Проект: "Коллапс и квантовые фазы вихрей в Бозе-Ферми смеси с притяжением между бозонами и фермионами"

аналитической ведомственной целевой программы "Развитие научного потенциала высшей школы (2006-2008 годы)

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Introduction

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- Temperatures 500 nK - 2 μK
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- – possible Fermi superfluidity, where the interaction between fermions is mediated by bosons;
- – sympathetic cooling of fermions via a buffer gas of bosons;
- – wealth of novel quantum phases in these mixtures;
- Example: mixture of ^{87}Rb (bosons) and ^{40}K (fermions).

Effective Hamiltonian

Grand-canonical partition function of the Bose-Fermi mixture:

$$Z = \int D[\phi^*]D[\phi]D[\psi^*]D[\psi] \exp\left\{-\frac{1}{\hbar}\left(S_B(\phi^*,\phi)+\right.\right.$$
$$\left.\left. + \quad S_F(\psi^*,\psi) + S_{int}(\phi^*,\phi,\psi^*,\psi)\right)\right\}.$$

Here Bose field $\phi(\tau,\mathbf{r})$ is periodic on the imaginary-time interval $[0,\hbar\beta]$, and Fermi (Grassmann) field $\psi(\tau,\mathbf{r})$ is antiperiodic on this interval.

$$S_B(\phi^*,\phi) = \int_0^{\hbar\beta} d\tau \int d\mathbf{r} \left\{\phi^*(\tau,\mathbf{r})\left(\hbar\frac{\partial}{\partial\tau} - \frac{\hbar^2\nabla^2}{2m_B} + V_B(\mathbf{r}) - \right.\right.$$
$$\left.\left. - \quad \mu_B\right)\phi(\tau,\mathbf{r}) + \frac{g_B}{2}|\phi(\tau,\mathbf{r})|^4\right\}.$$

Effective Hamiltonian

$$S_F(\psi^*,\psi) = \int_0^{\hbar\beta} d\tau \int d\mathbf{r} \left\{ \psi^*(\tau,\mathbf{r}) \left(\hbar\frac{\partial}{\partial\tau} - \frac{\hbar^2\nabla^2}{2m_F} + \right.\right.$$
$$\left.\left. + \quad V_F(\mathbf{r}) - \mu_F \right) \psi(\tau,\mathbf{r}) \right\}.$$

$$S_{int}(\phi^*,\phi,\psi^*,\psi) = g_{BF}\int_0^{\hbar\beta} d\tau \int d\mathbf{r} |\psi(\tau,\mathbf{r})|^2 |\phi(\tau,\mathbf{r})|^2,$$

where $g_B = 4\pi\hbar^2 a_B/m_B$, $g_{BF} = 2\pi\hbar^2 a_{BF}/m_I$, $m_I = m_B m_F/(m_B+m_F)$, a_B and a_{BF} - boson-boson and boson-fermion s wave scattering lengths respectively.

Effective Hamiltonian

Integral over Fermi fields is Gaussian, we can calculate this integral and obtain the partition function of the Fermi system as a functional of Bose field $\phi(\tau, \mathbf{r})$.

$$
\begin{aligned}
Z_F &= \int D[\psi^*]D[\psi]\exp\left(-\frac{1}{\hbar}\left(S_F(\psi^*,\psi) + S_{int}(\phi^*,\phi,\psi^*,\psi)\right)\right) = \\
&= \int D[\psi^*]D[\psi]\exp\left\{\int_0^{\hbar\beta} d\tau \int d\mathbf{r} \int_0^{\hbar\beta} d\tau' \int d\mathbf{r}' \times \right. \\
&\times \left. \psi^*(\tau,\mathbf{r})\mathbf{G}^{-1}(\tau,\mathbf{r},\tau',\mathbf{r}')\psi(\tau',\mathbf{r}')\right\},
\end{aligned}
$$

where

$$
\mathbf{G}^{-1} = \mathbf{G}_0^{-1} - \mathbf{\Sigma}
$$

is the Dyson equation, and $\mathbf{\Sigma}(\tau, \mathbf{r}, \tau', \mathbf{r}')$ is a selfenergy.

Effective Hamiltonian

$$\mathbf{G}_{\mathbf{0}}^{-\mathbf{1}}(\tau,\mathbf{r},\tau',\mathbf{r}') = -\frac{1}{\hbar}\left(\hbar\frac{\partial}{\partial\tau} - \frac{\hbar^2\nabla^2}{2m_F} + V_F(\mathbf{r}) - \right.$$
$$\left. - \mu_F\right)\delta(\mathbf{r}-\mathbf{r}')\delta(\tau-\tau').$$

$$\mathbf{\Sigma}(\tau,\mathbf{r},\tau',\mathbf{r}') = \frac{g_{BF}}{\hbar}|\phi(\tau,\mathbf{r})|^2\delta(\mathbf{r}-\mathbf{r}')\delta(\tau-\tau').$$

Gaussian integral over the Grassmann variables:

$$\int\prod_n d\psi_n^* d\psi_n \exp\left\{-\sum_{n,n'}\psi_n^* A_{n,n'}\psi_{n'}\right\} = \det A = e^{Sp[\ln A]}$$

$$Z_F = \exp\left(Sp\ln\left(-\mathbf{G}^{-\mathbf{1}}\right)\right) = \exp\left(-\frac{1}{\hbar}S_{eff}\right)$$

