This presents two models of the impact event of a microsphere with a surface. The first model considers an elastic impact, where the particle kinetic energy is related to the adhesion and dissipative forces acting during the surface contact. The second model, considers the elastic-plastic adhesion impact of ductile materials, where plastic deformation affects the impact event. Both models are based upon the governing equations of motion of the microsphere while in contact with the surface.

Impact events can be categorized according the initial strain rate that the microsphere experiences. The categories include:

1) A low-speed region (up to \(\sim 10\) m/s). It is generally believed here that the wave processes and inertial forces do not affect the properties of the materials and, therefore, can be neglected. This assumption, however, may not be true for all cases. Dynamic properties of materials can vary as much as up to 100% from static material properties. This is coupled with developing dislocations and also with the rate of diffusive, chemical and physical properties, both inside the stressed body and on its boundary with surrounding media.

2) A high-speed region (up to \(\sim 100\) m/s). In this case, the inertial resistance of the material must be considered and the material deformation is highly non-uniform, and

3) A supersonic and hypersonic speed region. It is generally believed that at such speeds the theory of hydrodynamics can be used.

1. The Elastic Impact Model

Brach and Dunn (1995) proposed an impact model that was based on Hertzian contact model and adhesion loading. Consider a spherical particle impacting a flat surface. The particle radius is \(r\), incident velocity, \(v\), rebound velocity, \(V\). The coordinate system is taken to be Cartesian in the impact plane with axes \(t\) directed along the surface and \(n\) perpendicular to the surface. It is assumed that elastic and adhesion forces are dominant forces and that gravity is neglected. Thus, energy is dissipated through material and adhesion damping. The general form of the equation for a dynamic system is \(M\ddot{q} = -F(t)\). In fact, the system of an impacting particle and a surface is nonlinear, as proposed by Dunn et. al. (1995) the expression for \(F(t)\) can be written as: \(F(t) = F_H + F_A + F_{HD} + F_{AD} + F_t\) where \(F_H\) is a Hertzian force, \(F_A\) is an adhesive force, \(F_{HD}\) and \(F_{AD}\) are damping forces and \(F_t\) is a tangential force. The final expression for the coefficient of restitution can be written as \(e_r = \sqrt{1 - \psi_d (1 + C_r \psi_n) - \psi_H C_H}\), where

\[
\psi_d = 2 C_A f_0 \left(\frac{4}{3 \pi}\right) \left(\frac{5}{4K}\right)^{3/5} \left(\frac{r}{m^*}\right)^{1/5} \psi_n^{4/5}, \quad \psi_H = 2 \left(\frac{2}{3}\right)^{5/2} \frac{a_m}{m^*} K v^{-1}. \]

Here \(C_R\) is a roughness coefficient, \(C_A\) and \(C_H\) are damping coefficients, \(f_0\) is an adhesion force per unit length, \(K\) is a Stiffness, \(a_m\) is a contact radius, \(r\) is a radius of the microparticle, \(m\) is a mass of the micropar-
particle particle, $v_n$ is a normal component of an incident velocity. A comparison of the given model with Wall’s (1990) experimental results is shown in Figure 1.

2. The Elastic-Plastic Impact Model

In the case of ductile-material impacts, plastic effects could play essential role even at rather relatively low velocities of deformation [Gorham, Kharaz (2000)]. The dependence of the local displacement $\delta(F)$ versus contact force can be taken as Biryukov, Kadomtsev (2002):

$$\delta = \begin{cases} 
    bF^{2/3}, & F_{\text{max}} < F_1, \ \text{d}F/\text{d}t > 0 \\
    b_f R^{2/3} + \delta_p (F_{\text{max}}), & F_{\text{max}} < F_1, \ \text{d}F/\text{d}t < 0, \\
    (1 + \beta) c_1 \sqrt{F} + (1 - \beta) F_d, & \text{d}F/\text{d}t > 0, \ F_{\text{max}} > F_1 
\end{cases},$$

where $b = R^{2/3} (3/(4E))^{2/3}$, $E = E_1 E_2 (1 - v_1^2) + E_2 (1 - v_2^2)^{-1} - F_1 = \eta (3R/4E)^{1/3}$, $\eta = \pi k \lambda$.

Here $\lambda = 5.7$ and $k$ is the smallest of two plastic constants of the particle and of the surface. $b_f = R_f^{2/3} (3/(4E))^{2/3}$, $R_f = (4/3)E^{1/3}(\eta)^{-1/2}$, $\delta_{\text{max}} = (1 - \beta) F_{\text{max}} (2\eta R_f)^{-1}$, $\eta = R_f^{1/3} - R_f$, $\beta = 0.33$, $c_1 = 3\eta^{1/2} (8E)^{-1}$, and $d = (2\eta R_f)^{-1}$. Solving the dynamic equation for the particle and taking into account for the adhesion effects the expression for the normal coefficient of restitution can be obtained: $e = \sqrt{e_{A}^2 + e_{p}^2} - 1$, where $e_{A} = \sqrt{1 - 2W_A/(m v^2)}$ and $e_{p} = (2/v_n)(1/(5m)) R_f^{2/4} (3/(4E))^{1/4} F_{\text{max}}^{2/4}$, where $F_{\text{max}}$ is the maximum contact force at the approach phase. The results of the given model and comparison with other theories and experimental data are shown in Figure 2.

![Figure 1](image1.png)

**Fig. 1.** Coefficient of restitution versus normal velocity. Silicon surface, ammonium fluorescent particle. Circles — experimental data, Wall (1990); Curves — present model.
Fig. 2. Coefficient of restitution versus normal velocity. Aluminum surface, aluminum oxide particle with the diameter of 5 mm. Circles are measurements for normal impact and squares are for oblique impact experiments. The dashed line is Tabor's (1990) model the dotted-dashed is Thornton's (1998) model and the solid line is the result of the present model.

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References