# Summaries of all articles

*G.G. Amosov and A.I. Dnestryan*

## On the spectrum of a set of integral operators determining the symplectic quantum tomogram

In this paper, we find eigenfunctions and eigenvalues of a set of integral operators determining the symplectic quantum tomogram. It is shown that given certain parameters, the studied operators define the fractional Fourier transform. The variances for canonical observables associated with spectrum states are calculated.

**Keywords:** symplectic quantum tomogram, fractional Fourier transform, spectrum of an integral operator.

*M.V. Balashov*

## On moduli of convexity of the set and the function

We consider a relationship between uniformly convex sets and functions in a Banach space. We prove that the level sets of a uniformly convex function are uniformly convex sets. We also prove that the boundary of a uniformly convex set is the graph of a uniformly convex function. The estimates for the moduli of convexity are obtained.

**Keywords:** modulus of convexity, uniformly convex function, uniformly convex set.

*A.A. Belolipetskiy and A.A. Ter-Krikorov*

## Solution of a singularly perturbed boundary initial problem for a linear parabolic equation

A singularly perturbed boundary initial problem for a linear parabolic equation is investigated. Approximations are constructed for small parameter values. We prove the existence and uniqueness theorem of solutions for which approximations are asymptotic. Such problems are of importance for laser targets technology in the thermonuclear synthesis problem.

**Keywords:** parabolic equation, singular perturbation, boundary layer solution, laser target.

*O.V. Besov*

## Sobolev's embedding theorem for anisotropically irregular domains

We prove a Sobolev-type embedding theorem, viz. an embedding of the Sobolev space $W_p^s(G)$ in the Lebesgue space $L_q(G)$ for anisotropically irregular domains $G \subset \mathbb{R}^n$ of various classes.

**Keywords:** Sobolev space, embedding theorem, irregular domain, anisotropy.

*A.I. Besportochnyy*

## Asymptotic regimes of the hydrodynamic contact of rigid cylinders having thin elastic coating

A one-dimensional lubricating fluid flow in a thin layer separating a cylinder with elastic coating and rigid half-space is considered. The fluid viscosity depends on pressure. The lubricant film thickness and pressure distribution within the contact area are investigated. Different asymptotic lubrication regimes are examined. The ranges of applicability of a number of formulas for calculating the film thickness are suggested.

**Keywords:** lubrication, hydrodynamic contact, elastic coating, qualitative analysis, asymptotic regimes.

*S.B. Gashkov and A.A. Burtsev*

## On hardware and software implementation of arithmetic in finite fields of characteristic 7 for pairing calculation

We study scheme (hardware) and program (software) methods for multiplying polynomials over fields of characteristic 7 in order to apply them to cryptographic protocols on elliptic curves based on parings. We consider hardware and software implementations of arithmetic in $GF(7)$, $GF(7^2)$, $GF(7^n)$, $GF(7^{7n})$, and $GF(7^{14n})$ and estimate the complexity of corresponding schemes and programs.

**Keywords:** schemes for arithmetic in finite fields, cryptographic reports.

*A.K. Volosova*

## On the theory of nonlinear diffusion and heat conductivity

It is shown that a wide class of equations in partial derivatives is equivalent to a system of functional linear algebraic equations. It permits us to construct exact and approximate solutions and determine the solution character of evolution with respect to "limit attracting solution" according to the eigenvalues of a matrix corresponding to the equation under study. We propose an alternative classification for PDE solutions on eigenvalues.

**Keywords:** transformation of variables, eigenvalues, classification for PDE solutions.

*B.I. Golubov and S.S. Volosivets*

## Generalized weighted integrability of the multiplicative Fourier transform

The multiplicative Fourier transform introduced by N.Ya. Vilenkin is defined by the sequence of natural numbers $\mathbf{P} = \{p_n\}_{n=1}^{\infty}$, $p_n \geqslant 2$. In the case $p_n \equiv 2$, it coincides with the known Walsh transform having numerous applications in numerical analysis and coding theory. The sufficient conditions on functions providing the weighted integrability of their multiplicative Fourier transforms on $\mathbb{R}_+$ are obtained for a bounded sequence $\mathbf{P}$. Some of them are sharp. The multiplicative counterparts of Hardy–Littlewood, Zygmund, Moricz, Onneweer, M. Izumi–S. Izumi and Aljancic–Tomic theorems are proved.

**Keywords:** multiplicative Fourier transform, multiplicative convolution, weighted integrability, function of bounded $s$-fluctuation, Lipschitz classes, monotone function.

*N.A. Gusev*

## Asymptotic properties of linearized equations of low compressible fluid motion

The initial boundary value problem for linearized equations of a viscous barotropic low compressible fluid in a bounded domain is considered. The convergence of solutions to this problem to the corresponding incompressible limit is studied. The sufficient conditions for the convergence to be weak and strong are given.