Effective Hamiltonian

$$\mathbf{G}^{-1} = \mathbf{G_0}^{-1} - \mathbf{\Sigma} = \mathbf{G_0}^{-1}(\mathbf{I} - \mathbf{G_0}\mathbf{\Sigma}),$$

$$Sp(\ln(-\mathbf{G}^{-1})) = Sp(\ln(-\mathbf{G_0}^{-1})) - \sum_{n=1}^{\infty}\frac{1}{n}Sp\left[(\mathbf{G_0}\mathbf{\Sigma})^n\right].$$

$$\mathbf{G_0}(\tau,\mathbf{r},\tau',\mathbf{r}') = \sum_{\omega,n}\frac{-\hbar}{-i\hbar\omega+\epsilon_n-\mu_F}\xi_n(\mathbf{r})\xi_n^*(\mathbf{r}')\frac{e^{-i\omega(\tau-\tau')}}{\hbar\beta},$$

where $\omega = \pi(2s+1)/\hbar\beta$; $s = 0, \pm 1, \ldots$ and

$$\left(-\frac{\hbar^2\nabla^2}{2m_F} + V_F(\mathbf{r})\right)\xi_n(\mathbf{r}) = \epsilon_n\xi_n(\mathbf{r}).$$

Effective Hamiltonian

In the semiclassical Thomas-Fermi approximation one has:

$$\sum_n \xi_n(\mathbf{r})\xi_n^*(\mathbf{r})F(\epsilon_n) = \frac{1}{(2\pi\hbar)^3}\int d\mathbf{p}F(H_0(\mathbf{p},\mathbf{r})),$$

where $H_0(\mathbf{p},\mathbf{r}) = p^2/(2m_F) + V_F(\mathbf{r})$ and $F(x)$ is an arbitrary function.

We suppose that all $|\phi(\tau_i,\mathbf{r}_i)|^2$ have one and the same argument $(\tau_1,\mathbf{r}_1)$. In this approximation we have:

$$S_{eff} = \int_0^{\hbar\beta} d\tau d\mathbf{r} f_{eff}(|\phi(\tau,\mathbf{r})|),$$

$$f_{eff}(|\phi(\tau,\mathbf{r})|) = -\frac{\beta^{-1}}{(2\pi\hbar)^3}\int d\mathbf{p}\ln\left(1 + e^{\beta(\mu_F - g_{BF}|\phi(\tau,\mathbf{r})|^2 - H_0(\mathbf{p},\mathbf{r}))}\right).$$

Effective Hamiltonian

The effective boson Hamiltonian:

$$H_{eff} = \int d\mathbf{r} \left\{ \frac{\hbar^2}{2m_B} |\nabla\phi|^2 + (V_B(\mathbf{r}) - \mu_B)|\phi|^2 + \right.$$

$$\left. + \frac{g_B}{2}|\phi|^4 + f_{eff}(|\phi|) \right\}.$$

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In low temperature limit $\tilde{\mu}/(k_B T) \gg 1$:

$$f_{eff}(|\phi|) = -\frac{2}{5}\kappa\tilde{\mu}^{5/2} - \frac{\pi^2}{4}\kappa(k_B T)^2\tilde{\mu}^{1/2},$$

where $\tilde{\mu} = \mu_F - V_F(\mathbf{r}) - g_{BF}|\phi(\tau, \mathbf{r})|^2$ and $\kappa = 2^{1/2} m_F^{3/2}/(3\pi^2\hbar^3)$.

Effective Hamiltonian

The effective boson Hamiltonian for $g_{BF} < 0$ at $T = 0$:

$$H_{eff} = \int d\mathbf{r} \left\{ \frac{\hbar^2}{2m_B} |\nabla\phi|^2 + (V_{eff}(\mathbf{r}) - \mu_B)|\phi|^2 + \right.$$

$$\left. + \frac{g_{eff}}{2}|\phi|^4 + \frac{\kappa}{8\mu_F^{1/2}} g_{BF}^3 |\phi|^6 \right\}.$$

$$V_{eff}(\mathbf{r}) = \left(1 - \frac{3}{2}\kappa\mu_F^{1/2} g_{BF}\right) \frac{1}{2} m_B \omega_B^2 \left(x^2 + y^2 + \lambda^2 z^2\right),$$

$$g_{eff} = g_B - \frac{3}{2}\kappa\mu_F^{1/2} g_{BF}^2,$$

$$\mu_F = \hbar\omega_F (6\lambda N_F)^{1/3}.$$

Variational approach

Variational boson wave function:

$$\phi(\mathbf{r}) = \sqrt{\frac{N_B\lambda}{\omega^3 a^3 \pi^{3/2}}} \exp\left(-\frac{(x^2+y^2+\lambda^2 z^2)}{2w^2a^2}\right).$$

$$a = \sqrt{\frac{\hbar}{m_B\omega_B}}.$$

Variational energy:

$$\begin{aligned} \frac{E_B}{N_B\hbar\omega_B} &= \frac{2+\lambda}{4}\frac{1}{w^2} + bw^2 + \frac{c_1N_B}{w^3} + \frac{c_2N_B^2}{w^6}, \\ b &= \frac{3}{4}\left(1 - \frac{3}{2}\kappa\mu_F^{1/2}g_{BF}\right), \end{aligned}$$

Variational approach

$$c_1 = \frac{1}{2}\left(g_B - \frac{3}{2}\kappa\mu_F^{1/2} g_{BF}^2\right)\frac{\lambda}{(2\pi)^{3/2}\hbar\omega_B a^3},$$

$$c_2 = \frac{\kappa}{8\mu_F^{1/2}} g_{BF}^3 \frac{\lambda^2}{3^{3/2}\pi^3\hbar\omega_B a^6}.$$

Experimental system: Mixture of fermionic ^{40}K and bosonic ^{87}Rb (Mogundo et al, Science 297, 2240 (2002)):

$a_B = 5.25\ nm,\ a_{BF} = -21.7^{+4.3}_{-4.8}\ nm$

Critical values: $N_{Bc} \approx 10^5$; $N_K \approx 2 \times 10^4$

Vortex state (variational approach)

