Federal State Autonomous Educational Institution of Higher Education "Moscow Institute of Physics and Technology (National Research University)"

APPROVED
Vice Rector for Academic Affairs

A.A. Voronov

Work program of the course (training module)

course: Ordinary Differential Equations/Дифференциальные уравнения
major: Applied Mathematics and Informatics
specialization: Computer Science/Информатика
Phystech School of Applied Mathematics and Informatics
Chair of Higher Mathematics
term: 2
qualification: Bachelor

Semesters, forms of interim assessment:
4 (spring) - Grading test
5 (fall) - Exam

Academic hours: 120 AH in total, including:
lectures: 60 AH.
seminars: 60 AH.
laboratory practical: 0 AH.

Independent work: 165 AH.

Exam preparation: 30 AH.

In total: 315 AH, credits in total: 7

Number of course papers, tasks: 4

Authors of the program:

A.M. Bishaev, doctor of physics and mathematical sciences, senior research professor, associate professor
V.N. Diesperov, doctor of physics and mathematical sciences, full professor, professor
V.S. Nikolaev, doctor of technical sciences, full professor, professor
A.E. Umnov, doctor of technical sciences, full professor, professor
V.Y. Dubinskaya, candidate of physics and mathematical sciences, associate professor, associate professor
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O.G. Podlipskaya, candidate of physics and mathematical sciences, assistant

The program was discussed at the Chair of Higher Mathematics 21.05.2020

Annotation

The discipline belongs to the basic part of the educational program. The development of the discipline is aimed at developing the ability to acquire new scientific and professional knowledge using modern educational and information technologies. Topics such as the Simplest types of differential equations, Linear differential equations and systems with constant coefficients, Elements of the calculus of variations, The study of the Cauchy problem, Autonomous systems of differential equations, First integrals and linear homogeneous partial differential equations of the first order, Linear differential equations and linear systems of differential equations with variable coefficients are considered.

1. Study objective

Purpose of the course

familiarization of students with the basics of differential equations and preparation for the study of other mathematical courses – the theory of functions of complex variables, equations of mathematical physics, optimization and optimal control, functional analysis, etc.

Tasks of the course

- the acquisition by students of theoretical knowledge and practical skills in the field of solutions of elementary differential equations, linear differential equations and systems, problems in calculus of variations, the study of Cauchy problems, the study of special solutions for the construction and study of phase trajectories of Autonomous systems, finding the first integrals and solutions with their help and nonlinear systems of equations, solving linear equations and systems with variable coefficients;
- preparation of students for the study of related mathematical disciplines;
- acquisition of skills in the application of methods of differential equations in physics and other natural Sciences.

2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

Code and the name of the competence	Competency indicators
UC-1 Search and identify, critically evaluate and synthesize information, apply a systematic approach to problem-solving	UC-1.1 Analyze problems, highlight the stages of their solution, plan the actions required to solve them
	UC-1.2 Find, critically assess, and select information required for the task in hand
	UC-1.3 Consider various options for solving a problem, assess the advantages and disadvantages of each option
	UC-1.4 Make competent judgments and estimates supported by logic and reasoning
UC-6 Use time-management skills, apply principles of self-development and lifelong learning	UC-6.2 Plan independent activities in professional problem-solving; critically analyze the work performed; find creative ways to use relevant experience for self-development

3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

- The simplest types of differential equations, methods of lowering the order of differential equations;
- basic formulas of General and particular solutions of linear systems and equations with constant coefficients, definition and properties of the matrix exponent;
- conditions of existence and uniqueness of the solution of the Cauchy problem for normal systems of differential equations and for the n-th order equation in the normal form, the nature of the dependence of the solutions on the initial conditions. The concept of a special solution;
- formulation of problems of variational calculus;
- basic concepts and properties of phase trajectories of Autonomous systems, classification of equilibrium positions of linear Autonomous systems of the second order;
- the concept of the first integral of nonlinear systems of differential equations, their application to the solutions of partial differential equations of the first order, the conditions of existence and uniqueness of the solution of the Cauchy problem for the equation in the first order partial prod-ucts;
- structure of the General solution of linear systems with variable coefficients, properties of the Vronsky determinant, Liouville-Ostrogradsky formula. Properties of zeros of solutions of differential equations of the second order (Sturm's theorem).

be able to:

- To solve the simplest differential equations, to apply methods of order reduction;
- solve linear equations and systems with constant coefficients, apply the matrix exponent to the solution of systems of linear equations with constant coefficients;
- investigate the Cauchy problem. Find special solutions of the equation of the first order, not resolved with respect to the derivative;
- to investigate various problems of variational calculus;
- find the equilibrium position, to build the linearized system in a neighborhood of an equilibrium, to determine the type of equilibrium and to construct the phase trajectories of linear systems of second order;
- find the first integrals of systems of differential equations, apply them to solve simple nonlinear systems. Solve first order linear partial differential equations;
- apply the Liouville-Ostrogradsky formula and the method of variation of constants to solve second-order equations with variable coefficients. To investigate the properties of solutions of differential equations of the second order using the theorem of Sturm.

master:

- Logical thinking, methods of proof of mathematical statements;
- skills in solving and researching differential equations and systems in mathematical and physical applications;
- the ability to use the necessary literature.

4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

4.1. The sections of the course (training module) and the complexity of the types of training sessions

№	Topic (section) of the course	Types of training sessions, including independent work			
		Lectures	Seminars	Laboratory practical	Independent work
1	The simplest types of differential equations	14	16		39
2	Linear differential equations and systems with constant coefficients	16	14		36
3	Elements of variational calculus	7	7		19
4	The study of the Cauchy problem	5	4		14
5	Autonomous systems of differential equations	7	7		19
6	First integrals and linear homogeneous partial differential equations of the first order	7	5		19

7	Linear differential equations and linear systems of differential equations with variable coefficients	4	7		19
AH in total		60	60		165
Exam preparation		30 AH.			
Total complexity		315 AH., credits in total 7			

4.2. Content of the course (training module), structured by topics (sections)

Semester: 4 (Spring)

1. The simplest types of differential equations

Basic concept. The simplest types of equations of the first order: equations with separating variables, homogeneous, linear, equations in complete differentials. Integrating factor. A method for introducing a parameter for a first-order equation that is unsolved with respect to a derivative. Methods of decreasing the order of differential equations. Using one-parameter transformation groups to lower the order of differential equations.

2. Linear differential equations and systems with constant coefficients

The formula for the General solution of a linear homogeneous equation of the n-th order. Finding solutions of linear inhomogeneous in the case when the right-hand side of the equation is a quasi multinomial. Euler equation. Study of boundary value problems for the second-order linear equation (in particular, in the presence of a small parameter for the highest derivative). The formula for the General solution of a linear homogeneous system of equations in the case of simple eigenvalues of the coefficient matrix of the system. Theorem on reduction of the matrix of linear transformation to Jordan form (without proof). The formula for the General solution of a linear homogeneous system in the case of multiple eigenvalues of the matrix of coefficients of the system. Finding a solution to a linear inhomogeneous system in the case where the free terms of the equations are vector-quasi-polynomials. Matrix exponent and its use to obtain the General solution formula and solution of the Cauchy problem for linear homogeneous and inhomogeneous systems. Laplace transform and its application to solving linear differential equations with constant coefficients.

Semester: 5 (Fall)

3. Elements of variational calculus

Basic concept. The simplest problem of variational calculus. A problem with free ends; a problem for functionals depending on several unknown functions, and a problem for functionals containing higher order derivatives. Isoperimetric problem. Lagrange's Task.

4. The study of the Cauchy problem

Theorem of existence and uniqueness of the solution of the Cauchy problem for normal systems of differential equations and for the n-th order equation in the normal form. Theorem on continuation of solutions of normal systems. The nature of the dependence of the Cauchy problem solution on the parameters and initial data: continuity, differentiability. The Cauchy problem for the first order equation unsolved with respect to the derivative. Special solution.

5. Autonomous systems of differential equations

Basic concepts and properties of phase trajectories. Classification of equilibrium positions of linear Autonomous systems of equations of the second order. The behavior of phase trajectories in the vicinity of the equilibrium position of Autonomous nonlinear systems of second-order equations. Stability and asymptotic stability of the equilibrium position of an Autonomous system. Sufficient conditions for asymptotic stability.

6. First integrals and linear homogeneous partial differential equations of the first order

Basic concepts and properties of phase trajectories. Classification of equilibrium positions of linear Autonomous systems of equations of the second order. The behavior of phase trajectories in the vicinity of the equilibrium position of Autonomous nonlinear systems of second-order equations. Stability and asymptotic stability of the equilibrium position of an Autonomous system. Sufficient conditions for asymptotic stability.

7. Linear differential equations and linear systems of differential equations with variable coefficients

The existence and uniqueness theorem of the Cauchy problem solution for normal linear systems of equations and for the n-th order equation in the normal form. Fundamental system and fundamental matrix of solutions of linear homogeneous system of equations. The structure of the General solution of a linear homogeneous and inhomogeneous system of equations. Vronsky's Determinant. Liouville-Ostrogradsky Formula. The method of variation of constants for a linear inhomogeneous system of equations. Consequences for linear equations of n-th order. Theorem of Assault and its consequences.

