Quantitative Description of Correlations Accompanying Mixing-Induced Quantum Non-Markovianity

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Brief outlook

We study the model of quantum non-Markovianity under mixing of Markovian processes developed by H.-P. Breuer, G. Amato, and B. Vacchini in their paper [1], applying a depolarizing map with a parameter p and constructing a new non-Markovianity measure.

Introduction

The criterion of non-Markovianity based on the increase of trace distance $D(\rho_S^1, \rho_S^2) = \frac{1}{2}||\rho_S^1 - \rho_S^2||_1$ is useful from the experimental point of view, giving an opportunity to reveal non-Markovian effects by means of an optimal strategy [2]. A corresponding non-Markovianity measure is defined as follows

$$N(\Phi) = \max_{\rho_s^{1,2}} \int_{\sigma>0} dt\, \sigma(t), \qquad (1)$$

where $\sigma(t) \equiv \frac{d}{dt} D(\Phi_t \rho_S^1, \Phi_t \rho_S^2)$. However, in some cases this criterion indicates growing distinguishability of states under mixing of two Markovian processes [3, 4, 5, 6], i.e. a process

$$\Phi_t = q_1 \Phi_t^{(1)} + q_2 \Phi_t^{(2)}, \quad q_1 + q_2 = 1,\ q_1 \geq 0,\ q_2 \geq 0 \qquad (2)$$

is non-Markovian. H.-P. Breuer et al. created a model explaining this phenomenon [1]. They consider the system S coupled to a qubit ancilla A serving as a classical degree of freedom and determining the choice of either $\Phi^{(1)}$, or $\Phi^{(2)}$. The reduced state of A is fixed and written as

$$\rho_A = q_1 \Pi_1 + q_2 \Pi_2, \qquad (3)$$

where Π_i, $i = 1, 2$, are orthogonal rank-one projections corresponding to some ancilla basis $|i\rangle$ (the projection Π_i is connected with the environment E_i conditioning one of the Markovian processes).

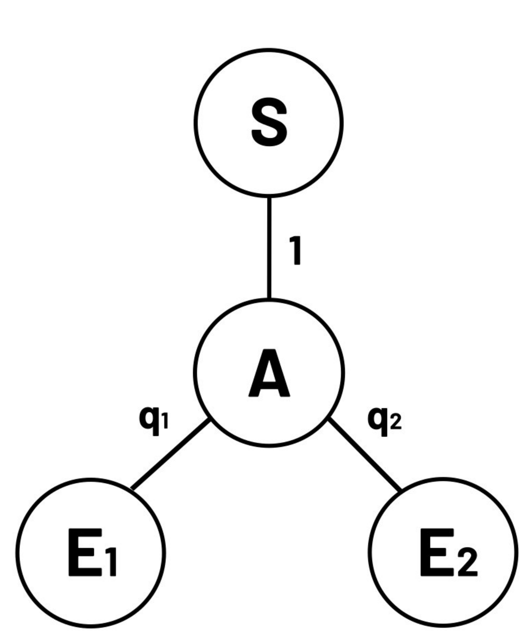

Figure 1: Graphical representation of Breuer-Amato-Vacchini model

Non-Markovianity Under Mixing: Example

The authors of the paper [1] suggest a mixture of two qubit dephasing processes as an example. For instance, dephasing describes spin-spin relaxation; its rate is set by the time $T2$. Both dynamical maps are accompanied by unitary rotations over spin z-axis.

Thus, we obtain a density operator ρ_S s.t.

$$\langle 0|\, \rho_S(t)\, |0\rangle = \langle 0|\rho_S(0)|0\rangle,$$
$$\langle 1|\, \rho_S(t)\, |1\rangle = \langle 1|\rho_S(0)|1\rangle,$$
$$\langle 1|\, \rho_S(t)\, |0\rangle = \kappa(t) \langle 1|\, \rho_S(0)\, |0\rangle,$$

where coherence $\kappa(t) = q_1\mu_1(t) + q_2\mu_2(t)$, $\mu_k(t) = \exp\{-(\gamma_k + i\lambda_k)t\}$, $k = 1, 2$.

