

**Federal State Autonomous Educational Institution of Higher Education "Moscow  
Institute of Physics and Technology  
(National Research University)"**

**APPROVED**

**Head of the Phystech School of  
Applied Mathematics and  
Informatics**

**A.M. Raygorodskiy**

**Work program of the course (training module)**

**course:** Introduction to Optimization/Введение в оптимизацию  
**major:** Applied Mathematics and Informatics  
**specialization:** Computer Science/Информатика  
Phystech School of Applied Mathematics and Informatics  
Chair of Discrete Mathematics  
**term:** 3  
**qualification:** Bachelor

Semester, form of interim assessment: 5 (fall) - Grading test

Academic hours: 60 AH in total, including:

lectures: 30 AH.

seminars: 30 AH.

laboratory practical: 0 AH.

Independent work: 75 AH.

In total: 135 AH, credits in total: 3

Number of course papers, tasks: 2

Author of the program: N.K. Tupitsa, associate professor

The program was discussed at the Chair of Discrete Mathematics 05.03.2020

## Annotation

Our course consists of two parts: The first part is devoted to an introduction to optimization. In this part, we will try to illustrate the basic ideas used in developing convex optimization methods. We will not so much analyze the known methods as illustrate the basic concepts using their example. At the end of the course, we'll look at linear optimization.

The second part, we will start with a discussion of convex optimization methods. Next, we will analyze the basic concepts of continuous optimization and analyze known methods. Finally, we will briefly review discrete optimization problems.

### 1. Study objective

#### Purpose of the course

- mastering theoretical and numerical methods for solving problems of finite-dimensional optimization (MO): the theory of necessary and sufficient conditions for a local extremum of a smooth function over a set and some numerical methods for finding local extrema in problems of unconditional and conditional optimization.

#### Tasks of the course

- mastering by students of basic knowledge (concepts, concepts and methods) in the field of ML;
- the acquisition of theoretical knowledge and practical skills in the field of ML;
- providing advice and assistance to students in conducting their own theoretical research in the field of ML.

### 2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

Code and the name of the competence	Competency indicators
Gen.Pro.C-1 Apply fundamental knowledge of physics, mathematics, and/or natural sciences in professional settings	Gen.Pro.C-1.1 Analyze the task in hand, develop approaches to complete it
Gen.Pro.C-4 Collect and process scientific and technical and/or technological data for fundamental and applied problem-solving	Gen.Pro.C-4.3 Prepare abstracts, reports, bibliographies, and reviews of information in professional settings

### 3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

- fundamental concepts, theorems, numerical algorithms of optimization methods (ML);
- modern problems of the corresponding sections of the Ministry of Defense;
- concepts, theorems, methods of proofs and proofs of the main theorems in the sections included in the basic part of the ML cycle;
- basic numerical algorithms for ML with justification of their convergence;
- basic properties of the corresponding mathematical objects;
- analytical and numerical approaches and methods for solving typical applied problems (ML).

be able to:

- understand the task at hand;
- use your knowledge to solve fundamental and applied problems of the Ministry of Defense;
- evaluate the correctness of the problem setting;
- strictly prove or disprove the statement;
- independently find algorithms for solving ML problems, including non-standard ones, and analyze them;
- independently see the consequences of the results obtained;
- accurately present mathematical knowledge in ML field orally and in writing.

master:

- skills of mastering a large amount of information and solving problems of medical science (including complex ones);
- skills of independent work and mastering new disciplines;
- culture of formulation, analysis and solution of mathematical and applied problems that require the use of mathematical approaches and methods of ML for their solution;
- the subject language of MO and the skills of competently describing the solution of problems and presenting the results obtained.

#### 4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

##### 4.1. The sections of the course (training module) and the complexity of the types of training sessions

№	Topic (section) of the course	Types of training sessions, including independent work			
		Lectures	Seminars	Laboratory practical	Independent work
1	Convex sets and affine sets.	6	6		14
2	Separation theorems. Convex functions	6	6		14
3	Duality theory	6	6		15
4	Subdifferential	6	6		16
5	Introduction to linear programming	6	6		16
AH in total		30	30		75
Exam preparation		0 AH.			
Total complexity		135 AH., credits in total 3			

##### 4.2. Content of the course (training module), structured by topics (sections)

Semester: 5 (Fall)

###### 1. Convex sets and affine sets.

Mathematical notation for sets, sequences, sums and products. Working with summation and product operators. Mathematical induction on integers.

