Federal State Autonomous Educational Institution of Higher Education "Moscow Institute of Physics and Technology (National Research University)"

APPROVED
Vice Rector for Academic Affairs

A.A. Voronov

Work program of the course (training module)

course:	Functions of One Complex Variable/Теория функций комплексного переменного
major:	Applied Mathematics and Informatics
specialization:	Computer Science/Информатика
	Phystech School of Applied Mathematics and Informatics
	Chair of Higher Mathematics
term:	3
qualification:	Bachelor

Semester, form of interim assessment: 6 (spring) - Exam

Academic hours: 90 AH in total, including:
lectures: 45 AH.
seminars: 45 AH.
laboratory practical: 0 AH.

Independent work: 60 AH.

Exam preparation: 30 AH.

In total: 180 AH, credits in total: 4

Number of course papers, tasks: 3

Authors of the program:

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The program was discussed at the Chair of Higher Mathematics 20.05.2020

Annotation

Discipline belongs to the basic part of the educational program. Mastering the discipline is aimed at developing the ability to acquire new scientific and professional knowledge using modern educational and information technologies. The following topics are considered: Elementary functions of a complex variable, their differentiability and integrability along a contour, Cauchy-Riemann conditions, Inverse function theorem, Multivalued functions, Principal regular branches of functions, Cauchy integral theorem, Cauchy integral formula, Power series, Taylor series for regular functions, Laurent series for a regular function in a ring, Isolated singular points. Deductions. Computation of integrals, Entire and meromorphic functions, Their properties, The concept of analytic continuation, Singular points of analytic functions, Argument principle, Rouche's theorem, Geometric principles of regular functions, Conformal mappings in the extended complex plane, The classical Dirichlet problem for the Laplace equation on the plane.

1. Study objective

Purpose of the course

- studying methods and mastering the apparatus for analyzing functions of a complex variable for their application in solving problems of mathematical physics, hydrodynamics, aerodynamics, etc.

Tasks of the course

- study of the properties of regular functions, expansion of regular functions in a ring in the form of a sum of a Laurent series;
- the ability to investigate isolated singular points of a function and apply the theory of residues to calculate integrals, including improper integrals of functions of a real variable;
- possession of the method of conformal mappings when solving problems of equations of mathematical physics on a plane.

2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

Code and the name of the competence	Competency indicators
UC-1 Search and identify, critically evaluate and synthesize information, apply a systematic approach to problem-solving	UC-1.1 Analyze problems, highlight the stages of their solution, plan the actions required to solve them
	UC-1.2 Find, critically assess, and select information required for the task in hand
	UC-1.3 Consider various options for solving a problem, assess the advantages and disadvantages of each option
	UC-1.4 Make competent judgments and estimates supported by logic and reasoning
UC-6 Use time-management skills, apply principles of self-development and lifelong learning	UC-6.2 Plan independent activities in professional problem-solving; critically analyze the work performed; find creative ways to use relevant experience for self-development

3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

- Cauchy-Riemann conditions, Cauchy integral theorem, Cauchy integral formula;
- criteria for the regularity of functions: the Morer and Weierstrass theorems, the representation of a regular function given in an annulus as a sum of a Laurent series; types of isolated feature points;
- the concept of a deduction at an isolated singular point;
- Cauchy's theorem on the calculation of integrals in terms of the sum of residues;
- the concept of a regular branch of a multivalued function;
- the concept of conformal mapping, linear fractional functions and Zhukovsky functions;
- Riemann's theorem on the conformal equivalence of simply connected domains;
- the solution of the classical Dirichlet problem for the Laplace equation on the plane by the method of conformal mappings.

be able to:

- represent a regular function defined in a ring as the sum of a Laurent series;
- find and investigate isolated singular points of a function;
- apply the theory of residues to calculate integrals, including improper integrals of functions of a real variable;
- find functions that carry out conformal mapping of given areas;
- to apply the method of conformal mappings when solving the Dirichlet problem for the Laplace equation on the plane.

master:
- methods of complex analysis used in calculating integrals using residues;
- methods of complex analysis used in solving problems of hydrodynamics, aerodynamics, mathematical physics, etc.