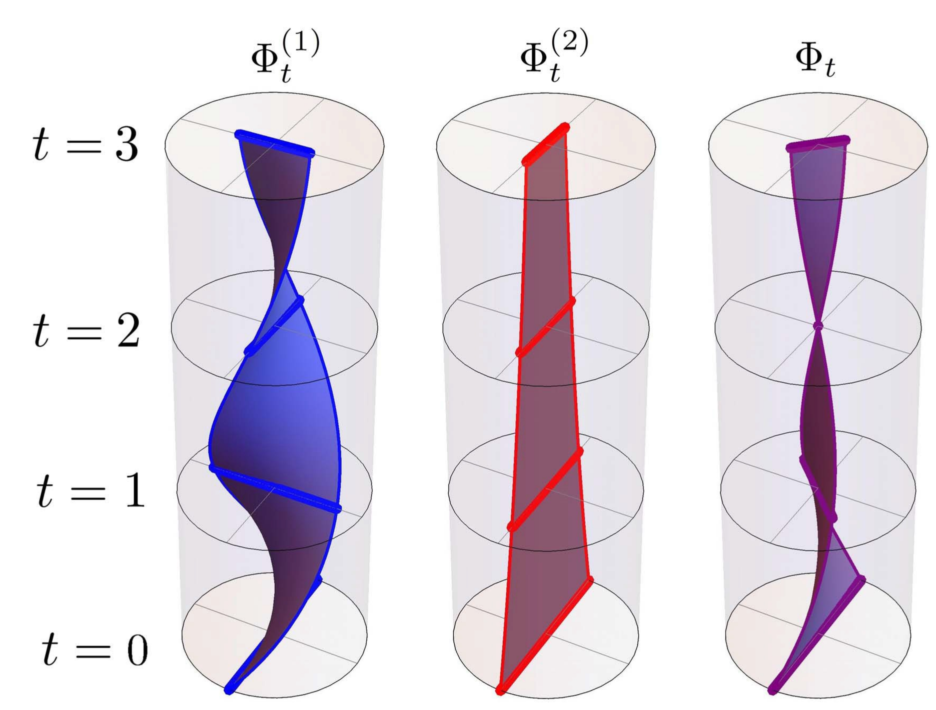

Figure 2: Distinguishabilities of states under $\Phi_t^{(1)}$ (left), $\Phi_t^{(2)}$ (middle), and their mixture Φ_t (right) (taken from [1])

Main Result

We obtain a new non-Markovianity measure based on the information backflow. The measure allows to find upper and lower bounds of the backflow, thereby simplifying the evaluation of that quantity.

Depolarization and Non-Markovianity

We apply a depolarizing map $D_{\tilde{p}}$, $\tilde{p} = 1 - p$, to the process

$$\Lambda(t)[\rho_S] = q_1 \Phi_t^{(1)}[\rho_s] \otimes \Pi_1 + q_2 \Phi_t^{(2)}[\rho_s] \otimes \Pi_2, \qquad (4)$$

which is Markovian and obtained from Stinespring dilation of the dynamical map Φ_t after tracing over two environments (so we have a state in the Hilbert space $S \otimes A$). Adding the second ancilla, A', with projections $\tilde{\Pi}_1$, $\tilde{\Pi}_2$, to describe depolarization as a dynamical map, we derive an expression

$$\Omega_t[\rho_{S+A}] := (D_{\tilde{p}} \circ \Lambda_t)[\rho_{S+A}] = p\Lambda_t[\rho_{S+A}] \otimes \tilde{\Pi}_1 + \frac{1-p}{2} \Phi_t[\rho_S] \otimes I \otimes \tilde{\Pi}_2. \qquad (5)$$

Considering the distinguishability of states in the case of dephasing processes introduced above ($q_1 = q_2$, $\gamma := \gamma_1 = \gamma_2$), we have two limiting cases: $p = 0$ (non-Markovian dynamics) and $p = 1$ (Markovian dynamics). Then, we find the value $p_* = p$ s.t. one type of the evolution replaces the other as p approaches p_*. In our case $p_* = \frac{1}{\xi+1}$, $\xi = \frac{2\gamma}{\Delta\lambda}$. The plot of non-Markovianity measure defined in Eq. (1) as the function of p and p_* is below on the right.

