Federal State Autonomous Educational Institution of Higher Education "Moscow Institute of Physics and Technology (National Research University)"

APPROVED
Vice Rector for Academic Affairs

A.A. Voronov

Work program of the course (training module)

course:	Multiple Integrals and Field Theory/Кратные интегралы и теория поля
major:	Applied Mathematics and Informatics
specialization:	Computer Science/Информатика
	Phystech School of Applied Mathematics and Informatics
	Chair of Higher Mathematics
term:	2
qualification:	Bachelor

Semester, form of interim assessment: 4 (spring) - Exam

Academic hours: 60 AH in total, including:
- lectures: 30 AH.
- seminars: 30 AH.
- laboratory practical: 0 AH.

Independent work: 90 AH.

Exam preparation: 30 AH.

In total: 180 AH, credits in total: 4

Authors of the program:

A.Y. Petrovich, candidate of physics and mathematical sciences, associate professor, associate professor

O.V. Besov, doctor of physics and mathematical sciences, full professor, professor

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V.Z. Sakbaev, doctor of physics and mathematical sciences, associate professor, associate professor

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O.E. Orel, candidate of physics and mathematical sciences, associate professor, associate professor

The program was discussed at the Chair of Higher Mathematics 20.05.2020

Annotation

Discipline belongs to the basic part of the educational program. Mastering the discipline is aimed at developing the ability to acquire new scientific and professional knowledge using modern educational and information technologies. Topics covered include Multiple Integrals, Curvilinear Integrals. Green's Formula, Surfaces. Surface integrals, Field theory: Ostrogradsky-Gauss and Stokes formulas.

1. Study objective

Purpose of the course

Further familiarization of students with the methods of mathematical analysis, the formation of their evidence-based and logical thinking.

Tasks of the course

- Formation of students' theoretical knowledge and practical skills in the problems of searching for unconditional and conditional extrema of a function of many variables, measure and integral theory, field theory;
- preparing students for the study of related mathematical disciplines;
- acquisition of skills in the application of methods of mathematical analysis in physics and other natural science disciplines.

2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

Code and the name of the competence	Competency indicators
UC-1 Search and identify, critically evaluate and synthesize information, apply a systematic approach to problem-solving	UC-1.1 Analyze problems, highlight the stages of their solution, plan the actions required to solve them
	UC-1.2 Find, critically assess, and select information required for the task in hand
	UC-1.3 Consider various options for solving a problem, assess the advantages and disadvantages of each option
	UC-1.4 Make competent judgments and estimates supported by logic and reasoning
UC-6 Use time-management skills, apply principles of self-development and lifelong learning	UC-6.2 Plan independent activities in professional problem-solving; critically analyze the work performed; find creative ways to use relevant experience for self-development

3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

- Definition of a multiple Riemann integral, a criterion for the integrability of a function, a sufficient condition for the integrability of a function, properties of integrable functions, a theorem on the reduction of a multiple integral to a repeated one, physical applications of the integral;
- basic facts and formulas of field theory (formulas of Green, Ostrogradsky-Gauss, Stokes), physical meaning of formulas of field theory.

be able to:

- Calculate the integral of a function of many variables over a set;
- be able to solve applied physical problems: calculate body mass, moments of inertia, volumes, etc.;
- apply field theory formulas for solving mathematical problems: calculating integrals, finding areas and volumes of bodies, areas of surfaces;
- to apply the formulas of field theory for solving physical problems: checking the potentiality and solenoidality of the field, finding the work of the field when a material point moves, etc.;
- be able to carry out calculations with the nabla operator.

master:

- Logical thinking, methods of proving mathematical statements;
- the skills of calculating integrals and the skills of applying field theory theorems in mathematical and physical applications;
- ability to use the necessary literature to solve problems.

4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

4.1. The sections of the course (training module) and the complexity of the types of training sessions

№	Topic (section) of the course	Types of training sessions, including independent work			
		Lectures	Seminars	Laboratory practical	Independent work
1	Curvilinear integrals. Green's formula	6	6		20
2	Surfaces. Surface integrals	8	8		24
3	Field theory: Ostrogradsky-Gauss and Stokes formulas	8	8		24
4	Multiple integrals	8	8		22
AH in total		30	30		90
Exam preparation		30 AH.			
Total complexity		180 AH., credits in total 4			

4.2. Content of the course (training module), structured by topics (sections)

Semester: 4 (Spring)

1. Curvilinear integrals. Green's formula

Definition of multiple integral and integrability criterion. Multiple integral properties.
Reduction of a multiple integral to a repeated one.
The geometric meaning of the modulus of the Jacobian of a mapping. Change of variables in multiple integrals.
Definition of multiple integral and integrability criterion. Multiple integral properties.
Reduction of a multiple integral to a repeated one.
The geometric meaning of the modulus of the Jacobian of a mapping. Change of variables in multiple integrals.