Variational function

$$\phi(\mathbf{r}) = \sqrt{\frac{\lambda N_B}{(\omega a)^5 \pi^{3/2}}} \rho \exp\left(-\frac{\rho^2 + \lambda^2 z^2}{2\omega^2 a^2}\right) e^{i\varphi l}$$

Variational energy

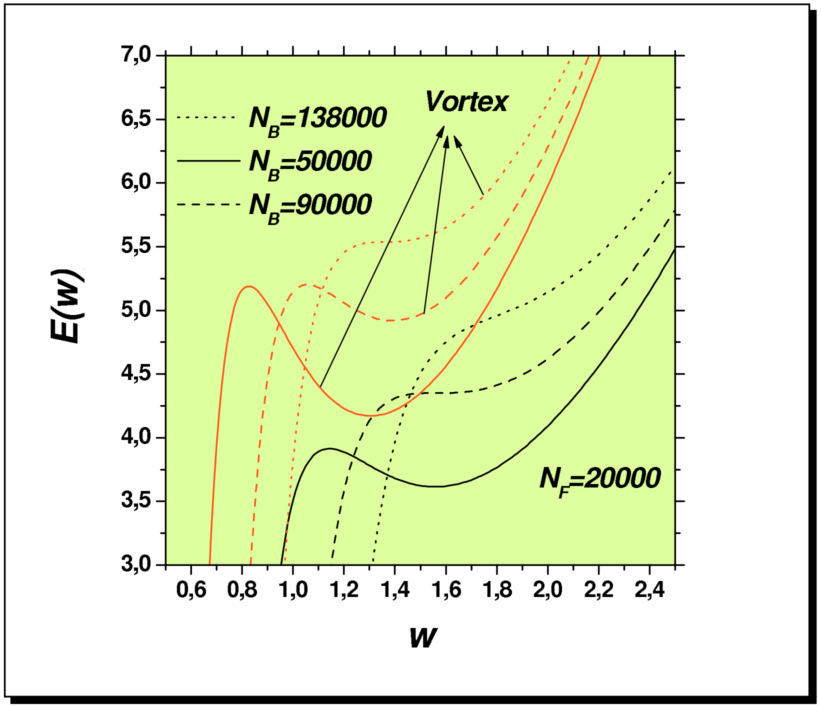
$$\frac{E_b}{N_B \hbar \omega_B} = \frac{2 + \lambda^2 + 2l^2}{4} \frac{1}{\omega^2} + B\omega^2 + \frac{C_1 N_B}{\omega^3} + \frac{C_2 N_B^2}{\omega^6},$$

$$B = \frac{5}{3} b,$$

$$C_1 = \frac{1}{2} c_1,$$

$$C_2 = \frac{2}{9} c_2.$$

Variational approach



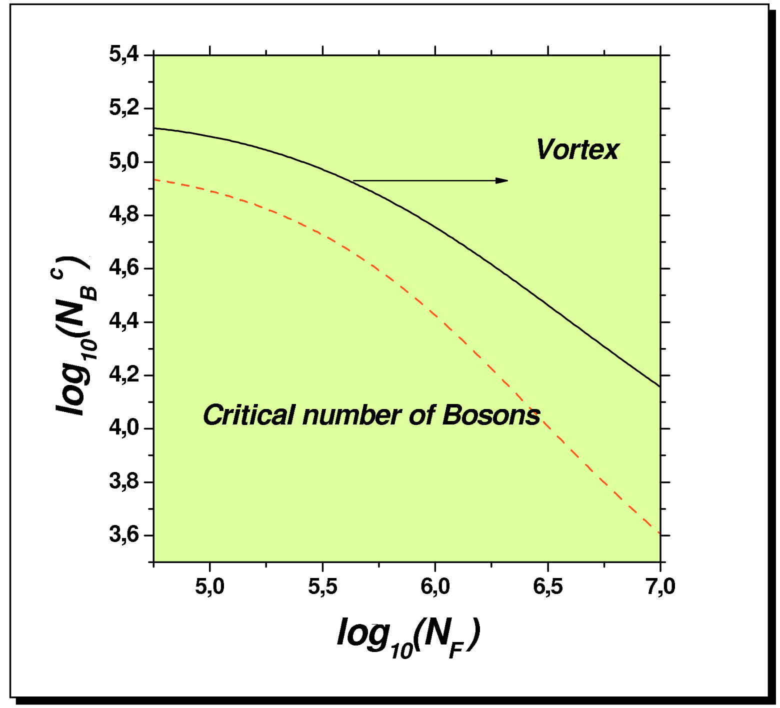


Fig. 1. Variational energy $\frac{E_B}{N_B \hbar \omega_B}$ as a function of w for various numbers of bosons.

Fig. 2. Critical number of bosons N_{Bc} as a function of the number of fermions N_F.