4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

4.1. The sections of the course (training module) and the complexity of the types of training sessions

№	Topic (section) of the course	Types of training sessions, including independent work			
		Lectures	Seminars	Laboratory practical	Independent work
1	Elementary functions of a complex variable, their differentiability and integrability along a contour. Cauchy-Riemann conditions. Inverse function theorem. Multivalued functions. Main regular branches of functions. Integral Cauchy theorem. Integral Cauchy formula.	8	10		6
2	Power series. Taylor series for a regular function. Laurent series for a regular function in a ring.	12	10		12
3	Isolated singular points. Deductions. Calculation of integrals.	15	12		20
4	Entire and meromorphic functions. Their properties. The concept of analytic continuation. Singular points of analytic functions. The principle of argument. Rouche's theorem.	2	5		4
5	Geometric principles of regular functions. Conformal mappings in the extended complex plane.	4	4		8
6	The classical Dirichlet problem for the Laplace equation in the plane.	4	4		10
AH in total		45	45		60
Exam preparation		30 AH.			
Total complexity		180 AH., credits in total 4			

4.2. Content of the course (training module), structured by topics (sections)

Semester: 6 (Spring)

1. Elementary functions of a complex variable, their differentiability and integrability along a contour. Cauchy-Riemann conditions. Inverse function theorem. Multivalued functions. Main regular branches of functions. Integral Cauchy theorem. Integral Cauchy formula.

Complex numbers. Extended complex plane. Riemann sphere. Sequences and Rows. The concept of a function of a complex variable. Continuous functions.

Differentiation with respect to a complex variable. Cauchy - Riemann conditions. The concept of a function that is regular in a domain. Conjugate harmonic functions of two variables.

Elementary functions of a complex variable: power, rational, exponential and trigonometric, their properties. Inverse function theorem (non-degenerate case). The concept of a multivalued function and its regular branches. Main regular branches of multivalued functions.

Integration over a complex variable. Integral Cauchy theorem for regular functions (proof for the case of a piecewise smooth contour in a simply connected domain). Cauchy integral formula (Cauchy integral). Integral of Cauchy type, its regularity.

Antiderivative. A sufficient condition for the existence of an antiderivative. Formula of Newton - Leibniz. Morer's theorem.

Increment of the argument z along a smooth contour, its integral representation and properties. Increment of the argument of the function f (z) along a continuous contour and its properties. General view of regular branches of multivalued functions in a simply connected domain that does not contain zero. Existence conditions and general form of regular branches of multivalued functions.

2. Power series. Taylor series for a regular function. Laurent series for a regular function in a ring.

Power series, Abel's first theorem, radius and circle of convergence. Expansion in a power series of a function regular in a circle. Weierstrass theorems for uniformly converging series of regular functions.

Laurent series and its ring of convergence. Laurent series expansion of a function regular in an annulus, its uniqueness and Cauchy inequality for the coefficients of the Laurent series. Uniqueness theorem for regular functions.

3. Isolated singular points. Deductions. Calculation of integrals.

Isolated singular points of an unambiguous nature, their classification. Determination of the nature of the singular point by the main part of the Laurent series.

Deductions. Calculation of integrals using residues. Lemma Jordan.

4. Entire and meromorphic functions. Their properties. The concept of analytic continuation. Singular points of analytic functions. The principle of argument. Rouche's theorem.

Entire functions. Liouville's theorem. The theorems of Sokhotskii-Weierstrass and Picard (the latter without proof) for entire functions.

Meromorphic functions. Expansion of meromorphic functions into a finite sum of elementary fractions.

The concept of the analytic continuation of elements into each other using a finite chain of circles and along a contour, the equivalence of these concepts. Uniqueness of the analytic continuation. The concept of an analytic function and its Riemann surface. The monodromy theorem (without proof).

Singular points of analytic functions, branch points. The Cauchy-Hadamard theorem on the presence of a singular point on the boundary of the circle of convergence of a power series.

The principle of argument. Rouche's theorem. The main theorem of algebra.

5. Geometric principles of regular functions. Conformal mappings in the extended complex plane.

Openness lemma. The principle of preserving the area. Univalence and multi-sheet in small. The principle of maximum modulus of a regular function. Principle of maximum and minimum of a harmonic function. Schwarz's lemma.

The geometric meaning of the modulus and argument of the derivative. The concept of conformal mapping in the extended complex domain.

Fractional linear functions and their properties.

Conformal mappings using elementary functions. Zhukovsky function and its properties. Riemann's theorem on the conformal equivalence of simply connected domains and the principle of boundary correspondence (without proof).

Cut erasure theorem. Symmetry principle for conformal mappings.

6. The classical Dirichlet problem for the Laplace equation in the plane.

The classical Dirichlet problem for the Laplace equation. Uniqueness of the solution. Poisson's integral for a circle. Existence of a solution to the Dirichlet problem for the Laplace equation.