5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)

Classroom equipped with multimedia projector, screen and microphone.

6. List of the main and additional literature, that is necessary for the course (training module) mastering

Main literature

Boyce, W. E.
Boyce's elementary differential equations and boundary value problems / W. E. Boyce, R. C. Diprima, D. B. Meade .— global edition .— Singapore : Wiley, 2017 .— 607 p. - Index: p. 602-607. - ISBN 978-1-119-38287-4.

Additional literature

1. Вариационное исчисление [Текст] : учебник для ун-тов / И. М. Гельфанд, С. В. Фомин .— М. : Физматгиз, 1961 .— 228 с.
2. Лекции по теории обыкновенных дифференциальных уравнений [Текст] : [учебник для вузов] / И. Г. Петровский .— М. : Физматлит, 2009 .— 208 с.

7. List of web resources that are necessary for the course (training module) mastering

1. http://lib.mipt.ru/catalogue/– the electronic library of the Institute, the section "Differential equations".
2. http://www.exponenta.ru educational mathematical site.
3. http://mathnet.ru – all-Russian mathematical portal.
4. http://www.edu.ru – Federal portal "Russian education".
5. http://benran.ru –library on natural Sciences of the Russian Academy of Sciences.
6. http://www.i-exam.ru – single portal of online testing in education.

8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)

The lectures use multimedia technologies, including presentations.
In the process of independent work of students it is possible to use such software as Mathcad, Scilab, etc.

9. Guidelines for students to master the course

Given in the annually developed homework.

SUPPLEMENT

Assessment funds for course (training module)

major: Applied Mathematics and Informatics
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Phystech School of Applied Mathematics and Informatics
Chair of Higher Mathematics
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qualification: Bachelor

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O.A. Pyrkova, candidate of physics and mathematical sciences, associate professor, associate professor
O.G. Podlipskaya, candidate of physics and mathematical sciences, assistant

1. Competencies formed during the process of studying the course

Code and the name of the competence	Competency indicators
UC-1 Search and identify, critically evaluate and synthesize information, apply a systematic approach to problem-solving	UC-1.1 Analyze problems, highlight the stages of their solution, plan the actions required to solve them
	UC-1.2 Find, critically assess, and select information required for the task in hand
	UC-1.3 Consider various options for solving a problem, assess the advantages and disadvantages of each option
	UC-1.4 Make competent judgments and estimates supported by logic and reasoning
UC-6 Use time-management skills, apply principles of self-development and lifelong learning	UC-6.2 Plan independent activities in professional problem-solving; critically analyze the work performed; find creative ways to use relevant experience for self-development

2. Competency assessment indicators

As a result of studying the course the student should:

know:

- The simplest types of differential equations, methods of lowering the order of differential equations;
- basic formulas of General and particular solutions of linear systems and equations with constant coefficients, definition and properties of the matrix exponent;
- conditions of existence and uniqueness of the solution of the Cauchy problem for normal systems of differential equations and for the n-th order equation in the normal form, the nature of the dependence of the solutions on the initial conditions. The concept of a special solution;
- formulation of problems of variational calculus;
- basic concepts and properties of phase trajectories of Autonomous systems, classification of equilibrium positions of linear Autonomous systems of the second order;
- the concept of the first integral of nonlinear systems of differential equations, their application to the solutions of partial differential equations of the first order, the conditions of existence and uniqueness of the solution of the Cauchy problem for the equation in the first order partial prod-ucts;
- structure of the General solution of linear systems with variable coefficients, properties of the Vronsky determinant, Liouville-Ostrogradsky formula. Properties of zeros of solutions of differential equations of the second order (Sturm's theorem).

be able to:

- To solve the simplest differential equations, to apply methods of order reduction;
- solve linear equations and systems with constant coefficients, apply the matrix exponent to the solution of systems of linear equations with constant coefficients;
- investigate the Cauchy problem. Find special solutions of the equation of the first order, not resolved with respect to the derivative;
- to investigate various problems of variational calculus;
- find the equilibrium position, to build the linearized system in a neighborhood of an equilibrium, to determine the type of equilibrium and to construct the phase trajectories of linear systems of second order;
- find the first integrals of systems of differential equations, apply them to solve simple nonlinear systems. Solve first order linear partial differential equations;
- apply the Liouville-Ostrogradsky formula and the method of variation of constants to solve second-order equations with variable coefficients. To investigate the properties of solutions of differential equations of the second order using the theorem of Sturm.

master:

- Logical thinking, methods of proof of mathematical statements;
- skills in solving and researching differential equations and systems in mathematical and physical applications;
- the ability to use the necessary literature.