Variational approach

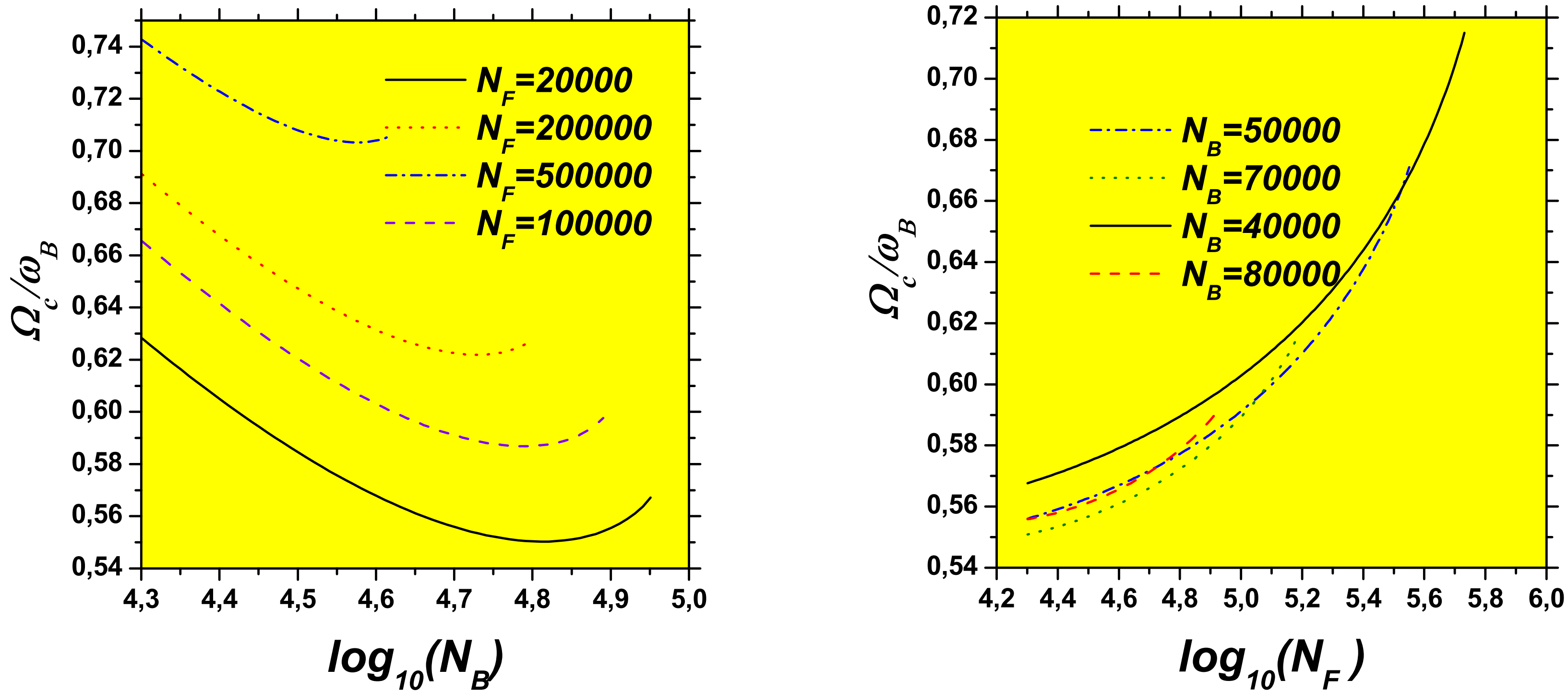


Fig. 3. Critical angular vortex velocity as a function of a number of bosons N_B.

Fig. 4. Critical angular vortex velocity as a function of a number of bosons N_F.

Variational approach

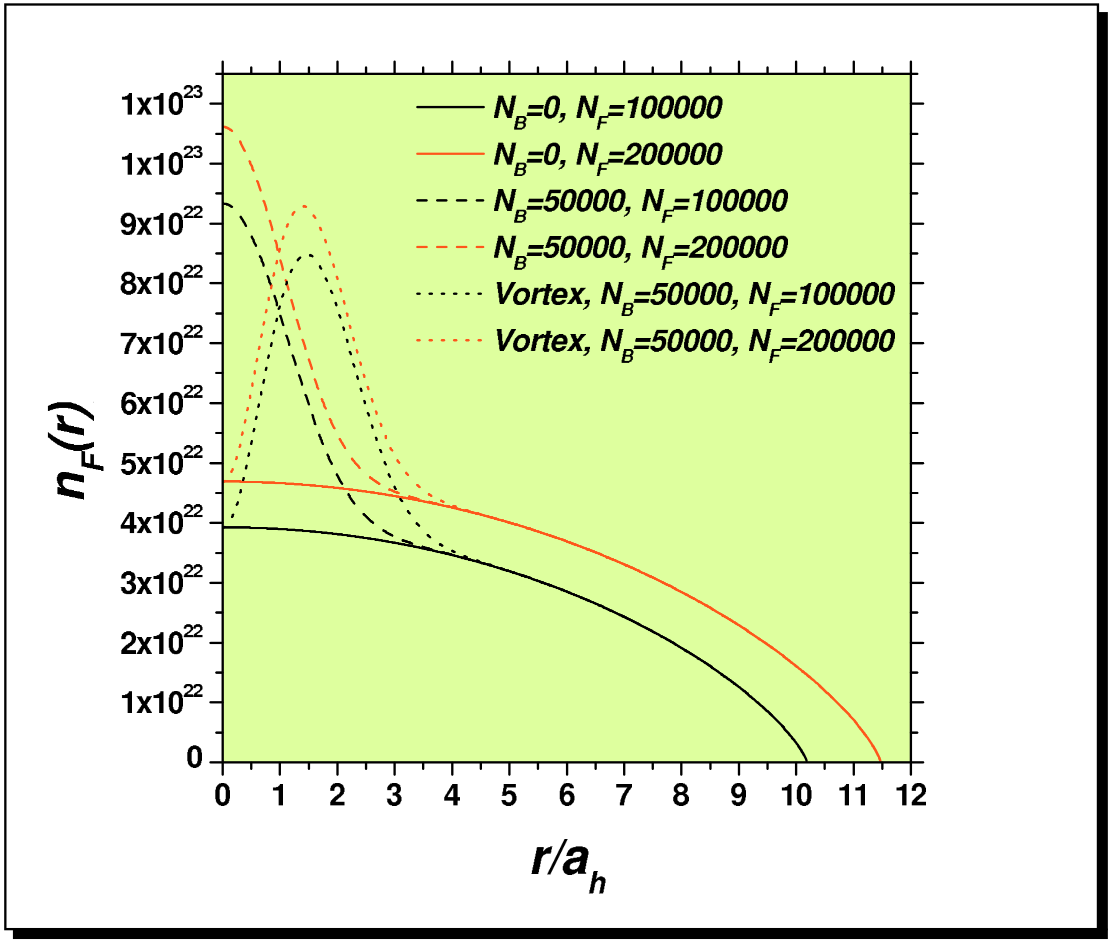


Fig. 5. Density distribution of Fermions.