5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)

- Classroom equipped with a multimedia projector and screen.
- Provision of independent work - the availability of textbooks and problem books for the TFKP course in the library of the institute, Internet access to obtain auxiliary educational material on the website of the Department of Higher Mathematics

6. List of the main and additional literature, that is necessary for the course (training module) mastering

Main literature

1. A collection of problems on complex analysis /L. I. Volkovyskii, G. L. Lunts, I. G. Aramanovich ; translated by J. Berry ; translation edited by T. Kovari. New York, Dover publications, inc., 2018
2. Complex analysis /T. W. Gamelin. New York, Springer, 2001

Additional literature

1. Лекции по теории функций комплексного переменного [Текст] / В. В. Горяйнов, Е. С. Половинкин ; М-во образования и науки РФ, Моск. физ.-техн. ин-т (гос. ун-т) - М.МФТИ,2017

Шабунин, М. И.

Теория функций комплексного переменного [Текст] : учебник для вузов / М. И. Шабунин, Ю. Сидоров .— М. : БИНОМ. Лаб. знаний, 2010, 2012, 2013, 2015 .— 248 с. : ил. - Библиогр.: с. 247. - 5000 экз. - ISBN 978-5-94774-005-9 (в пер.).

7. List of web resources that are necessary for the course (training module) mastering

1. http://univertv.ru/video/matematika/ Open educational video portal UniverTV.ru ahhh! Educational films on various topics. Lectures at leading Russian and foreign universities. A scientific conference or scientific-popular lecture on your issue.
2. http://www.iqlib.ru/ Electronic library IQlib educational and prosvetitelskaia. Educational resource, combining the Internet and library user services to complete the work with the library funds.
Free access to electronic textbooks, reference and teaching AIDS. The audience of the IQlib electronic library is students, teachers of educational institutions, researchers and all those who want to improve their level of knowledge.
3. http://www.edu.ru/modules.php?op=modload&name=Web_Links&file=index&l_op=viewlink&cid=1314 Federal portal "Russian education". Catalog of educational resources.
4. http://www.i-exam.ru – single portal of online testing in education.

8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)

The lectures use multimedia technologies, including the demonstration of presentations.

To control and correct knowledge, students can use computer testing, including on the portal www.i-exam.ru

In the process of independent work of students, it is possible to use software such as Mathcad, Scilab, etc.

9. Guidelines for students to master the course

A student studying the theory of functions of a complex variable must, on the one hand, master the general conceptual apparatus, and on the other hand, must learn to apply theoretical knowledge in practice.

When studying theoretical material, the student needs to restore knowledge in the course of mathematical analysis of functions of one and two real variables, since many of the course results are based on this knowledge. On the other hand, this will allow a deeper understanding of the differences in the properties of functions of complex and real variables.

The main object of study is a holomorphic (regular) function of a complex variable in some circle of the complex plane and its properties. Since there are several equivalent definitions of a function holomorphic in a circle, it is recommended to adhere to one main source (better - a course of lectures) when studying a subject.

Successful mastering of the course requires intense independent work of the student. The course program provides the minimum required time for a student to work on a topic.

Independent work includes:

- reading and taking notes of the recommended literature,
- study of educational material (based on lecture notes, educational and scientific literature),

preparation of answers to questions intended for self-study,

proof of individual statements, properties;

- solving problems offered to students in lectures and practical classes,
- preparation for practical exercises, colloquia.

Guidance and control over the student's independent work is carried out in the form of individual consultations.

An indicator of mastery of the material is the ability to solve problems. To form the ability to apply theoretical knowledge in practice, the student needs to solve as many problems as possible. When solving problems, each action must be argued, referring to the known theoretical information.

In preparation for practical exercises, it is necessary to repeat the previously studied basic definitions, theorem formulations. At the beginning of the lesson, as a rule, a short (10-15 minutes) survey is conducted on the material of the past lessons in oral or written form.

Usually they adhere to the following scheme: study of the material of the lecture on the synopsis on the same day when the lecture was listened to (10-15 minutes); repetition of the material on the eve of the next lecture (10-15 minutes), study of educational material based on lecture notes, educational and scientific literature, preparation of answers to questions intended for self-study (1 hour a week), preparation for a practical lesson, problem solving (1 hour) ... It is important to achieve an understanding of the material being studied, not its mechanical memorization. If you find it difficult to study certain topics, issues, you should seek advice from a lecturer or teacher leading practical classes.