**Keywords:** linearized equations of compressible fluid, compressibility factor, low compressible fluid.

*P.E. Dvurechensky and G.E. Ivanov*

## Optimal strategy algorithm in a nonlinear differential game using convolution

We develop a method for computing quasioptimal strategies in a nonlinear differential game on a fixed time interval with a goal set. In the two-dimensional case, the play-attainable sets are calculated by an algorithm similar to that for convolution of Minkowski's sum of two polygons. We give in detail the error estimates of the algorithm.

**Keywords:** differential game, optimal strategy algorithm.

*N.O. Ermilov*

## Tetrahedrons with the same suite of length ribbings and volume inscribed into the same sphere

The recovering of a tetrahedron on the specified length ribbings depends on the allocation of the length ribbings. In the capacity of the tetrahedron length ribbings, other characteristics such as volume and the radius of a circumscribed sphere can define the tetrahedron. But there is an example of the tetrahedron defined ambiguously by these eight parameters.

**Keywords:** tetrahedron, suite of the length ribbings, volume, radius of a circumscribed sphere.

*G.E. Ivanov and G.M. Ivanov*

## Relationship between supporting conditions in Banach spaces

We consider two classes of weakly convex sets in a Banach space. The classes are characterized by $N$-supporting and $P$-supporting conditions, respectively. We prove that the two classes coincide provided that the Banach space is uniformly convex. The proof is based on the theorem of the smooth equivalent renorm of a uniformly convex Banach space.

**Keywords:** supporting condition, weak convexity.

*V.M. Ipatova*

## Attractors of finite difference schemes for the Lorenz system with time dependent coefficients

The explicit and implicit finite difference schemes for the Lorenz system with time dependent coefficients are considered. For each of these schemes the existence of its uniform attractor is established. It is shown that with the decrease of a time step the attractors of schemes become closer to the genuine attractor of the system.

**Keywords:** attractor, nonautonomous system, finite difference scheme.

*V.V. Maksimenko, L.Yu. Kupriyanov, V.A. Zagaynov and A.A. Hasanov*

## Self-organization elements in gas diffusion inside a system of nanogeterogeneities

The ideal gas diffusion in a system of small spherical heterogeneities is considered. It can be a collective of particles or a system of small spherical cavities in some homogeneous medium. The effective diffusion coefficient is calculated using the methods of multiple scattering theory. Contribution to the diffusion of multiple passing closed loops on a molecule trajectory is considered. Interest in such loops is associated with the constructive interference of amplitudes of two alternative ways of the loop passing (clockwise and counterclockwise). This interference always exists irrespective of medium disorder. It is shown that interference corrections to classical diffusion reduce to the appearance of low frequency macroscopic oscillations of gas concentration and the total diffusion stopping. The latter resembles the Anderson localization of an electron in the system of impurities inside an ideal lattice.

**Keywords:** diffusion, nanogeterogeneities, oscillations, localization.

*N.A. Markitantova and A.P. Chernyaev*

## Nonlinear filtration to a horizontal well in the case of special nonlinearity

The model of stationary incompressible fluid filtration for the special nonlinear flow to a horizontal nonsymmetric well is considered. The hodograph plane map is used. The equation in hodograph plane variables may be reduced to Laplace's equation.

**Keywords:** stationary filtration, nonlinear filtration flow, incompressible liquid, nonsymmetric well.

*G.I. Marchuk, V.I. Agoshkov and V.M. Ipatova*

## Theory of solvability of initial boundary value problems and data assimilation problems for the primitive equations of the ocean

This paper is a review of progress in the solvability analysis of initial boundary value problems and data assimilation problems for the primitive equations of the ocean. The historical sketch and the description of the earliest results are given. The existence of weak solutions and the solvability of data assimilation problems for the primitive equations with a free surface condition on the upper boundary are studied. The existence and uniqueness theorems for a strong solution to the simplified system of primitive equations are considered.

**Keywords:** partial differential equations, ocean dynamics models, solvability, inverse and variational problems, data assimilation problems.

*M.N. Matveev*

## Invisible facets and face polytopes

The notion of a polytopally complete fan is well-known, viz. any cone of a fan is the conical hull of a polytope facet. The paper generalizes this notion to incomplete fans. The key feature of this generalization is that the definition of a polytopally incomplete fan uses the notion of a polytoplly invisible facet. A criterion for an unnecessarily complete fan to be polytopal is proved. An application of the criterion is shown.

**Keywords:** polytopes, cones, fans, polytope facets, systems of linear equations and inequalities.

*Yu.N. Orlov*

## Asymptotic diagonalization of polynomial quantum Hamiltonians

The method of asymptotic diagonalization of polynomial quantum Hamiltonians for large numbers in terms of the second quantization approach is described. The asymptotic spectrum distribution function and the corresponding system of special polynomials as Hamiltonian eigenfunctions are constructed.