Variational approach

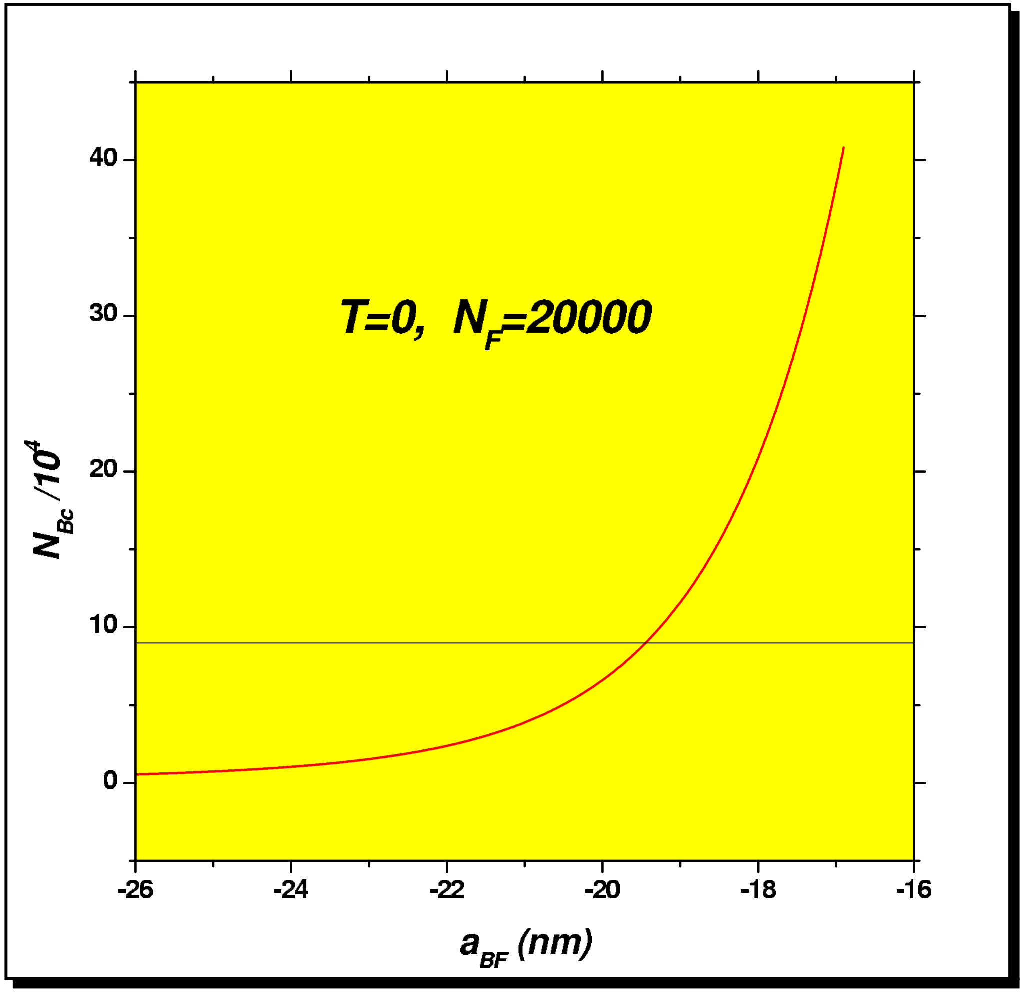


Fig. 5. Critical number of bosons as a function of boson-fermion scattering length a_{BF}.

Thomas-Fermi Approximation

The Gross-Pitaevski equation that follows from the effective hamiltonian:

$$\left(-\frac{\hbar^2}{2m_B}\triangle\phi + (V_{\text{eff}} - \mu_B) + g_{\text{eff}}|\phi|^2 + \frac{3\kappa}{8\mu_F^{1/2}} g_{BF}^3 |\phi|^4\right)\phi = 0.$$

At high enough densities one can neglect the kinetic energy.

$$|\phi|^2 = \theta(\mu_B - V_{\text{eff}}) g_{\text{eff}} \frac{-1 + \sqrt{1 + (\mu_B - V_{\text{eff}})\frac{3\kappa}{g_{\text{eff}}^2 2\mu_F^{1/2}} g_{BF}^3}}{\frac{3\kappa}{4\mu_F^{1/2}} g_{BF}^3} \rightarrow$$

$$\rightarrow \theta(\mu_B - V_{\text{eff}}) \frac{(\mu_B - V_{\text{eff}})}{g_{\text{eff}}}, \text{ if } g_{BF} \rightarrow 0.$$

Thomas-Fermi Approximation

Density of Bosons:

$$n(\mathbf{r}) = n(0)\left(1 - \sqrt{1 + \frac{\bar{x}^2 + \bar{y}^2 + \lambda^2\bar{z}^2 - R^2}{R_{max}^2}}\right).$$