A prerequisite is the performance of household chores, which are drawn up in a notebook specially designated for this and are systematically submitted for verification.

Intermediate control of knowledge is carried out in the form of written tests, in which the student is asked to answer in writing a theoretical question and solve several problems on the test topic, as well as the student, in the course of mastering the course, must complete three individual homework with their subsequent defense:

1. “Complex numbers. Holomorphic functions, their representation in the form of series. Special points of an unambiguous nature and their classification. "
2. “Residues and calculation of integrals. Regular branches of multivalued functions and their expansion in Taylor and Laurent series. The principle of argument and Rusche's theorem. "
3. “Conformal mappings. The principle of symmetry. "

SUPPLEMENT

Assessment funds for course (training module)

major:	Applied Mathematics and Informatics
specialization:	Computer Science/Информатика
	Phystech School of Applied Mathematics and Informatics
	Chair of Higher Mathematics
term:	3
qualification:	Bachelor

Semester, form of interim assessment: 6 (spring) - Exam

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1. Competencies formed during the process of studying the course

Code and the name of the competence	Competency indicators
UC-1 Search and identify, critically evaluate and synthesize information, apply a systematic approach to problem-solving	UC-1.1 Analyze problems, highlight the stages of their solution, plan the actions required to solve them
	UC-1.2 Find, critically assess, and select information required for the task in hand
	UC-1.3 Consider various options for solving a problem, assess the advantages and disadvantages of each option
	UC-1.4 Make competent judgments and estimates supported by logic and reasoning
UC-6 Use time-management skills, apply principles of self-development and lifelong learning	UC-6.2 Plan independent activities in professional problem-solving; critically analyze the work performed; find creative ways to use relevant experience for self-development

2. Competency assessment indicators

As a result of studying the course the student should:

know:

- Cauchy-Riemann conditions, Cauchy integral theorem, Cauchy integral formula;
- criteria for the regularity of functions: the Morer and Weierstrass theorems, the representation of a regular function given in an annulus as a sum of a Laurent series; types of isolated feature points;
- the concept of a deduction at an isolated singular point;
- Cauchy's theorem on the calculation of integrals in terms of the sum of residues;
- the concept of a regular branch of a multivalued function;
- the concept of conformal mapping, linear fractional functions and Zhukovsky functions;
- Riemann's theorem on the conformal equivalence of simply connected domains;
- the solution of the classical Dirichlet problem for the Laplace equation on the plane by the method of conformal mappings.

be able to:

- represent a regular function defined in a ring as the sum of a Laurent series;
- find and investigate isolated singular points of a function;
- apply the theory of residues to calculate integrals, including improper integrals of functions of a real variable;
- find functions that carry out conformal mapping of given areas;
- to apply the method of conformal mappings when solving the Dirichlet problem for the Laplace equation on the plane.

master:

- methods of complex analysis used in calculating integrals using residues;
- methods of complex analysis used in solving problems of hydrodynamics, aerodynamics, mathematical physics, etc.

3. List of typical control tasks used to evaluate knowledge and skills

Current control is carried out on the basis of a point-rating system (BRS) for evaluating knowledge in the discipline being studied. The BRS takes into account the students ' performance of a set of homework assignments and tests in accordance with the curriculum. Data on attendance and current academic performance are entered by teachers in special journals and recorded in the BRS.

Current control on the basis of homework is carried out during the academic semester in the terms set by the Educational Department, in accordance with the curriculum.
To pass the task, the student must provide a solution to the homework problem in writing, answer the questions of the teacher and write a test paper on the task, which checks the knowledge of concepts and statements on the topics of the task and the ability to solve problems.
You can't use other people's help, computers, or mobile phones during the test.

* A BAR is attached to the subject being studied.

4. Evaluation criteria

Certification in the discipline "Functions of One Complex Variable/Теория функций комплексного переменного" is carried out in the form of an exam.
The exam is conducted taking into account the control tasks previously completed by the students.

