



Квантовая метрология на трансмоне в режиме кутрита

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Долгопрудный

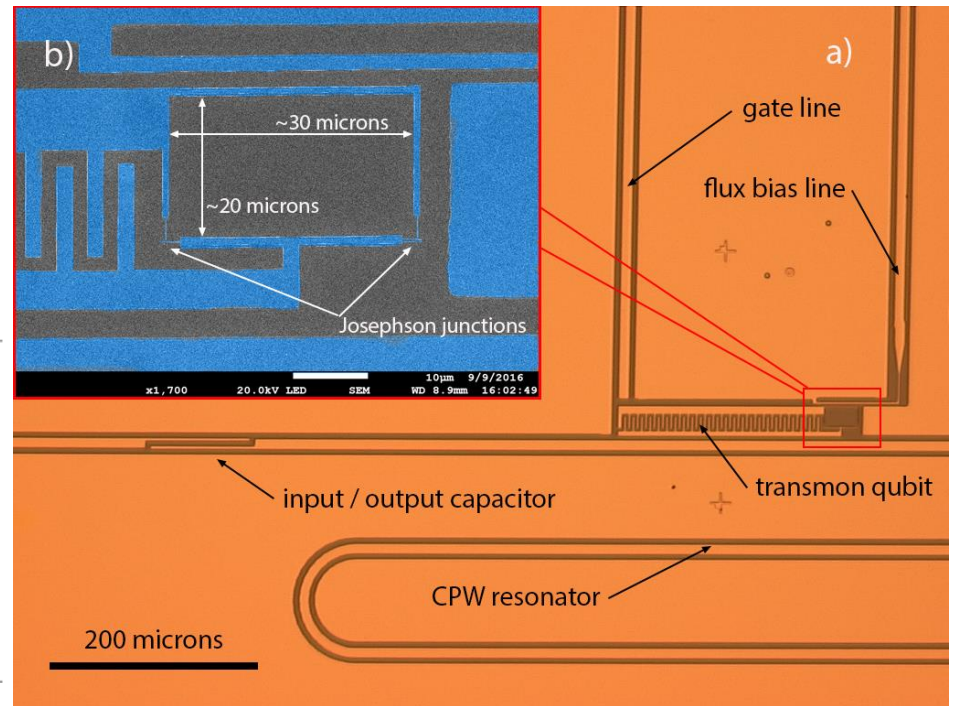
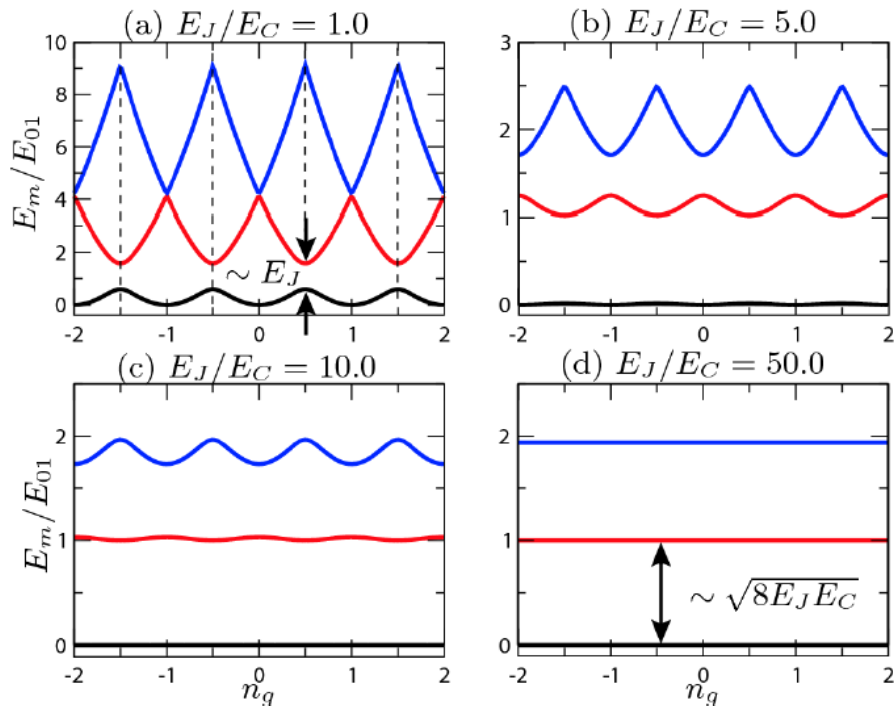
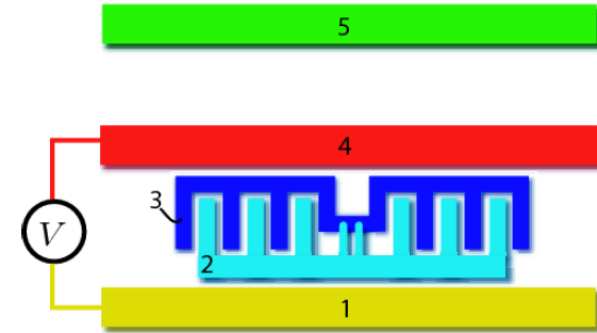
Классические измерения