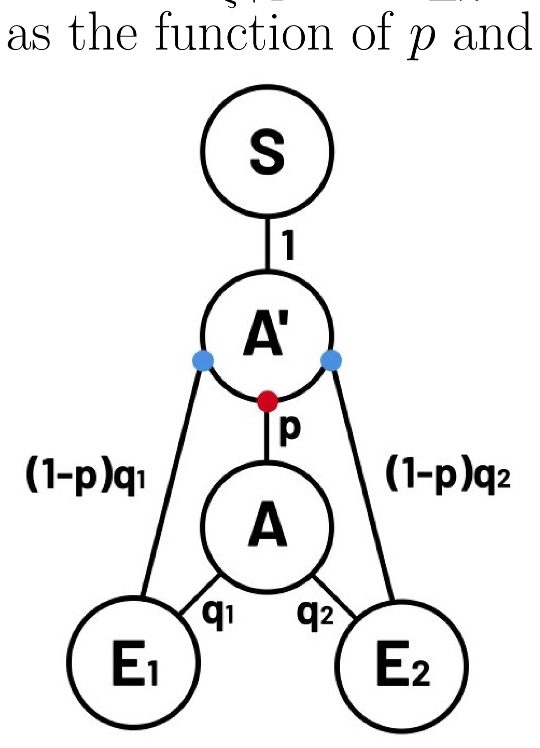

Figure 3: Graphical representation of depolarized Breuer-Amato-Vacchini model (red circle corresponds to $\tilde{\Pi}_1$, blue circles correspond to $\tilde{\Pi}_2$)

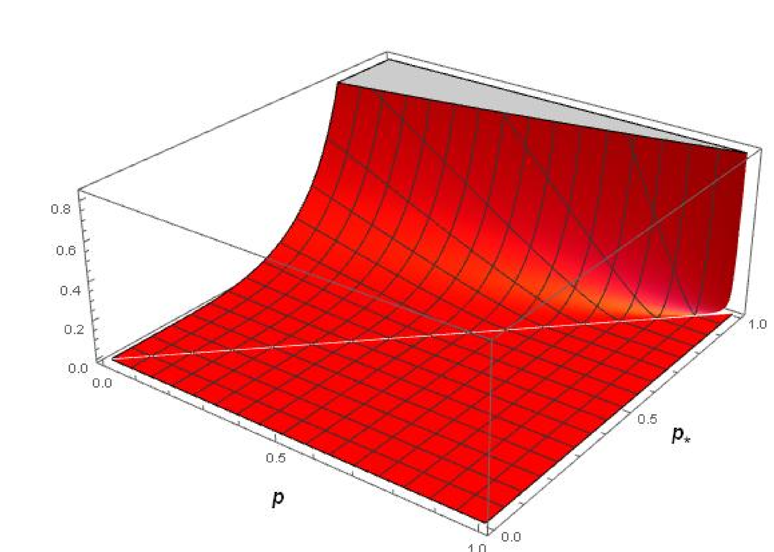

Figure 4: $N(\Omega)$ as the function of p, p_*

Conclusion

As we see in Fig. 4, $N(\Phi)$ is a monotonic function of p_*. That means this value serves as a suitable non-Markovianity measure. Moreover, calculation of p_* is based on the search of local minima of distinguishability D. This procedure is easier (at least, in our example) than integration and maximization in Eq. (1). Nevertheless, calculation of $N(\Omega)$ as a function of p and p_* gives various lower and upper bounds of $N(\Phi)$. We obtain a general upper bound $N(\Phi) \leq \frac{p_*}{1-p_*}$. A lower bound depends on the parameters of the problem.

References

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