Here

$$\bar{r_i} = r_i/a_h;\;\; R^2 = \frac{2\mu_B}{c_0 m_B \omega_B^2 a_h^2};$$

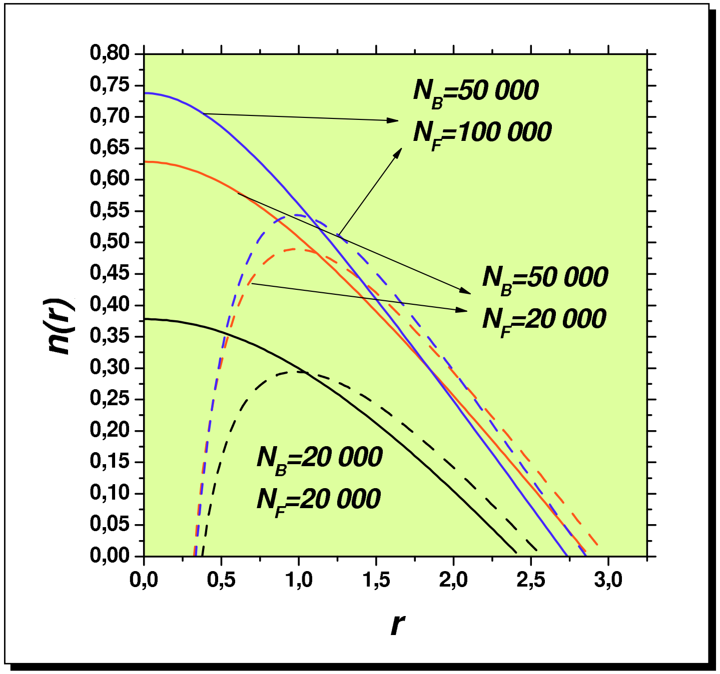
$$n(0) = -\frac{4}{3}\frac{g_{eff}\mu_F^{1/2}}{\kappa g_{BF}^3};\;\; R_{max}^2 = -\frac{4}{3}\frac{g_{eff}^2\mu_F^{1/2}}{\kappa g_{BF}^3 c_0 m_B \omega_B^2 a_h^2};$$

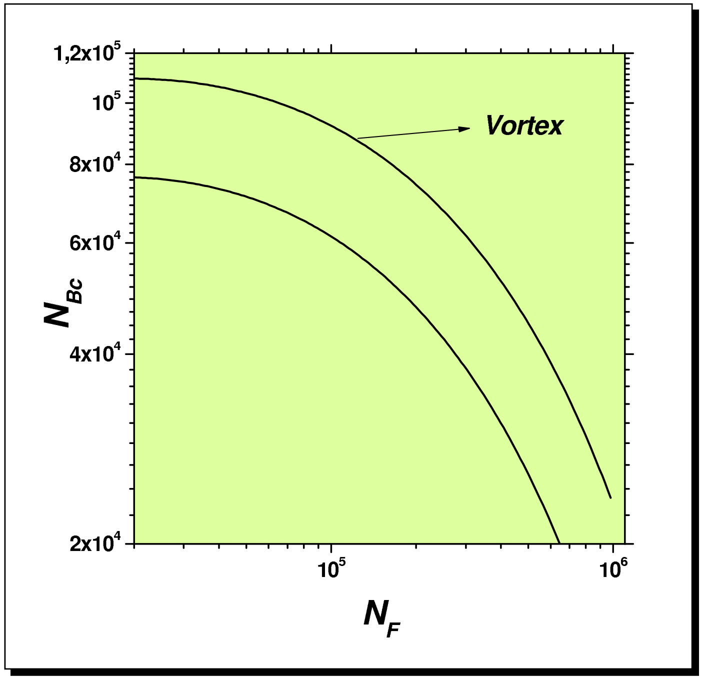
$$c_0 = \left(1 - \frac{3}{2}\kappa\mu_F^{1/2} g_{BF}\right).$$

Thomas-Fermi Approximation

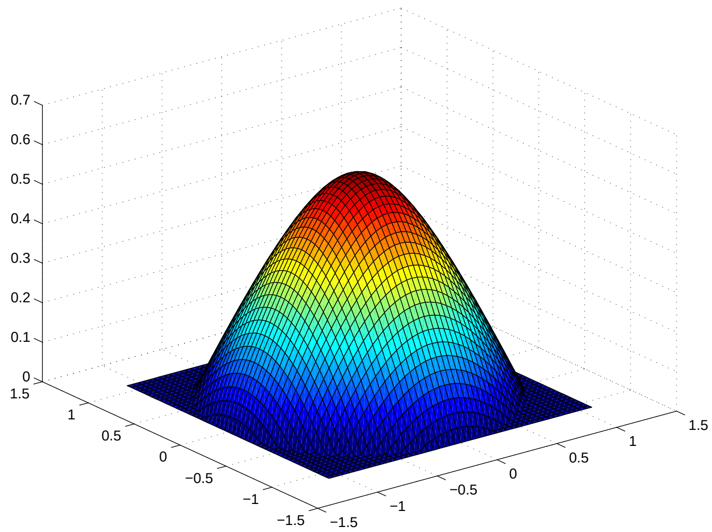
Vortex state in TFA: $\phi(\mathbf{r}) = \sqrt{n(\mathbf{r})}e^{i\varphi l}$,

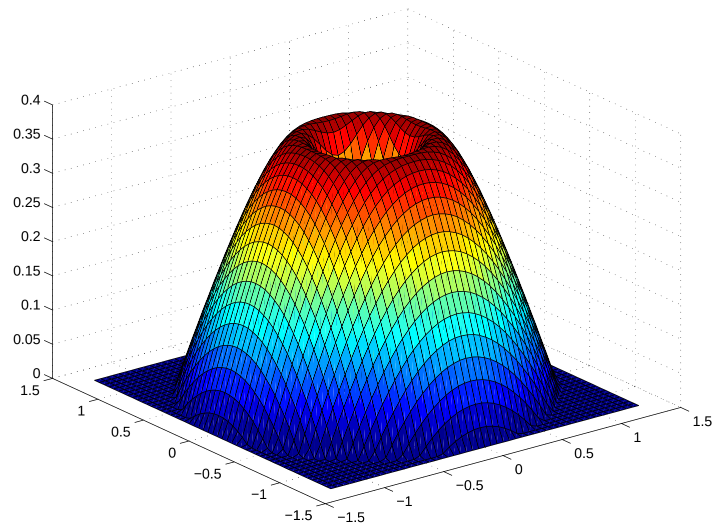
where $n(\mathbf{r}) = n(0)\left(1 - \sqrt{1 + \frac{\bar{\rho}^2 + \lambda^2 \bar{z}^2 + \frac{l^2}{c_0 \bar{\rho}^2} - R^2}{R^2_{max}}}\right)$.