###### 2. Separation theorems. Convex functions

Division with remainder. Divisibility. Representation of integers. Positional number systems. Conversion between binary, hexadecimal and octal systems.

###### 3. Duality theory

Prime numbers. Facts on distribution of primes. Greatest common divisor and least common multiple. Euclidean algorithm.

###### 4. Subdifferential

The Fundamental theorem of arithmetic. Linear Diophantine equations. Linear congruences. Chinese remainder theorem. Applications of congruences.

###### 5. Introduction to linear programming

Wilson's theorem, Fermat's little theorem and Euler's theorem. Primitive roots. Existence of primitive roots. Corollaries. Quadratic residues. The Jacobi symbol. Applications of number theory in cryptography.

#### **5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)**

Classroom equipped with a computer and multimedia equipment (projector, sound system).

#### **6. List of the main and additional literature, that is necessary for the course (training module) mastering**

##### Main literature

1. Численные методы оптимизации [Текст] : [учеб.пособие для вузов] / А.Ф.Измайлов, М.В.Солодов .— М. : Физматлит, 2003, 2005 .— 304 с.
2. Методы оптимизации [Текст]. Ч. 2 : Численные алгоритмы : учеб. пособие для вузов / Жадан, В. Г. ; М-во образования и науки РФ, Моск. физ.-техн. ин-т (гос. ун-т) .— М. : МФТИ, 2015 .— 320 с.

##### Additional literature

1. Численные методы, алгоритмы и программы. Введение в распараллеливание [Текст] : учеб. пособие для вузов / В. Е. Карпов, А. И. Лобанов .— М. : Физматкнига, 2014 .— 192 с.

#### **7. List of web resources that are necessary for the course (training module) mastering**

<http://dm.fizteh.ru>

#### **8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)**

The lectures use multimedia technologies, including the demonstration of presentations. Electronic textbooks are used.

#### **9. Guidelines for students to master the course**

1. It is recommended to successfully pass test papers, as this simplifies the final assessment of the subject.
2. To prepare for the final certification in the subject, it is best to use the lecture materials.

**Assessment funds for course (training module)**

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**term:** 3  
**qualification:** Bachelor

Semester, form of interim assessment: 5 (fall) - Grading test

**Author:** N.K. Tupitsa, associate professor

## 1. Competencies formed during the process of studying the course

Code and the name of the competence	Competency indicators
Gen.Pro.C-1 Apply fundamental knowledge of physics, mathematics, and/or natural sciences in professional settings	Gen.Pro.C-1.1 Analyze the task in hand, develop approaches to complete it
Gen.Pro.C-4 Collect and process scientific and technical and/or technological data for fundamental and applied problem-solving	Gen.Pro.C-4.3 Prepare abstracts, reports, bibliographies, and reviews of information in professional settings

## 2. Competency assessment indicators

As a result of studying the course the student should:

### know:

- fundamental concepts, theorems, numerical algorithms of optimization methods (ML);
- modern problems of the corresponding sections of the Ministry of Defense;
- concepts, theorems, methods of proofs and proofs of the main theorems in the sections included in the basic part of the ML cycle;
- basic numerical algorithms for ML with justification of their convergence;
- basic properties of the corresponding mathematical objects;
- analytical and numerical approaches and methods for solving typical applied problems (ML).

### be able to:

- understand the task at hand;
- use your knowledge to solve fundamental and applied problems of the Ministry of Defense;
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### master:

- skills of mastering a large amount of information and solving problems of medical science (including complex ones);
- skills of independent work and mastering new disciplines;
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- the subject language of MO and the skills of competently describing the solution of problems and presenting the results obtained.

## 3. List of typical control tasks used to evaluate knowledge and skills

A list of typical questions and topics for the intermediate certification of students.