$$\sigma_P \propto \frac{1}{\sqrt{N}} \propto \frac{1}{\sqrt{t}}$$

Гейзенберговский предел

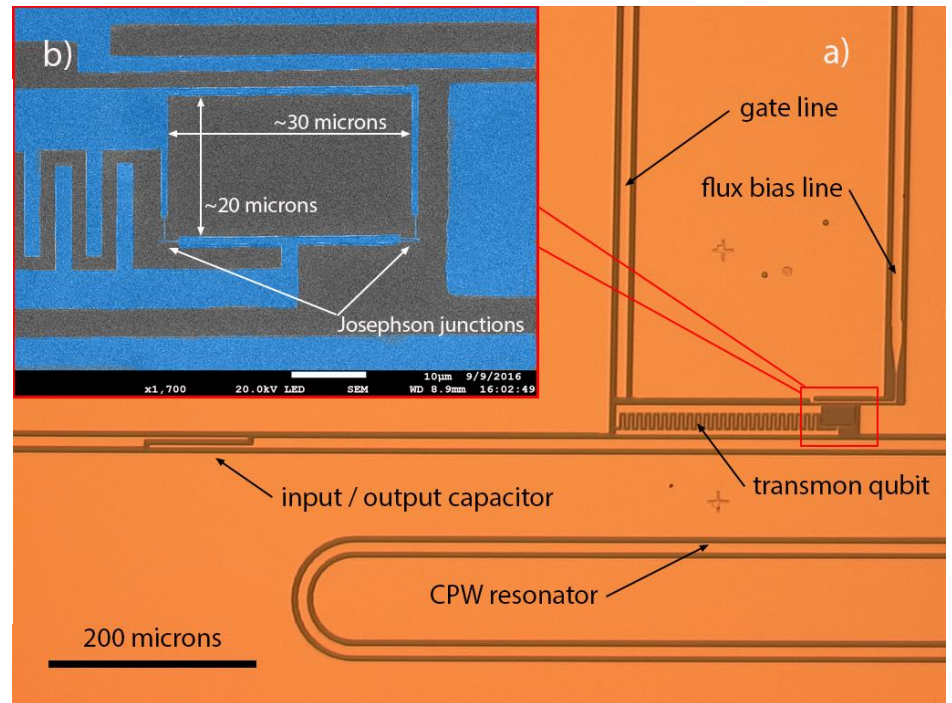
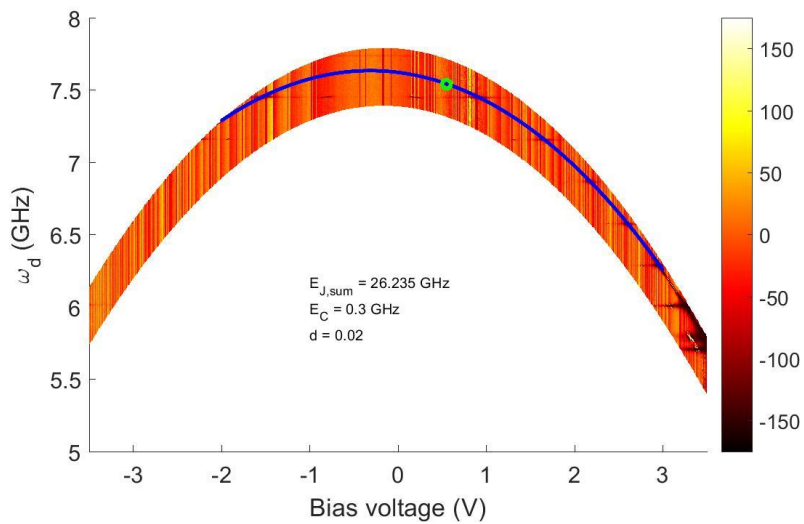
$$\sigma_P \propto \frac{1}{N} \propto \frac{1}{t}$$

$$H = 4E_C \left(n - \frac{C_g V_g}{2e} \right)^2 |n\rangle\langle n| - \frac{E_J}{2} (|n+1\rangle\langle n| + |n\rangle\langle n+1|)$$



$$\hbar\omega_{01} = \sqrt{8E_J E_C} \sqrt{\cos^2\left(\pi \frac{\Phi(V)}{\Phi_0}\right) + d^2 \sin^2\left(\pi \frac{\Phi(V)}{\Phi_0}\right)} - E_C, \quad d = \frac{|E_{J1} - E_{J2}|}{E_{J1} + E_{J2}}$$

$$\delta\Phi \rightarrow \delta H \propto \begin{pmatrix} 0 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & 2\omega \end{pmatrix}$$



$$H_{JC} = \sum_{j=1}^N \hbar \tilde{\omega}_j |j\rangle\langle j| + \sum_{j=0}^{N-1} \hbar \Omega_{j,j+1} (|j+1\rangle\langle j| e^{-i\omega_{j,j+1}^{(\Omega)} t} + h.c.)$$

