b) not surjective, injective, not a mapping; c) not surjective, not injective, mapping.
2. State the defining properties of correspondences inverse to injective and to surjective.
3. The correspondence is both injective and surjective. Is it necessarily a bijection?
4. Prove that the composition of mappings, injective correspondences, surjective correspondences and bijections is a mapping, injective correspondence, surjective correspondence, bijection, respectively.

5. Show that in any infinite set there is a countable subset.
6. Prove that any subset of a countable set is at most countable.

Topics for coursework:
- Prove that the union of two countable sets is countable.

Ticket 1:
1. State the defining properties of correspondences inverse to injective and to surjective.
2. The correspondence is both injective and surjective. Is it necessarily a bijection?

Ticket 2:
1. Prove that the composition of mappings, injective correspondences, surjective correspondences and bijections is a mapping, injective correspondence, surjective correspondence, and bijection, respectively.
2. Show that any infinite set contains a countable subset.

- the mark "excellent (10)" is given to a student who has shown comprehensive, systematized, in-depth knowledge of the curriculum of the discipline and the ability to confidently apply them in practice when solving specific problems, free and correct justification of the decisions made

- the mark "excellent (9)" is given to a student who has shown comprehensive, systematized, in-depth knowledge of the curriculum of the discipline and the ability to apply them in practice in solving specific problems, free and correct justification of the decisions

- the mark "excellent (8)" is given to a student who has shown comprehensive systematized, deep knowledge of the curriculum of the discipline and the ability to apply them in practice in solving specific problems, and the correct justification of the decisions

- the mark "good (7)" is given to a student if he firmly knows the material, expresses it competently and to the point, knows how to apply the acquired knowledge in practice, but makes some inaccuracies in the answer or in solving problems;

- the mark "good (6)" is given to the student if he knows the material, presents it competently and in essence, knows how to apply the knowledge gained in practice, but makes some inaccuracies in the answer or in solving problems;

- the mark "good (5)" is given to the student if he knows the material, and essentially expounds it, knows how to apply the knowledge gained in practice, but makes some inaccuracies in the answer or in solving problems;

- the mark "satisfactory (4)" is given to a student who has shown a fragmented, scattered nature of knowledge, insufficiently correct formulations of basic concepts, a violation of the logical sequence in the presentation of the program material, but at the same time he owns the main sections of the curriculum necessary for further education and can apply the obtained knowledge by model in a standard situation;

- the mark "satisfactory (3)" is given to a student who has shown a fragmentary, scattered nature of knowledge, insufficiently correct formulations of basic concepts, violation of the logical sequence in the presentation of program material, but at the same time he has fragmentary knowledge of the main sections of the curriculum necessary for further education and can apply the knowledge gained by the model in a standard situation;

- the mark "unsatisfactory (2)" is given to a student who does not know most of the main content of the curriculum of the discipline, makes gross mistakes in the formulation of the basic concepts of the discipline and does not know how to use the knowledge gained in solving typical practical problems;

- grade "unsatisfactory (1)" is given to a student who does not know the formulations of the basic concepts of the discipline.

5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience

During the exam, students can use the discipline program.

3. Перечень типовых контрольных заданий, используемых для оценки знаний, умений, навыков

Для оценки знаний в конце семестра проводится семестровая контрольная работа (типовой вариант – в приложении А).

Промежуточная аттестация по дисциплине «Дискретная математика» осуществляется в форме экзамена. Экзамен проводится в устной форме.

Примеры контрольных вопросов:

1. Сформулировать определения операций над множествами, свойства операций, основные тождества алгебры множеств.
2. Сформулировать теоремы сложения и умножения для конечных множеств.
3. Дать определения основных видов конечных выборок (перестановки, размещения, сочетания, размещения и сочетания с повторениями, перестановки с повторениями) и выражения для подсчета их количеств.
4. Записать обобщение формулы включения и исключения для подсчета количества элементов, обладающих ровно *r* свойствами.
5. Дать определение булевой функции, сформулировать способы задания булевых функций, перечислить все элементарные булевы функции от одной и двух переменных.
6. Дать определения канонических видов булевой функции (СДНФ, СКНФ, полином Жегалкина).
7. Описать методы построения СДНФ, СКНФ, полинома Жегалкина.
8. Сформулировать определение двойственной функции.
9. Сформулировать принцип двойственности.
10. Дать определения замкнутых и полных систем булевых функций.
11. Сформулировать теорему Поста о полноте.
12. Дать определение функции проводимости переключательной схемы.
13. Описать способ реализации булевой функции в виде функции проводимости последовательно-параллельной переключательной схемы.
14. Дать определения сокращенной, тупиковой и минимальной ДНФ, описать процедуру минимизации функции в классе ДНФ.
15. Дать определения операций над высказываниями, свойства операций, основные тождества алгебры высказываний.
16. Дать определения основных видов графов (граф, мультиграф, ориентированный и неориентированный графы), описать способы их задания с помощью матриц смежности и инцидентности
17. Дать определение изоморфизма графов.
18. Дать определения основных видов подграфов (пути, цепи, циклы).
19. Сформулировать определения эйлеровых, гамильтоновых, полугамильтоновых, планарных графов.
20. Сформулировать критерии эйлеровости и планарности графов.
21. Описать алгоритм построения эйлерова цикла.
22. Дать определение взвешенного графа, описать алгоритм «фронта волны» нахождения кратчайших путей от выделенной вершины графа до остальных.
23. Описать алгоритм Дейкстры нахождения кратчайших путей от выделенной вершины графа до остальных.
24. Дать определения «дерева», «леса», «остовного дерева» графа.
25. Описать «жадный» алгоритм построения минимального «остовного дерева» взвешенного неориентированного графа.
26. Дать определение транспортной сети, полного и максимального потоков в ней.
27. Описать методы построения полного и максимального потока в транспортной сети.
28. Дать определение разреза и сформулировать теорему о минимальном разрезе.
29. Дать определения кода, алфавитного кода, свойства взаимной однозначности кода.