3. List of typical control tasks used to evaluate knowledge and skills

Current control is carried out on the basis of a point-rating system (BRS) for evaluating knowledge in the discipline being studied. The BRS takes into account the students ' performance of a set of homework assignments and tests in accordance with the curriculum. Data on attendance and current academic performance are entered by teachers in special journals and recorded in the BRS.

Current control on the basis of homework is carried out during the academic semester in the terms set by the Educational Department, in accordance with the curriculum.
To pass the task, the student must provide a solution to the homework problem in writing, answer the questions of the teacher and write a test paper on the task, which checks the knowledge of concepts and statements on the topics of the task and the ability to solve problems.
You can't use other people's help, computers, or mobile phones during the test.

* A BRS is attached to the subject being studied.

4. Evaluation criteria

Certification in the discipline "Ordinary Differential Equations" is carried out in the form of a differentiated test and exam.
The grading test and examination shall be conducted in accordance with the previously performed by the students in the control tasks.

Control tasks:

1. The simplest types of equations of the first order: equations with separating variables, homogeneous, linear, equations in full differentials. Integrating factor. Bernoulli and Riccati equations.
2. A method of introducing a parameter for a first order equation not resolved with respect to a derivative.
3. Order reduction methods for differential equations.
4. General solution of a linear homogeneous equation of n-th order with constant coefficients.
5. The General solution of linear differential equations of n-th order with constant coefficients and right-hand side of the equation in the form of a quasi multinomial.
6. The General solution of a normal linear homogeneous system of equations with constant coefficients in the case where there is a basis from the eigenvectors of the matrix of the system.
7. The General solution of a normal linear homogeneous system of equations with constant coefficients in the case where there is no basis from the eigenvectors of the matrix of the system.
8. Finding normal solutions of linear inhomogeneous system of equations with constant coefficients in the case when the free term is a vector quasi multinomial.
9. Exponent of a square matrix; matrix formulas for solving the Cauchy problem for normal linear systems with constant coefficients.
10. The simplest problem of variational calculus.
11. Generalizations of the simplest problem of variational calculus: the free-end problem, the problem for functionals depending on several unknown functions, and the problem for functionals containing higher-order derivatives.
12. Isoperimetric problem.
13. Existence and uniqueness theorems for the solution of the Cauchy problem for normal systems of differential equations and for the n-th order equation in the normal form.
14. Theorem on continuation of solutions of normal systems of ordinary differential equations and its consequences.
15. Continuous dependence on the parameters of the Cauchy problem solution for normal systems of ordinary differential equations. Differentiability of the decision on parameters, the equation in variations.
16. A theorem of existence and uniqueness of the solution of the Cauchy problem for a first-order equation not resolved with respect to the derivative. Special solution.
17. Autonomous systems of differential equations. Properties of phase trajectories of normal Autonomous systems. A theorem on the straightening of trajectories.

18. Classification of equilibrium positions of a linear Autonomous homogeneous system of differential equations of the second order. The behavior of phase trajectories in the vicinity of the equilibrium position for Autonomous nonlinear systems of the second order.

19. The first integrals of Autonomous systems of differential equations. Criterion of the first integral. Application of the first integrals to reduce the order of the system of equations.

20. Theorem on the number of independent first integrals of an Autonomous system of differential equations.

21. First order linear homogeneous partial differential equation; General solution formula. The theorem of existence and uniqueness of solutions of the Cauchy problem.

22. Existence and uniqueness theorems for the solution of the Cauchy problem for normal linear systems of ordinary differential equations with variable coefficients and for the linear equation of n-th order.

23. The fundamental system of solutions, the fundamental matrix and the structure of the General solution of a normal linear homogeneous system of equations with variable coefficients. The fundamental system of solutions and the structure of the General solution of a linear homogeneous equation of n-th order.

24. Vronsky determinant and Liouville--Ostrogradsky formula for solutions of a normal linear homogeneous system of equations and for solutions of a linear homogeneous equation of n-th order.

25. A method of variation of constants for a normal linear inhomogeneous system of equations and for a linear inhomogeneous equation of n-th order.

26. Theorem of Assault and its consequences.

27. Lyapunov stability of the equilibrium position of an Autonomous system. Sufficient conditions for asymptotic stability of the equilibrium position of an Autonomous system.

