Federal State Autonomous Educational Institution of Higher Education "Moscow Institute of Physics and Technology
(National Research University)"

APPROVED
Head of the Phystech School of Applied Mathematics and Informatics
A.M. Raygorodskiy

Work program of the course (training module)

course:	Probability Theory/Теория вероятностей
major:	Applied Mathematics and Informatics
specialization:	Computer Science/Информатика
	Phystech School of Applied Mathematics and Informatics
	Chair of Discrete Mathematics
term:	2
qualification:	Bachelor

Semesters, forms of interim assessment:

4 (spring) - Exam
5 (fall) - Exam

Academic hours: 120 AH in total, including:
lectures: 60 AH.
seminars: 60 AH.
laboratory practical: 0 AH.

Independent work: 90 AH.

Exam preparation: 60 AH.

In total: 270 AH, credits in total: 6

Author of the program: A.A. Glibichuk, candidate of physics and mathematical sciences, associate professor

The program was discussed at the Chair of Discrete Mathematics 06.03.2023

Annotation

The course includes all the basic definitions and statements of probability theory, from the Kolmogorov axiomatics to multidimensional limit theorems. The course is intended for mathematicians and assumes knowledge of the basics of measure theory, combinatorics and mathematical analysis.

1. Study objective

Purpose of the course

- mastering the basic modern methods of probability theory.

Tasks of the course

- students mastering basic knowledge (concepts, concepts, methods and models) in probability theory;
- acquisition of theoretical knowledge and practical skills in probability theory;
- providing advice and assistance to students in conducting their own theoretical research in probability theory.

2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

Code and the name of the competence	Competency indicators
Gen.Pro.C-1 Apply fundamental knowledge of physics, mathematics, and/or natural sciences in professional settings	Gen.Pro.C-1.1 Analyze the task in hand, develop appproaches to complete it
Gen.Pro.C-5 Participate in fundamental and applied research and development activities; independently devise new theoretical research methods (including mathematical research methods) and work with cutting-edge scientific equipment (measuring, analytical, technological)	Gen.Pro.C-5.1 Perform tasks in the field of theoretical and experimental research and development activities
Pro.C-1 Assign, formalize, and solve tasks, develop and research mathematical models of studied phenomena and processes, systematically analyze scientific problems, obtain new scientific outcomes	Pro.C-1.2 Make hypotheses, build mathematical models of the studied phenomena and processes, evaluate the quality of the developed model

3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

- fundamental concepts, laws of probability theory;
- modern problems of the corresponding sections of probability theory;
- concepts, axioms, methods of proof and proof of the main theorems in the sections included in the basic part of the cycle;
- basic properties of the corresponding mathematical objects;
- analytical and numerical approaches and methods for solving typical applied problems of probability theory.

be able to:

- understand the task;
- use your knowledge to solve fundamental and applied problems;
- evaluate the correctness of the problem statements;
- strictly prove or disprove the statement;
- independently find algorithms for solving problems, including non-standard ones, and conduct their analysis;
- independently see the consequences of the results;
- accurately represent mathematical knowledge in probability theory in oral and written form.

master:

- skills of mastering a large amount of information and solving problems (including complex ones);
- skills of independent work and mastering new disciplines;
- the culture of the formulation, analysis and solution of mathematical and applied problems that require the use of mathematical approaches and methods for their solution;
- the subject language of probability theory and the skills of competent description of problem solving and presentation of the results.

4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

4.1. The sections of the course (training module) and the complexity of the types of training sessions

№	Topic (section) of the course	Types of training sessions, including independent work			
		Lectures	Seminars	Laboratory practical	Independent work
1	Discrete probability spaces.	4	4		8
2	Independence of an arbitrary set of random variables.	4	4		7
3	Random variables in discrete probability spaces.	6	6		8
4	Bernoulli test design	5	5		7
5	Random elements, random variables and vectors.	6	6		8
6	Systems of sets (semirings, rings, algebras, sigma-algebras)	5	5		7
7	Carathéodory's theorem on the continuation of a probability measure (proof of uniqueness).	5	5		8
8	Completeness and continuity of measures	5	5		7
9	Immeasurable sets.	5	5		8
10	Convergence. Cauchy Convergence Criterion	5	5		7
11	Conditional probabilities.	5	5		8
12	Lebesgue integral	5	5		7
AH in total		60	60		90
Exam preparation		60 AH.			
Total complexity		270 AH., credits in total 6			

4.2. Content of the course (training module), structured by topics (sections)

Semester: 4 (Spring)

1. Discrete probability spaces.

Discrete probability spaces. The classic definition of probability. Examples.