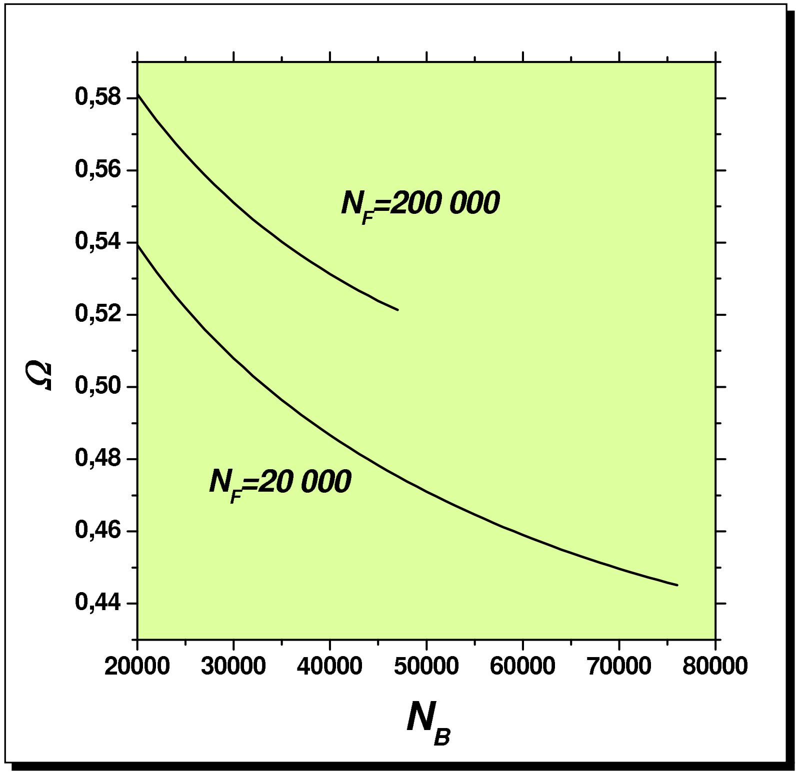
Расчет





Распределение бозонной плотности в магнитной ловушке для $l = 0$ и $l = 1$.

Thomas-Fermi Approximation



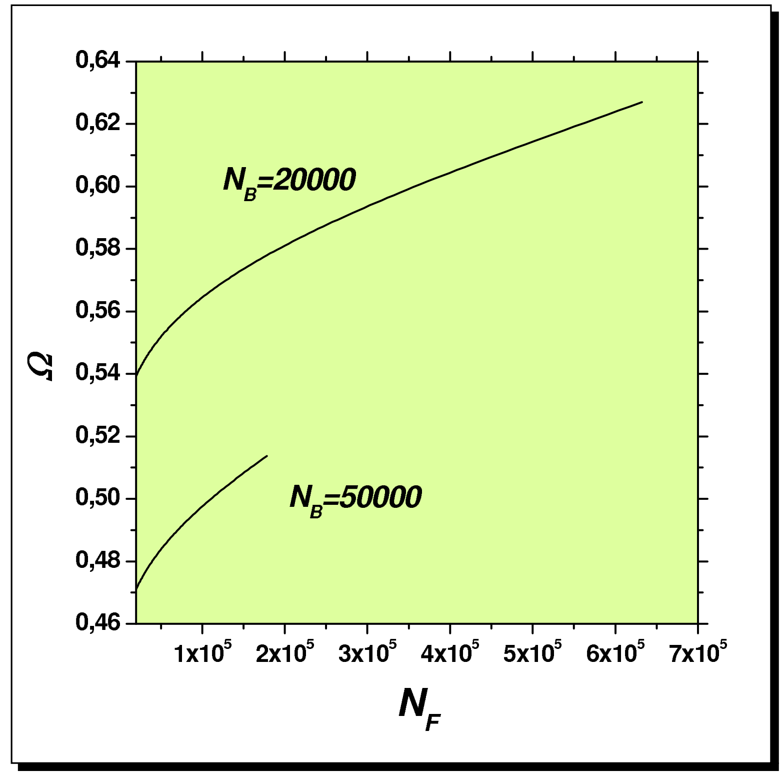


Fig. 9. Critical angular vortex velocity as a function of a number of bosons N_B in TFA.

Fig. 10. Critical angular vortex velocity as a function of a number of fermions N_F in TFA.