1. Convex sets
2. The concept of an affine variety.
3. The concept of a convex function. Function epigraph.
4. Hahn-Banach theorem. Separation theorems.
5. The concept of a conjugate function. Properties of the conjugate function.

An indicative list of questions for monitoring:

1. Convex sets
2. The concept of an affine variety.
3. The concept of a convex function. Function epigraph.
4. Hahn-Banach theorem. Separation theorems.

5. The concept of the conjugate function. Properties of the conjugate function.
6. Support function of a set. Properties of the support function of the set.
7. Directional derivative.
8. Subgradient of a function at a point. Subdifferential of a function at a point. Properties of the subdifferential of a function.
9. The problem of linear programming. D
10. The military problem of linear programming.
11. Simplex method.

#### 4. Evaluation criteria

1. Introduction. General formulation of the problem. Numerical undecidability of the optimization problem in general form. Black box concept. Complexity estimates for the class of Lipschitz functions. Optimization areas business cards.
2. Elements of convex analysis. Ben-Tal-Nemirovski Convex sets and operations on them. Convex hull. Relative interior. Topological properties of convex sets.
3. Theorems of Carathéodory, Radon, Helly. Polyhedral sets. Theorem on alternatives and duality of the linear programming problem. Separation theorem. Polara. Double cone. Lemma Milyutin-Dubovitsky.
4. Definitions and properties of convex functions. Overplot. Jensen's inequality. Convex Preserving Operations. Lipschitz continuity of convex functions. Extrema of convex functions.
5. Subdifferential. Supporting hyperplanes. Legendre transform and its properties. Convex programming problem. The theorem on alternatives in the convex case. Lagrangian. Duality of the convex programming problem. Optimality conditions in saddle form. Karush-Kuhn-Tucker conditions and Slater's condition. Duality in linear and quadratic programming.
6. Saddle points. Interpretations in game theory. Minimax lemma and the Sion-Kakutani theorem.
7. A general mathematical programming problem. Linearization. Optimality conditions of the 1st and 2nd orders.

- the mark "excellent (10)" is given to a student who has shown comprehensive, systematized, in-depth knowledge of the curriculum of the discipline and the ability to confidently apply them in practice when solving specific problems, free and correct justification of the decisions

- the mark "excellent (9)" is given to a student who has shown comprehensive, systematized, in-depth knowledge of the curriculum of the discipline and the ability to apply them in practice in solving specific problems, free and correct justification of decisions

- the mark "excellent (8)" is given to a student who has shown comprehensive systematized, deep knowledge of the curriculum of the discipline and the ability to apply them in practice in solving specific problems, and the correct justification of the decisions made

- the mark "good (7)" is given to a student if he firmly knows the material, expresses it competently and to the point, knows how to apply the knowledge gained in practice, but makes some inaccuracies in the answer or in solving problems;

- the mark "good (6)" is given to the student if he knows the material, presents it competently and in essence, knows how to apply the knowledge gained in practice, but makes some inaccuracies in the answer or in solving problems;

- the mark "good (5)" is given to a student if he knows the material, and essentially expounds it, knows how to apply the knowledge gained in practice, but makes some inaccuracies in the answer or in solving problems;

- the mark "satisfactory (4)" is given to a student who has shown a fragmented, scattered nature of knowledge, insufficiently correct formulations of basic concepts, a violation of the logical sequence in the presentation of the program material, but at the same time he owns the main sections of the curriculum necessary for further education and can apply the received knowledge by model in a standard situation;

- the mark "satisfactory (3)" is given to a student who has shown a fragmentary, scattered nature of knowledge, insufficiently correct formulations of basic concepts, violation of the logical sequence in the presentation of program material, but at the same time he has fragmentary knowledge of the main sections of the curriculum necessary for further education and can apply the knowledge gained by the model in a standard situation;

- the mark "unsatisfactory (2)" is given to a student who does not know most of the main content of the curriculum of the discipline, makes gross errors in the formulation of the basic concepts of the discipline and does not know how to use the knowledge gained in solving typical practical problems

- grade "unsatisfactory (1)" is given to a student who does not know the formulations of the basic concepts of the discipline.

## **5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience**

During the differential test, students can use the discipline program.