Summaries of all articles

A. R. Yarmukhametov

On some properties of random graphs of a special type

In this paper, we consider random subgraphs of a complete distance graph. The graph vertices are vectors $\mathbf{x} \in \{0,1\}^n$ satisfying the condition $\|\mathbf{x}\| = \sqrt{\frac{n}{2}}$. The graph edges are those pairs of vectors whose elements are $\sqrt{\frac{n}{2}}$ apart. The threshold probability is earlier known for the property of connectivity of such random graphs. Also the threshold probability is known for the emergence of a giant component. We now prove that, as in the classical Erdős–Rényi model, the phase transition from the connectivity to its absence coincides with the transition from connectivity to the presence of isolated vertices. Also, we give a result on the limit probability of connectivity assuming that the edge probability is “inside” the phase transition.

Keywords: random graph, connectivity, isolated vertex, giant component, distance graph, threshold probability.

A. A. Kokotkin, A. M. Raigorodskii

On the realization of random graphs by graphs of diameters

This work lies on the interface between combinatorial geometry and theory of random graphs. We study the conditions under which a random graph in the Erdős–Rényi model contains subgraphs isomorphic to some graphs of diameters on the plane having their chromatic numbers equal to three. For the appropriate extremal characteristic of a random graph, we succeed in obtaining tight bounds and even asymptotics.

Keywords: random graph, graph of diameters, chromatic number.

L. A. Ostroumova

Expectations of the *k*th indegrees of vertices of random graphs in the Bollobás–Riordan model

This work is concerned with the Bollobás–Riordan model of a random web graph. This model is good for describing the behaviour of the real WWW. The expectation of the kth degree of a vertex in this model is considered. We obtain new upper and lower bounds.

Keywords: random graph, web graph, vertex degree.

L. A. Ostroumova

On the *r*-diameters of random graphs in the Bollobás–Riordan model

This work is concerned with the Bollobás–Riordan model of a random web graph. This model is good for describing the behaviour of the real WWW. We consider a generalization of the notion of the diameter of a graph — the so-called r-diameter which is defined as a maximum value taken over all sets of vertices of size r of a minimum distance between a pair of vertices in a given set. We prove a theorem which states that a web graph on n vertices is almost sure to have the r-diameter as close as presumed to the value $\frac{\ln n - \ln r}{\ln \ln n}$.

Keywords: random graph, web graph, diameter.

E. A. Grechnikov

Arithmetic properties of the second degrees of vertices of a random web-graph in the Bollobás–Riordan model

In this paper, we study the Bollobás–Riordan model of a random graph. We consider the expectation of the number of vertices having a given second degree in this random graph. We also prove that one of the main parts of this expectation is a linear combination with rational coefficients of the numbers 1, $\ln 2$ and π.

Keywords: random graph, web graph, vertex degree.

V. V. Bulankina

On dividing planar sets into five parts

In 1933, K. Borsuk proposed to partition sets of diameter 1 into parts of smaller diameter. Borsuk's problem is now one of the most popular in combinatorial geometry. In 1956, H. Lenz refined Borsuk's problem by asking a question of a minimum diameter of a part in a partition of a set into a given number of parts. In 2010, V.P. Filimonov replaced the question of a minimum diameter by that of a minimum distance which is absent among the points of each part. Filimonov showed that in partitioning a set into five parts one can always avoid the distance $\frac{1}{\sqrt{3}} = 0.577\ldots$. We succeed in showing that the same is true of the distance $\sqrt{2-\sqrt{3}} = 0.517\ldots$ To do this, we develop a new technique for studying infinite universal covering systems, which is also of interest.

Keywords: Borsuk's problem, diameter, forbidden distance, partition, universal covering system.

E. Yu. Voronetskii

On dividing planar sets into four, five, and six parts without sufficiently small distances

In this paper, we improve the previously known upper bound on a minimum distance, which is absent among the points of each part in a partition of an arbitrary set of diameter 1 on the plane into five parts.

Keywords: Borsuk's problem, diameter, forbidden distance, partition, universal cover.

D. A. Belov, N. A. Aleksandrov

On dividing planar sets into six parts of smaller diameter

In this paper, we improve the previously known upper bound on a minimum diameter of any of the six parts in a partition of an arbitrary set of diameter 1 on the plane into six parts.

Keywords: Borsuk's problem, diameter, partition, universal covering system.

A. B. Kupavskii, E. I. Ponomarenko, A. M. Raigorodskii

On some analogs of Borsuk's problem in the space $\mathbb{Q}^n$

In 1933, K. Borsuk conjectured that each set of diameter 1 in $\mathbb{R}^n$ can be partitioned into $n+1$ parts of smaller diameter. This conjecture was disproved in 1993. We consider various generalizations of Borsuk's problem to the cases of sets which lie in the space $\mathbb{Q}^n$ with the Euclidean metric and general metric l_p.