Переход в представление вращающейся волны

$$U = \sum_{j=1}^N \exp \left[i \sum_{k=0}^{j-1} \omega_{k,k+1}^{(\Omega)} t \right] |j\rangle\langle j|, \quad H \rightarrow U H U^\dagger + i\hbar \frac{dU}{dt} U^\dagger$$

$$H_{JC} = \sum_{j=1}^N \hbar \left(\tilde{\omega}_j - \tilde{\omega}_0 - \sum_{k=0}^{j-1} \omega_{k,k+1}^{(\Omega)} \right) |j\rangle\langle j| + \sum_{j=0}^{N-1} \frac{\hbar \Omega_{j,j+1}}{2} (|j+1\rangle\langle j| + h.c.)$$

Ограничившись первыми $3m$ уровнями

$$H = \begin{pmatrix} 0 & \Omega_{01}(t) & 0 \\ \Omega_{01}(t) & 2\delta_{01} & \Omega_{12}(t) \\ 0 & \Omega_{12}(t) & 2(\delta_{01} + \delta_{12}) \end{pmatrix}$$

$\delta_{j,j+1} = \tilde{\omega}_{j+1} - \tilde{\omega}_j - \omega_{j,j+1}^{(\Omega)}$ - отстройка

$$|\Psi_i\rangle = |0\rangle$$

$$\hat{U}_{prep} = \exp \left[-i \begin{pmatrix} 0 & \Delta & 0 \\ \Delta & \epsilon & \Delta \\ 0 & \Delta & 0 \end{pmatrix} \right]$$



$$\frac{1}{\sqrt{3}} \left(e^{i\frac{\pi}{6}} |0\rangle + ie^{-i\epsilon} |1\rangle + e^{i\frac{5\pi}{6}} |2\rangle \right)$$

$$\hat{U}_{count}(\omega) = \exp \left[i \begin{pmatrix} 0 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & 2\omega \end{pmatrix} t \right]$$



$$\frac{1}{\sqrt{3}} \left(e^{i\frac{\pi}{6}} |0\rangle + ie^{-i(\epsilon-\omega t)} |1\rangle + e^{i\frac{5\pi}{6}} e^{2i\omega t} |2\rangle \right)$$

$$\hat{U}_{meas} = \exp \left[i \begin{pmatrix} 0 & \Delta & 0 \\ \Delta & \epsilon & \Delta \\ 0 & \Delta & 0 \end{pmatrix} \right]$$



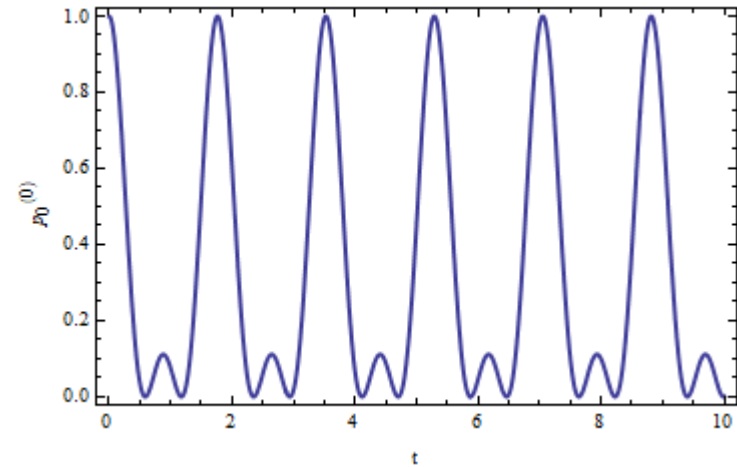
$$\begin{aligned} \epsilon &= 0.85246, \\ \Delta &= -1.2953 \end{aligned}$$

$$|\Psi_f\rangle = \frac{1}{3} \begin{pmatrix} 1 + e^{i\omega t} + e^{2i\omega t} \\ -ie^{i\epsilon} e^{i\frac{\pi}{6}} (1 + e^{i\frac{4\pi}{3}} e^{i\omega t} + e^{i\frac{2\pi}{3}} e^{2i\omega t}) \\ e^{-i\frac{2\pi}{3}} (1 + e^{i\frac{2\pi}{3}} e^{i\omega t} + e^{i\frac{4\pi}{3}} e^{2i\omega t}) \end{pmatrix}$$