Numerical solution of GGPE

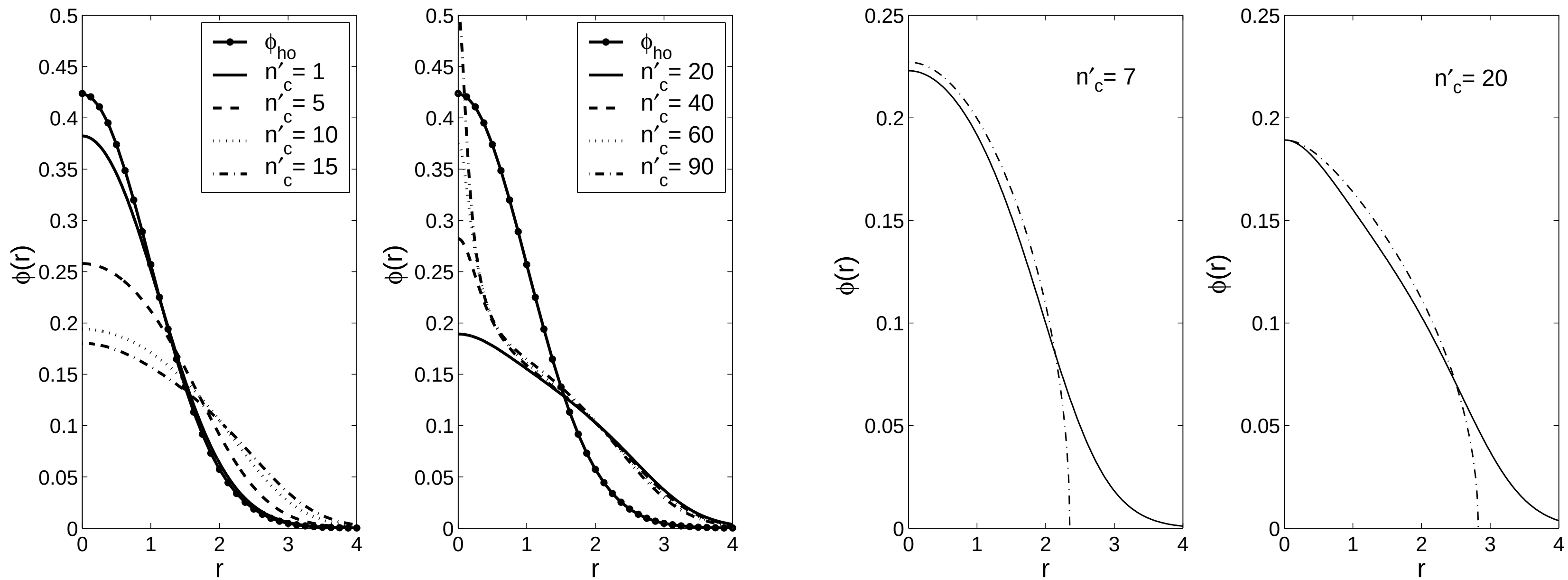
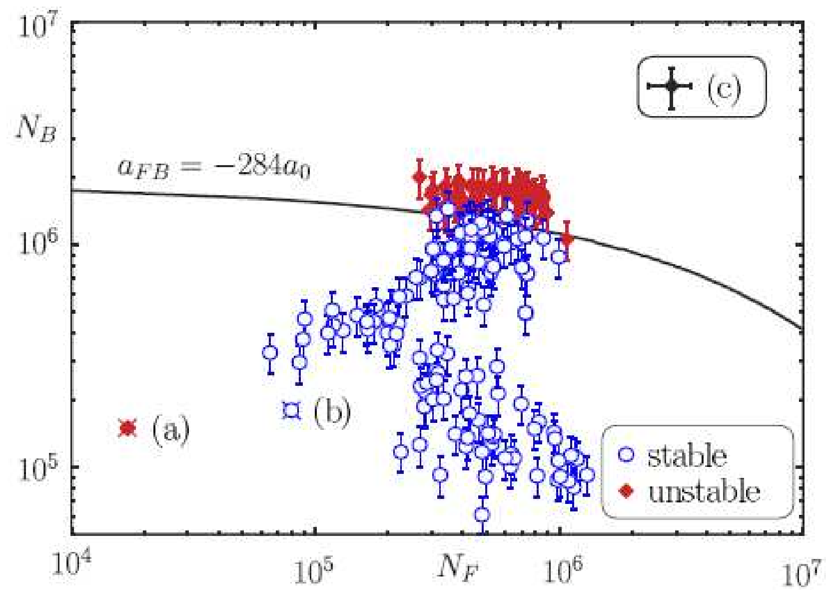


Fig. 11. Evolution of the profile of the condensate wave function with increasing central density n_c'. $\phi_{ho} = \pi^{-3/4} exp(-r^2/2)$ corresponds to the ground state of the ideal Bose.

Fig. 12 . The profile of the condensate wave function $\phi(r)$, found from the numeric solution of GGPE (solid line), and the TF approximation (dashed-dotted line).

Experiment

Fig. 6. Experimental stability diagram for the ^{40}K-^{87}Rb mixture(C. Ospelkaus, S. Ospelkaus, K. Sengstock, and K. Bongs, Phys. Rev. Lett. 96, 020401 (2006)). The solid line is based on the present theory and $a_{BF} = -284a_0$.



Nature of the collapse transition.

A strong rise of density of bosons and fermions in the collapsing condensate enhances intrinsic inelastic processes, in particular, the recombination in 3-body interatomic collisions, as is the case for the well-known ^{7}Li condensates. In the presence of a vortex there appears a hole in the middle of the condensate. This reduces the maximum density of the condensate and increases the critical number of bosons. However, for the Bose-Fermi mixtures with attraction between components the formation of the boson-fermion bound states is also possible (M. Yu. Kagan et al, cond-mat/0209481). It seems that the description of the evolution of the collapsing condensate should include both these mechanisms.

Summary

- Using the effective Hamiltonian for the Bose system, the instability and collapses of the trapped boson-fermion mixture due to the boson-fermion attractive interaction in the presence of the quantized vortices was investigated.

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- References: A.M. Belemuk, N.M. Chtchelkatchev, V.N. Ryzhov, S.-T. Chui, "Vortex state in a Bose-Fermi mixture with attraction between bosons and fermions", Phys. Rev. A 73, 053608 (2006); А.М. Белемук, В.Н. Рыжов, С.Т. Чуи, "Механизм коллапса конденсатной волновой функции в бозе-ферми-смеси с притяжением между компонентами", Письма в ЖЭТФ, 2006, т. 84, вып. 6, стр.354-359; A.M. Belemuk, V.N. Ryzhov, S.-T. Chui, "Stable and unstable regimes in Bose-Fermi mixtures with attraction between

Заключение

- "Сильно коррелированные электронные системы и квантовые критические явления"(Троицк, 2007); "23rd International Conference on Statistical Physics"(Genova, Italy, 2007).

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- Thank you!