2. Surfaces. Surface integrals

Green's formula. Potential vector fields on the plane. Condition of independence of a curvilinear integral of the second kind from the path of integration.

3. Field theory: Ostrogradsky-Gauss and Stokes formulas

Plain smooth surface. Surface integral of the first kind. Independence of the expression of the integral through the parametrization of the surface from the admissible parameter change Surface area.
Orientation of a simple smooth surface. Surface integral of the second kind, expression in terms of surface parametrization. Piecewise smooth surfaces, their orientation and integrals over them.

4. Multiple integrals

Gauss-Ostrogradsky formula. Divergence of a vector field, its independence from the choice of a rectangular coordinate system and geometric meaning. Solenoidal vector fields. Connection of solenoidality with turning the field divergence into the rudder. The concept of vector potential.

Stokes formula. The rotor of a vector field, its independence from the choice of a rectangular coordinate system and its geometric meaning. Potential vector fields. Conditions for the independence of the curvilinear integral from the path of integration. Connection of potentiality with the vanishing of the rotor of the field.

Vector "nabla" and actions with it. Basic relations containing the nabla vector. Laplacian and vector gradient for scalar and vector fields.

5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)

Classroom equipped with a multimedia projector, screen and microphone.

6. List of the main and additional literature, that is necessary for the course (training module) mastering

Main literature

1. Mathematical analysis II /V. A. Zorich. Berlin, Springer, 2016

Additional literature

1. Advanced calculus, A. Friedman ; The Ohio State University. Mineola ; New York, Dover publications, inc., 2016

7. List of web resources that are necessary for the course (training module) mastering

1. http://lib.mipt.ru/catalogue/1195/?page=0 – электронная библиотека Физтеха, раздел «Анализ. Учебники по элементарному анализу».
2. http://www.exponenta.ru – образовательный математический сайт.
3. http://mathnet.ru – общероссийский математический портал.
4. http://www.edu.ru – федеральный портал «Российское образование».
5. http://benran.ru –библиотека по естественным наукам Российской академии наук.
6. http://www.i-exam.ru – единый портал Интернет-тестирования в сфере образования.

8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)

The lectures use multimedia technologies, including the demonstration of presentations.
In the process of independent work of students, it is possible to use software such as Mathcad, Scilab, etc.

9. Guidelines for students to master the course

Provided in the annually developed homework assignments.

SUPPLEMENT

Assessment funds for course (training module)

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1. Competencies formed during the process of studying the course

Code and the name of the competence	Competency indicators
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2. Competency assessment indicators

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know:

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be able to:

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- to apply the formulas of field theory for solving physical problems: checking the potentiality and solenoidality of the field, finding the work of the field when a material point moves, etc.;
- be able to carry out calculations with the nabla operator.

master:

- Logical thinking, methods of proving mathematical statements;
- the skills of calculating integrals and the skills of applying field theory theorems in mathematical and physical applications;
- ability to use the necessary literature to solve problems.

3. List of typical control tasks used to evaluate knowledge and skills

Current control is carried out on the basis of a point-rating system (BRS) for evaluating knowledge in the discipline being studied. The BRS takes into account the students ' performance of a set of homework assignments and tests in accordance with the curriculum. Data on attendance and current academic performance are entered by teachers in special journals and recorded in the BRS.

Current control on the basis of homework is carried out during the academic semester in the terms set by the Educational Department, in accordance with the curriculum.
To pass the task, the student must provide a solution to the homework problem in writing, answer the questions of the teacher and write a test paper on the task, which checks the knowledge of concepts and statements on the topics of the task and the ability to solve problems.
You can't use other people's help, computers, or mobile phones during the test.

* A BAR is attached to the subject being studied.

4. Evaluation criteria

Certification in the discipline "Multiple Integrals and Field Theory/Кратные интегралы и теория поля" is carried out in the form of an exam.

The exam is conducted taking into account the control tasks previously completed by the students.