**Keywords:** quantum Hamiltonian, conservation laws, spectrum asymptotic, special polynomials.

*O.K. Podlipsky*

## On methods of expert knowledge bases for development of applied consulting and training systems

Methods for constructing expert knowledge bases for development of applied consulting and training systems proposed by a group of experts are studied. The formal model of the expert in malignant glomerulonephritis is constructed.

**Keywords:** decision-making theory, expert knowledge base.

*E.S. Polovinkin*

## Riemannian integrability of set-valued maps

In this paper, we obtain new results for the Riemannian integration on a segment of set-valued maps with nonconvex compact values belonging to uniformly smooth Banach spaces. We prove the convexity of these Riemann integrals and obtain the common properties of Riemann integrals. We find the necessary and sufficient condition for Riemannian integrability of a nonconvex set-valued map belonging to a separable uniformly smooth Banach space. This condition is the almost everywhere continuity of the convex hull of map values.

**Keywords:** Minkowski sum of sets, Hausdorff metric, Riemann integral of set-valued map, uniformly smooth Banach space, necessary and sufficient condition of Riemannian integrability.

*O.A. Pyrkova, A.A. Onufriev and A.T. Onufriev*

## Initial time behavior of the velocity in a homogeneous and isotropic turbulent flow

In this paper, the initial time behavior of the velocity in a homogeneous and isotropic turbulent flow is considered, with account taken of an intermitting flow phenomenon. The flow is considered as a mixture of turbulent and viscous regimes. For both regimes there are Loitsansky invariants and Kolmogorov (turbulent regime) and Millionschikov's (viscous regime) similarities. Velocity decays coincide with experimental data at initial time.

**Keywords:** homogeneous and isotropic turbulent flow, homogeneous and isotropic turbulent flow, velocity decays.

*V.Zh. Sakbaev*

## On the regularization of a degenerate Schrödinger operator and the minimizaton of a seminorms set

The Cauchy problem for the Shrödinger equation with degenerate Hamiltonian is studied. The Hamiltonian is a second- order linear differential operator of nonnegative characteristic form. We obtain the necessary and sufficient conditions for well-posedness of the Cauchy problem. In the case of ill-posedness, we use the elliptic regularization method and the quazisolution method for studying the Cauchy problem. We establish the following correspondence between these methods: any sequence of regularized solutions and any minimizing sequence of the set of deviation seminorms have a unique limit.

**Keywords:** operator of nonnegative characteristic form, elliptic regularization, quazisolution.

*V.B. Trushin*

## Existence of solutions of some variational inequalities

The paper deals with the solvability conditions for noncoercive variational inequalities.

**Keywords:** monotonicity, pseudomonotonicity, variational inequality, coercivity.

*A.I. Tyulenev*

## Description of the traces of weight Sobolev spaces

In this paper, we derive the generalization of the weight Sobolev space Gagliardo's theorem on the description of the traces of Sobolev spaces $W_p^1(Q)$. Here $Q$ is a square (cube) and the trace is considered on the fixed side (face) of this square (cube). If the domain studied is a square, we consider the weight function monotonically dependent on the distance to the fixed side of the square. If the domain is a cube, we consider the weight function monotonically decreasing on some variables, with others fixed, and monotonically increasing on the remainder variables, with others fixed.

**Keywords:** Sobolev space, weight function.

*E.A. Umnov and A.E. Umnov*

## Parametric linearization method using penalty functions with everywhere invertable derivation

An approach to parametric primal-dual linear programming problems is considered. The approach is based on the smooth penalty function method. A be-level optimization scheme is given. The suggested method is demonstrated for a set of test problems.

**Keywords:** mathematical programming, be-level optimization, penalty function method, primal-dual linear programming problems.

*A.A. Fonarev*

## Functional minimization on the convex set of a normed space

The functional minimization on the convex set of a real normed space in the absence of the reexivity of space and the coercivity of the functional is investigated. A relaxational sequence is constructed using the iterative process. This sequence minimizes the functional in the presence of the convexity of the functional.

**Keywords:** functional, minimization, iterative process.

*A.P. Chernyaev*

## Nonlinear filtration for the special and degree laws of the resistance medium

The general kind of incompressible liquid filtration fluids are considered. The hodograph plane equations obtained from the physical plane fluids are linearized. They are reduced to Laplace's equation according to the special filtration law and to the known equations according to the degree filtration law. An example of fluid to a horizontal well shows the solvability of the initial problem in the comfortable conditions for numerical and quality investigations.

**Keywords:** filtration, filtration fluids, incompressible liquid, physical plane, hodograph, horizontal well, resistance medium.

*M.E. Shirokov*

## On properties of probability measures on the set of quantum states

Two properties of the set of probability measures on the space of quantum states endowed with weak convergence topology are considered. Their corollaries related to the particular functional constructions used in the convex analysis and their applications are discussed.

**Keywords:** quantum state, probability measure, barycenter map, convex hull and convex closure of a function.