2. Independence of an arbitrary set of random variables.

Independence of an arbitrary set of random variables. Independence criterion, a theorem on the independence of Borel functions from disjoint sets of independent random variables.

3. Random variables in discrete probability spaces.

Random variables in discrete probability spaces. Independence of random variables. The mathematical expectation of a random variable, its basic properties. Dispersion, covariance and their properties.

4. Bernoulli test design

Mathematical model, limit theorems: Poisson and Muavre-Laplace

5. Random elements, random variables and vectors.

Random elements, random variables and vectors. A sufficient condition for the measurability of a mapping, a corollary for random variables and vectors. Actions on random variables.

6. Systems of sets (semirings, rings, algebras, sigma-algebras)

Minimal ring containing a half ring. The concept of the smallest ring, algebra, sigma-algebra containing a system of sets. Measures on half rings. The classical Lebesgue measure on the half ring of spaces and its sigma additivity.

Semester: 5 (Fall)

7. Carathéodory's theorem on the continuation of a probability measure (proof of uniqueness).

Carathéodory's theorem on the continuation of a probability measure (proof of uniqueness). Lebesgue theorem on distribution function

8. Completeness and continuity of measures

Theorems on the relation between continuity and sigma additivity. Borel measure. Lebesgue-Stieltjes measures on the line and their sigma additivity.

9. Immeasurable sets.

Theorem on the structure of measurable sets. Measurable functions. Their properties. Measurable functions and passage to the limit.

10. Convergence. Cauchy Convergence Criterion

Convergence in measure and almost everywhere. Their properties (Cauchy criterion for convergence in measure, arithmetic, connection of convergence, Riesz theorem). Theorems of Egorov and Luzin.

11. Conditional probabilities.

Conditional probabilities. The formula for total probability. Bayes formula. Examples

12. Lebesgue integral

The Lebesgue integral and its properties. Definition of the Lebesgue integral in the general case. The main properties of the Lebesgue integral.

5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)

A standard classroom.

6. List of the main and additional literature, that is necessary for the course (training module) mastering

Main literature

1. Вероятность [Текст] : в 2 т. Т. 1 : Элементарная теория вероятностей. Математические основания. Предельные теоремы : учебник для вузов / А. Н. Ширяев .— 4-е изд., перераб. и доп. — М. : Изд-во МЦНМО, 2007, 2011 .— 552 с.
2. Курс теории вероятностей [Текст] : учеб. пособие для вузов / В. П. Чистяков .— 7-е изд., испр. — М : Дрофа, 2007 .— 253 с.

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Additional literature

1. Задачи по теории вероятностей [Текст] : учеб. пособие для вузов / А. Н. Ширяев .— М. : МЦНМО, 2006 .— 416 с.

7. List of web resources that are necessary for the course (training module) mastering

http://dm.fizteh.ru/

8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)

Multimedia technologies can be employed during lectures and practical lessons, including presentations.

9. Guidelines for students to master the course

1. It is recommended to successfully pass the test papers, as this simplifies the final certification in the subject.
2. To prepare for the final certification in the subject, it is best to use the lecture materials.

SUPPLEMENT

Assessment funds for course (training module)

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5 (fall) - Exam

Author: A.A. Glibichuk, candidate of physics and mathematical sciences, associate professor

1. Competencies formed during the process of studying the course

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Pro.C-1 Assign, formalize, and solve tasks, develop and research mathematical models of studied phenomena and processes, systematically analyze scientific problems, obtain new scientific outcomes	Pro.C-1.2 Make hypotheses, build mathematical models of the studied phenomena and processes, evaluate the quality of the developed model

2. Competency assessment indicators

As a result of studying the course the student should:

know:

- fundamental concepts, laws of probability theory;
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- concepts, axioms, methods of proof and proof of the main theorems in the sections included in the basic part of the cycle;
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3. List of typical control tasks used to evaluate knowledge and skills

Homework:

1. On a chessboard of size $n \times n$ n rooks are randomly placed. Find the probabilities of the following events: 1. A = {rooks do not beat each other}; 2. B = {the rooks do not beat each other and there are no pieces on the main diagonal}.

2. Give examples showing that, generally speaking, the equalities P (B | A) + P (B | A) = 1, P (B | A) + P (B | A) = 1 are false. 3. Show that each of the functions G1 (x, y) = I (x + y ≥ 0), G2 (x, y) = [x + y], where [·] is the integer part of a number, is right continuous, increasing in each variable, but is not a distribution function in R2.