Control tasks:

1. Differentiation of functions with respect to a complex variable.
2. Cauchy-Riemann conditions.
3. Inverse function theorem.
4. Cauchy's integral theorem for a regular function in a simply connected domain.
5. Integral Cauchy formula.
6. Integral of Cauchy type and its properties.
7. Taylor series. Expansion of a regular function in a power series.
8. Weierstrass' theorems.
9. Laurent series expansion of a function regular in a ring. Uniqueness of the Laurent expansion.
10. The uniqueness theorem for a regular function.
11. The concept of antiderivative. A sufficient condition for the existence of an antiderivative for a continuous function. Newton-Leibniz formula.
12. Morer's theorem. Cut erasure theorem.
13. Classification of isolated singular points of a single-valued nature according to the structure of the main part of the Laurent expansion.
14. Residue theorem. Calculation of improper integrals using residues. Lemma Jordan.
15. General view of the regular branches of multivalued functions in a simply connected domain.
16. Criterion for the selection of regular branches of a multivalued function in a given area, their general form.
17. Criterion for the selection of regular branches of a multivalued function in a given area, their general form.
18. Cauchy inequality for the coefficients of the Laurent series. Liouville's theorem.
19. The principle of argument. Rouche's theorem. The main theorem of algebra.
20. The concept of an analytical function. Singular points of analytic functions. Cauchy-Hadamard theorem.
21. The geometric meaning of the modulus and argument of the derivative. The concept of conformal mapping in a domain on the complex plane. Conformity criterion at a point.
22. The concept of conformal mapping in extended complex. Riemann's theorem on the existence of a conformal mapping and the principle of correspondence of boundaries. (no proof). General view of the conformal mapping of the unit circle onto itself.
23. Fractional linear function and its properties.
24. Zhukovsky function and its properties.
25. Conformal mappings carried out by power and exponential functions.
26. The classical Dirichlet problem for the Laplace equation. Uniqueness of the solution. Poisson's integral for a circle.

Examples of exam tickets:

Ticket 1
1. Cauchy-Riemann conditions.
2. Criterion for the selection of regular branches of a multivalued function in a given area, their general form.

Ticket 2
1. Integral of Cauchy type and its properties.

2. The concept of an analytical function. Singular points of analytic functions. Cauchy-Hadamard theorem.

Grade "excellent (10)" is given to a student who has exhibited extensive and deep knowledge of the course and ability to apply skills when solving specific tasks;

Grade "excellent (9)" is given to a student who has exhibited extensive and deep knowledge of the course and ability to apply skills when solving specific tasks, but he has made minor errors that were independently found and corrected;

Grade "excellent (8)" is given to a student who has exhibited extensive and deep knowledge of the course and ability to apply skills when solving specific tasks, but he has made minor errors that were independently corrected after the instructions of an examiner;

Grade "good (7)" is given to a student who has a good command of the course and is able to apply skills when solving specific tasks, but has made minor mistakes when answering questions or solving problems;

Grade "good (6)" is given to a student who has a good command of the course and is able to apply skills when solving specific tasks, but has made rare mistakes when answering questions or solving problems;

Grade "good (5)" is given to a student who has a good command of the course and is able to apply skills when solving specific tasks, but has made mistakes when answering questions or solving problems;

Grade "satisfactory (4)" is given to a student who has exhibited fragmented knowledge, has made inaccurate formulation of the basic concepts, but understands the subject well, is able to apply the knowledge in standard situations and possesses skills necessary for the future study;

Grade "satisfactory (3)" is given to a student who has exhibited fragmented knowledge, has made inaccurate formulation of the basic concepts, has inconsistencies in understanding the course, but is able to apply the knowledge in standard situations and possesses skills necessary for the future study;

Grade "unsatisfactory (2)" is given to a student who does not possess knowledge of the essential concept of the course, has made gross mistakes in formulations of basic concepts and cannot use the knowledge in solving typical tasks;

Grade "unsatisfactory (1)" is given to a student who has exhibited total lack of knowledge of the course.

5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience

During the oral exam, the student is given one hour (astronomical) to prepare. The questioning of the student card for the oral exam should not exceed two hours.
During the exam, students can use the discipline program.

Scoring for Exams without Written Part
Department of Higher Mathematics

NN	Types of Control	Points
1	Control test 1	0—9
2	Control test 2	0—9
3	Homework 1	0—3
4	Homework 2	0—3
5	Theory checking	0—3
6	Class attendance and activity during seminars	0—3
7	Oral exam	0—70
	Total Score	0—100

The score for the oral exam is computed by the formula $7 \cdot N$, where $N \geq 3$ is a grade gained by a student during the exam. If $N = 1$ or 2, then the student grade is equal to N.

Conversion Scale between the total score and the student grade

Total Score	Student Grade	
93—100	10	Excellent
86—92	9	
79—85	8	
72—78	7	Good
65—71	6	
58—64	5	
51—57	4	Satisfactory
44—50	3	
30—43	2	Unsatisfactory
0—29	1	