Keywords: Borsuk's problem, coloring and partitioning, graph of diameters.

V. B. Goldshteyn

On Borsuk's problem for $(0,1)$- and $(-1,0,1)$-polytopes in spaces of small dimensions

Classical Borsuk's conjecture is studied on dividing sets into parts of smaller diameters. The conjecture is proved for $(0,1)$-vectors with $n \leqslant 9$ and for $(-1,0,1)$-vectors with $n \leqslant 6$. Here n is the dimension.

Keywords: Borsuk's problem, diameter, coloring algorithms.

A. B. Kupavskii, A. M. Raigorodskii, M. V. Titova

On densest sets omitting distance 1 in spaces of small dimensions

In this paper, we study the value of the maximum upper density of a subset of the space $\mathbb{R}^d$, which avoids distance 1, for $d \leqslant 8$. We obtain new lower bounds on this value and apply the obtained results to solve a problem of geometric Ramsey theory.

Keywords: upper density, sets omitting distance one, chromatic number of a space, Ramsey numbers, distance graphs, lattices, packings.

A. E. Zvonarev, A. M. Raigorodskii

On distance graphs with large chromatic numbers and small clique numbers

The work is concerned with the study of the chromatic number $\chi(\mathbb{R}^n)$ of the Euclidean space. The quantity is defined as the minimum number of colors needed for a coloring of points in $\mathbb{R}^n$ such that any two points at distance 1 from each other have different colors. It is known that $\chi(\mathbb{R}^n) \geqslant (\zeta + o(1))^n$, where $\zeta = 1.239\ldots$ This is equivalent to the existence of an n-dimensional distance graph (vertices are points and edges are segments of length 1) with chromatic number $(\zeta + o(1))^n$. We prove much more: there exists a distance graph with chromatic number $(\zeta + o(1))^n$ and without cliques of growing size.

Keywords: chromatic number of a space, distance graph, graph without cliques, Ramsey theory.

E. I. Ponomarenko, A. M. Raigorodskii

On the chromatic number of the space $\mathbb{Q}^n$

This work deals with the classical Nelson–Hadwiger problem of the chromatic number of space. We consider a generalization of this problem to the case of the space $\mathbb{Q}^n$. We introduce the new quantity $\chi_{\text{aff}}(\mathbb{Q}^n)$ equal to the maximum value of the chromatic number of a distance graph whose vertices lie in an affine subspace of dimension n in a space $\mathbb{Q}^m$ and whose edges are generated by a rational distance. We prove new bounds on this quantity.

Keywords: chromatic number of a space, rational space.

D. A. Shabanov

On a generalization of the Erdős–Lovász problem

We study a generalization of the classical Erdős–Lovász problem dealing with coloring nonuniform hypergraphs. Let $H = (V, E)$ be an arbitrary hypergraph whose minimum edge size is n and its girth is at least 4. We obtain a new sufficient condition for r-colorability of the hypergraph H in terms of some bounds on the function $f_r(H) = \sum_{e \in E} r^{1-|e|}$.

Keywords: hypergraph colorings, Erdős–Lovász problem, hypergraphs with large girth.

S. M. Teplyakov

Recurrent upper bounds in the Erdős–Hajnal problem on hypergraph coloring and its generalizations

In 1961, P. Erdős and A. Hajnal posed a problem of finding the quantity $m(n)$ equal to the minimum number of edges in an n-uniform hypergraph with chromatic number greater than 2. Different asymptotic bounds are now known for $m(n)$. However, exact values are found only for $n \leqslant 3$. For other small values of n, only recurrent estimates are obtained. We consider an important generalization of the problem. Namely, we are interested in a quantity $m_k(n)$ equal to the minimum number of edges in an n-uniform hypergraph that does not admit a bichromatic coloring of the set of its vertices such that any edge contains in it at least k vertices of the first color and at least k vertices of the second color. We can find a series of recurrent bounds on $m_k(n)$. These bounds considerably strengthen all the previously known estimates for most values of n and k.

Keywords: hypergraph, chromatic number.

D. D. Cherkashin, A. B. Kulikov, A. M. Raigorodskii

On hypergraph cliques with chromatic number 3 and a given number of vertices

In 1973, P. Erdős and L. Lovász pointed out that any hypergraph with pairwise intersecting edges has chromatic number 2 or 3. In the first case, this hypergraph can have any number of edges. However, Erdős and Lovász proved that in the second case, the number of edges is bounded from above. For example, if a hypergraph is n-uniform, has pairwise intersecting edges and chromatic number 3, the number of its edges is less than n^n. Recently, D.D. Cherkashin improved this bound (see [2]). In this paper, we further improve it, when the number of vertices of an n-uniform hypergraph is bounded from above by the value n^m with some $m = m(n)$.

Keywords: hypergraph clique, chromatic number.