30. Сформулировать определение префиксного кода. Должен ли взаимно однозначный алфавитный код обязательно быть префиксным?

31. Сформулировать неравенство Крафта – Макмиллана.

32. Описать алгоритмы построения кодов Фано и Хаффмена.

33. Дать определение самокорректирующегося кода, его геометрическую интерпретацию на единичном *n*-мерном кубе.

34. Сформулировать метод шифрования, поиска и исправления ошибки в коде Хэмминга.

Примеры контрольных заданий:

1. Исследовать полноту системы булевых функций $\{(x+y)\cdot(\bar{x}+\bar{y});\ \ x\cdot y\oplus z;\ \ (x\cdot y)\square z;\ \ x\cdot y+x\cdot z+y\cdot z\}$.

2. Для булевой функции $f(x_1,x_2,x_3,x_4)=x_1+(x_2\oplus x_3)+\bar{x}_4$ исследовать принадлежность к каждому из пяти классов Поста T_0, T_1, S, L, M.

3. Функция $f^*(x_1,x_2,x_3)$ задана строкой (0,0,0,1,1,0,0,0). Для функции $f(x_1,x_2,x_3)$ построить канонический многочлен Жегалкина, СКНФ, сокращенную и все минимальные ДНФ.

4. Для булевой функции $f(x,y,z)$, заданной строкой $(1,0,1,1,1,1,0,1)$, построить сокращенную, все тупиковые и минимальные ДНФ.

5. Построить префиксный код с минимальной избыточностью (код Хаффмена) с помощью алфавита $\{0;1;2;3\}$ для передачи девятибуквенных сообщений с относительными частотами появления букв: 0,22; 0,20; 0,12; 0,11; 0,10; 0,09; 0,08; 0,06; 0,02.

Дальнейшие примеры контрольных вопросов и заданий приведены в приложении Б.

4. Критерии оценивания

Оценка «отлично (10)» выставляется обучающемуся, если он показал всесторонние, систематизированные, глубокие знания учебной программы дисциплины и умение уверенно применять их на практике при решении конкретных задач, свободное и правильное обоснование принятых решений;

оценка «отлично (9)» выставляется обучающемуся, если он показал всесторонние, систематизированные, глубокие знания учебной программы дисциплины и умение уверенно применять их на практике при решении конкретных задач, свободное и правильное обоснование принятых решений, но при этом были допущены небольшие неточности, которые были самостоятельно обнаружены и исправлены;

оценка «отлично (8)» выставляется обучающемуся, если он показал всесторонние, систематизированные, глубокие знания учебной программы дисциплины и умение уверенно применять их на практике при решении конкретных задач, свободное и правильное обоснование принятых решений, но при этом были допущены небольшие неточности, которые после указания экзаменатора были самостоятельно исправлены;

оценка «хорошо (7)» выставляется обучающемуся, если он твердо знает материал, грамотно и по существу излагает его, умеет применять полученные знания на практике, но допускает неточности в ответе или делает несущественные ошибки при решении задач;

оценка «хорошо (6)» выставляется обучающемуся, если он твердо знает материал, грамотно и по существу излагает его, умеет применять полученные знания на практике, но допускает небольшие ошибки в ответе и (или) при решении задач;

оценка «хорошо (5)» выставляется обучающемуся, если он твердо знает материал, грамотно и по существу излагает его, умеет применять полученные знания на практике, но отвечает неуверенно и (или) допускает ошибки при решении задач;

оценка «удовлетворительно (4)» выставляется обучающемуся, показавшему фрагментарный, разрозненный характер знаний, неточные формулировки базовых понятий, нарушения логической последовательности в изложении программного материала, если при этом он владеет основными разделами учебной программы,

необходимыми для дальнейшего обучения и может применять полученные знания по образцу в стандартной ситуации;

оценка «удовлетворительно (3)» выставляется обучающемуся, показавшему фрагментарный, разрозненный характер знаний, неточные формулировки базовых понятий, нарушения логической последовательности в изложении программного материала, не владеющему некоторыми разделами учебной программы, но умеющему применять полученные знания по образцу в стандартной ситуации;

оценка «неудовлетворительно (2)» выставляется обучающемуся, который не знает большей части основного содержания учебной программы дисциплины, допускает грубые ошибки в формулировках основных понятий дисциплины и не умеет использовать полученные знания при решении типовых практических задач;

оценка «неудовлетворительно (1)» выставляется обучающемуся, показавшему полное незнание учебной программы дисциплины.

5. Методические материалы, определяющие процедуры оценивания знаний, умений, навыков и (или) опыта деятельности

При проведении устного экзамена обучающемуся предоставляется 1,5 астрономических часа на подготовку. Опрос обучающегося по билету на экзамене не должен превышать двух астрономических часов.

Во время проведения экзамена обучающиеся могут пользоваться только программой дисциплины.