4. The random variable ξ has the standard Cauchy distribution. Find the distribution densities of the random variables ξ2 1 + ξ2, 1 1 + ξ2, 2ξ 1 − ξ2, 1 ξ. 5. Let ξ_1, ξ_2 be random variables, each of which does not depend on the random variable ξ. Is it true that the vector (ξ_1, ξ_2) also does not depend on the random variable ξ?

Objectives of the first test:

1. Is F (x, y) = (1 − e − x − y) I (x + y ≥ 0) a distribution function?

2. Let P1, P2, P3 be exponential distributions with parameter 1, P = P1 × P 2 × P3. Find P ({(x, y, z): from segments of lengths x, y, z you can make a triangle}).

3. Find the mathematical expectation and variance of a random variable whose distribution function is F (x) = □ 0, x ≤ − 2π 3; (1/5) cos: 0.

4. Given are independent random variables X, Y~ U [−1,1]. Let ξ = X / (X + Y), η = X + Y. Find cov (ξ, η). 5. Let ξ1, ξ2, ... be independent identically distributed random variables with density cx2e − xI (x> 0). Find c and prove that P (limn → ∞ ξn lnn = 1) = 1.

Objectives of the second control:

1. The random variables ξ, η are independent. The density ξ is equal to (x / 2) I ((0,2]), the random variable η has an exponential distribution with parameter 1. Find the probability that the vector (η, ξ) falls into a circle of radius 1 centered at the point (1, 0).

2. The characteristic function of the vector (ξ, η) is equal to φ (ξ, η) = e − 2yi − 1/2 (2x2−2xy + y2). Find a number α such that the random variables ξ + αη and η are independent. Find Eξ2η using this number.

3. The random variable ξ has a Poisson distribution with the parameter λ. Find the mathematical expectation and variance of the random variable f (ξ) if f (x) = (−1) x − 1x.

4. For the simplest symmetric random walk Sn, find the probability that it never crosses level 2 in N steps: P (S2 6 = 2, ..., SN 6 = 2) =?

5. The die is rolled N times. With each throw, 1 is added to the dropped out number, if it is different from 6. If 6, then the number does not change. What should be N for a number divisible by 3 to appear (after recalculation) at least 700 times with a probability exceeding 0.98?

4. Evaluation criteria

List of questions for semester 4:

1. The probability space. Axioms of Kolmogorov. The continuity theorem at the “zero” of a probability measure.
2. Discrete probability spaces. The classic definition of probability. Examples.
3. Geometric probabilities. Examples.
4. Conditional probabilities. The formula for total probability. Bayes Formulas Examples.
5. The theorem on monotone classes.
6. Independence of events and systems of events. Bernstein example. A lemma on a sufficient condition for the independence of sigma-algebras.
7. The Caratheodory theorem on the continuation of a probability measure (proof of uniqueness).
8. Random variables in discrete probability spaces. Independence of random variables. The mathematical expectation of a random variable, its basic properties. Dispersion, covariance and their properties.
9. Random elements, random variables and vectors. A sufficient condition for the measurability of the mapping, the corollary for random variables and vectors. Actions on random variables.
10. Independence of an arbitrary set of random variables. Independence criterion, a theorem on the independence of Borel functions from disjoint sets of independent random variables.
11. Mathematical expectation of a random variable (Lebesgue integral in a probabilistic measure): definition for simple, non-negative and arbitrary random variables. Validation of definitions.
12. The main properties of mathematical expectation. The mathematical expectation theorem for the product of independent random variables.
13. Dispersion, covariance and their properties. The Cauchy-Bunyakovsky inequality. Dispersion of the sum of independent random variables. The covariance matrix of a random vector, its non-negative definiteness.