28. Group properties of solutions of Autonomous systems of differential equations. The concept of phase volume. The Formula Of Liouville. Poincare Theorem.

Examples of examination tickets:

Ticket 1

1. General solution of a linear homogeneous equation of n-th order with constant coefficients.

2. First order linear homogeneous partial differential equation; General solution formula. The theorem of existence and uniqueness of solutions of the Cauchy problem.

Ticket 2

1. The General solution of linear differential equations of n-th order with constant coefficients and right-hand side of the equation in the form of a quasi multinomial.

2. Existence and uniqueness theorems for the solution of the Cauchy problem for normal linear systems of ordinary differential equations with variable coefficients and for the linear equation of n-th order.

Grade "excellent (10)" is given to a student who has exhibited extensive and deep knowledge of the course and ability to apply skills when solving specific tasks;

Grade "excellent (9)" is given to a student who has exhibited extensive and deep knowledge of the course and ability to apply skills when solving specific tasks, but he has made minor errors that were independently found and corrected;

Grade "excellent (8)" is given to a student who has exhibited extensive and deep knowledge of the course and ability to apply skills when solving specific tasks, but he has made minor errors that were independently corrected after the instructions of an examiner;

Grade "good (7)" is given to a student who has a good command of the course and is able to apply skills when solving specific tasks, but has made minor mistakes when answering questions or solving problems;

Grade "good (6)" is given to a student who has a good command of the course and is able to apply skills when solving specific tasks, but has made rare mistakes when answering questions or solving problems;

Grade "good (5)" is given to a student who has a good command of the course and is able to apply skills when solving specific tasks, but has made mistakes when answering questions or solving problems;

Grade "satisfactory (4)" is given to a student who has exhibited fragmented knowledge, has made inaccurate formulation of the basic concepts, but understands the subject well, is able to apply the knowledge in standard situations and possesses skills necessary for the future study;

Grade "satisfactory (3)" is given to a student who has exhibited fragmented knowledge, has made inaccurate formulation of the basic concepts, has inconsistencies in understanding the course, but is able to apply the knowledge in standard situations and possesses skills necessary for the future study;

Grade "unsatisfactory (2)" is given to a student who does not possess knowledge of the essential concept of the course, has made gross mistakes in formulations of basic concepts and cannot use the knowledge in solving typical tasks;

Grade "unsatisfactory (1)" is given to a student who has exhibited total lack of knowledge of the course.

5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience

To take a test is carried out according to the results of the ongoing monitoring and delivery of tasks defined by the program of discipline, taking into account points at BRS. When you set the number of points BRS not less than the threshold student is set off.

The time of the written examination is four astronomical hours. During the written examination, students can only use pen, pencil and paper.

During the oral examination, the student is given 1 astronomical hour to prepare. The survey of the student on the ticket on the oral exam should not exceed two astronomical hours. During the examination, students can only use the discipline program.

«СОГЛАСОВАНО»

Проректора по учебной работе и довузовской подготовке

_____ А.А. Воронов

«____» _____ 2020

Балльно-рейтинговая система оценки знаний студентов

Дисциплина: **Ordinary Differential Equations/Дифференциальные уравнения**
3 курс, 5 семестр, экзамен

Кафедра: **высшей математики**

№	Вид занятий	Сумма баллов
1.	Посещение лекций	0–3
2.	Проверка теоретических знаний	0–3
3.	Контрольная работы, проводимые в классе	0–18
4.	Домашняя работа	0–6
5.	Итоговый контроль Экзамен (устный ответ)	0–70
	ИТОГО	**0–100**

*Если при учете этого вида работы итоговая сумма за работу в семестре превосходит 30 баллов, то считать ее равной 30 баллам.

Сумма баллов за устный ответ начисляется по формуле $N * 7$, где $N \geq 3$ — предварительная оценка за устный ответ по десятибалльной шкале. Если $N = 1, 2$, то итоговая оценка совпадает с N.

Соответствие оценок итоговой академической успеваемости балльно-рейтинговой системы.

Баллы БРС	Оценки	
93–100	10	отлично
86–92	9	
79–85	8	
72–78	7	хорошо
65–71	6	
58–64	5	
51–57	4	удовлетворительно
44–50	3	
30–43	2	неудовлетворительно
0–29	1	

Регламент принятия домашних заданий и проведения экзамена определяется «Положением о текущем контроле успеваемости и промежуточной аттестации студентов на кафедре высшей математики».

Зав.кафедрой _____ Г. Е. Иванов