Control tasks:

1. Multiple Riemann integral. Criteria for the integrability of a function. Integrability of a function that is continuous on a closed measurable set.
2. The measure of the graph of a function of several variables, the measure of the subgraph of a non-negative function.
3. Properties of integrable functions: linearity of an integral, additivity of an integral over sets, monotonicity of an integral, continuity of an integral, mean value theorem.
4. Integrability of a function that is continuous and bounded on an open measurable set.
5. Reduction of a multiple integral to a repeated one.
6. The theorem on the measure of the image and the theorem on the change of variables in a multiple integral under a simple mapping; without proof: the mapping splitting theorem. Geometric meaning of the modulus of the Jacobian and the sign of the Jacobian of the mapping in the two-dimensional case.
7. The theorem on the change of variables in a multiple integral.
8. Green's formula.
9. Potential vector fields. Conditions for the independence of a curvilinear integral of the second kind from the path of integration.
10. Plain smooth surface. Tangent plane and surface normal. Surface orientation.
11. Surface area, surface integrals of the first and second kind.
12. The Gauss-Ostrogradsky formula.
13. Geometric definition of divergence. Solenoidal vector fields.
14. Stokes formula.
15. Geometric definition of a vortex. Connection of potentiality and irrotationality of a vector field.
16. Approximation of a curvilinear integral of the second kind by an integral over an inscribed broken line.

Examples of exam tickets:

Ticket 1
1. Reduction of a multiple integral to a repeated one.
2. Approximation of a curvilinear integral of the second kind by an integral over an inscribed broken line.

Ticket 2
1. The theorem on the change of variables in a multiple integral.
2. Geometric definition of divergence. Solenoidal vector fields.

Grade "excellent (10)" is given to a student who has exhibited extensive and deep knowledge of the course and ability to apply skills when solving specific tasks;

Grade "excellent (9)" is given to a student who has exhibited extensive and deep knowledge of the course and ability to apply skills when solving specific tasks, but he has made minor errors that were independently found and corrected;

Grade "excellent (8)" is given to a student who has exhibited extensive and deep knowledge of the course and ability to apply skills when solving specific tasks, but he has made minor errors that were independently corrected after the instructions of an examiner;

Grade "good (7)" is given to a student who has a good command of the course and is able to apply skills when solving specific tasks, but has made minor mistakes when answering questions or solving problems;

Grade "good (6)" is given to a student who has a good command of the course and is able to apply skills when solving specific tasks, but has made rare mistakes when answering questions or solving problems;

Grade "good (5)" is given to a student who has a good command of the course and is able to apply skills when solving specific tasks, but has made mistakes when answering questions or solving problems;

Grade "satisfactory (4)" is given to a student who has exhibited fragmented knowledge, has made inaccurate formulation of the basic concepts, but understands the subject well, is able to apply the knowledge in standard situations and possesses skills necessary for the future study;

Grade "satisfactory (3)" is given to a student who has exhibited fragmented knowledge, has made inaccurate formulation of the basic concepts, has inconsistencies in understanding the course, but is able to apply the knowledge in standard situations and possesses skills necessary for the future study;

Grade "unsatisfactory (2)" is given to a student who does not possess knowledge of the essential concept of the course, has made gross mistakes in formulations of basic concepts and cannot use the knowledge in solving typical tasks;

Grade "unsatisfactory (1)" is given to a student who has exhibited total lack of knowledge of the course.

5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience

When conducting an oral exam, the student is given 1 astronomical hour for preparation. The poll of a student on a ticket for an oral exam should not exceed two astronomical hours. During the exam, students can use only the discipline program.

Scoring for Exams without Written Part
Department of Higher Mathematics

NN	Types of Control	Points
1	Control test 1	0—9
2	Control test 2	0—9
3	Homework 1	0—3
4	Homework 2	0—3
5	Theory checking	0—3
6	Class attendance and activity during seminars	0—3
7	Oral exam	0—70
	Total Score	0—100

The score for the oral exam is computed by the formula $7 \cdot N$, where $N \geq 3$ is a grade gained by a student during the exam. If $N = 1$ or 2, then the student grade is equal to N.

Conversion Scale between the total score and the student grade

Total Score	Student Grade	
93—100	10	Excellent
86—92	9	
79—85	8	
72—78	7	Good
65—71	6	
58—64	5	
51—57	4	Satisfactory
44—50	3	
30—43	2	Unsatisfactory
0—29	1	