List of questions for semester 5:

1. Markov inequality, Chebyshev inequality. The law of large numbers in the form of Chebyshev. Jensen's inequality.
2. Types of convergence of random variables, their relationship. Cauchy criterion for convergence with probability 1.
3. The inequality of Kolmogorov. A convergence theorem for an almost certainly series of random variables.
4. The strengthened law of large numbers for independent random variables with limited variances.
5. The passage to the limit under the sign of the Lebesgue integral. Monotone convergence theorem, Fatou lemma, Lebesgue theorem on majorized convergence.
6. Borel Cantelli Lemma. The strengthened law of large numbers for independent identically distributed random variables with limited mathematical expectation.
7. The formula for recalculation of mathematical expectations. Variable replacement theorem in Lebesgue integral.
8. Direct product of probability spaces. Fubini's theorem (b / d). Joint distribution of a finite set of random variables. Convolution of distributions.
9. Weak convergence and convergence of mostly probabilistic measures. Alexandrov's theorem (b / d). A theorem on the equivalence of convergence in the distribution of random variables and the convergence of distribution functions at all continuity points of the limit function.
10. Bernoulli test design and polynomial design. Limit theorems for the Bernoulli scheme: Poisson's theorem and Muavre-Laplace theorem (b / d).
11. Characteristic functions of probability measures. distribution functions, random variables and vectors. Examples. The main properties of the characteristic functions of random variables.
12. The uniqueness theorem for characteristic probability functions. The independence of the components of a random vector in terms of characteristic functions.
13. The continuity theorem for characteristic functions (b / d). Central limit theorem for independent identically distributed random variables.
14. Berry Esseen's theorem on the rate of convergence in the central limit theorem (b / d). Estimates for the constant in Berry Esseen's theorem.

Assessment “excellent (10)” is given to a student who has displayed comprehensive, systematic and deep knowledge of the educational program material, has independently performed all the tasks stipulated by the program, has deeply studied the basic and additional literature recommended by the program, has been actively working in the classroom, and understands the basic scientific concepts on studied discipline, who showed creativity and scientific approach in understanding and presenting educational program material, whose answer is characterized by using rich and adequate terms, and by the consistent and logical presentation of the material;

Assessment “excellent (9)” is given to a student who has displayed comprehensive, systematic knowledge of the educational program material, has independently performed all the tasks provided by the program, has deeply mastered the basic literature and is familiar with the additional literature recommended by the program, has been actively working in the classroom, has shown the systematic nature of knowledge on discipline sufficient for further study, as well as the ability to amplify it on one’s own, whose answer is distinguished by the accuracy of the terms used, and the presentation of the material in it is consistent and logical;

Assessment “excellent (8)” is given to a student who has displayed complete knowledge of the educational program material, does not allow significant inaccuracies in his answer, has independently performed all the tasks stipulated by the program, studied the basic literature recommended by the program, worked actively in the classroom, showed systematic character of his knowledge of the discipline, which is sufficient for further study, as well as the ability to amplify it on his own;

Assessment “good (7)” is given to a student who has displayed a sufficiently complete knowledge of the educational program material, does not allow significant inaccuracies in the answer, has independently performed all the tasks provided by the program, studied the basic literature recommended by the program, worked actively in the classroom, showed systematic character of his knowledge of the discipline, which is sufficient for further study, as well as the ability to amplify it on his own;

Assessment “good (6)” is given to a student who has displayed a sufficiently complete knowledge of the educational program material, does not allow significant inaccuracies in his answer, has independently carried out the main tasks stipulated by the program, studied the basic literature recommended by the program, showed systematic character of his knowledge of the discipline, which is sufficient for further study;

Assessment “good (5)” is given to a student who has displayed knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, who while not being sufficiently active in the classroom, has nevertheless independently carried out the main tasks stipulated by the program, mastered the basic literature recommended by the program, made some errors in their implementation and in his answer during the test, but has the necessary knowledge for correcting these errors by himself;

Assessment “satisfactory (4)” is given to a student who has discovered knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, who while not being sufficiently active in the classroom, has nevertheless independently carried out the main tasks stipulated by the program, learned the main literature but allowed some errors in their implementation and in his answer during the test, but has the necessary knowledge for correcting these errors under the guidance of a teacher;

Assessment “satisfactory (3)” is given to a student who has displayed knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, not showed activity in the classroom, independently fulfilled the main tasks envisaged by the program, but allowed errors in their implementation and in the answer during the test, but possessing necessary knowledge for elimination under the guidance of the teacher of the most essential errors;

Assessment “unsatisfactory (2)” is given to a student who showed gaps in knowledge or lack of knowledge on a significant part of the basic educational program material, who has not performed independently the main tasks demanded by the program, made fundamental errors in the fulfillment of the tasks stipulated by the program, who is not able to continue his studies or start professional activities without additional training in the discipline in question;

Assessment “unsatisfactory (1)” is given to a student when there is no answer (refusal to answer), or when the submitted answer does not correspond at all to the essence of the questions contained in the task.

5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience

During the test the student are allowed to use the program of the discipline.