- $H = h_0 \left(\frac{t_0}{3^0} + \frac{t_1}{3^1} + \dots + \frac{t_{K-1}}{3^{K-1}} \right)$
- $p_i^{(0)} = \frac{1}{3} \left[1 + 2 \cos \left(\omega(H)t - \frac{2\pi}{3} i \right) \right]^2, \quad i = 0, 1, 2$

Байесовская схема

- $P^{(i)}(\omega(H)|i) = \frac{P_i(\omega(H))P^{(i-1)}(\omega(H))}{P(i)}$
- $\omega \in \{\omega_m\}_{m=1}^M$
- $P^{(0)}(\omega_m) = \frac{1}{M}$
- $P(\omega(H) - \tilde{\omega}|\vec{t}) = \frac{1}{2\pi} \frac{\sin^2[3^K(\omega(H) - \tilde{\omega})t_K/2]}{3^K \sin[(\omega(H) - \tilde{\omega})t_K/2]}$



$$|\Psi_i\rangle = |0\rangle$$

$$\hat{U}_{prep} = \exp \left[-i \begin{pmatrix} 0 & \Delta & 0 \\ \Delta & \epsilon & \Delta \\ 0 & \Delta & 0 \end{pmatrix} \right]$$



$$|\Psi_{prep}\rangle = \frac{1}{\sqrt{3}} \left(e^{i\frac{\pi}{6}} |0\rangle + i e^{-i\epsilon} |1\rangle + e^{i\frac{5\pi}{6}} |2\rangle \right)$$

$$\hat{U}_{count}(\omega) = \exp \left[i \begin{pmatrix} 0 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & 2\omega \end{pmatrix} t \right]$$



ρ

$$\hat{U}_{meas} = \exp \left[i \begin{pmatrix} 0 & \Delta & 0 \\ \Delta & \epsilon & \Delta \\ 0 & \Delta & 0 \end{pmatrix} \right]$$



ρ_f

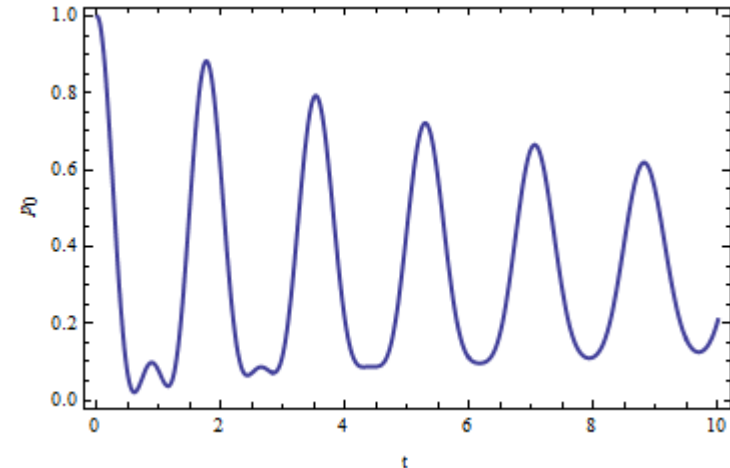
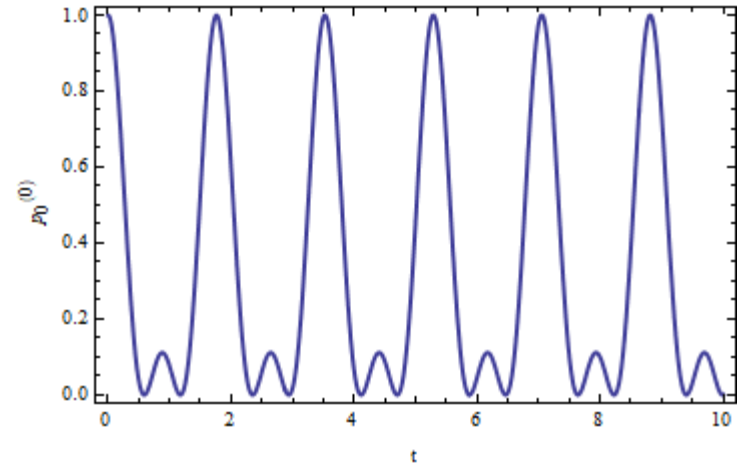
$$\begin{aligned} \epsilon &= 0.85246, \\ \Delta &= -1.2953 \end{aligned}$$

$$\frac{d\rho}{dt} = -i[\rho, \hat{H}_{sys}] + \Gamma_{01}D[\sigma_{01}]\rho + \Gamma_{12}D[\sigma_{12}]\rho,$$

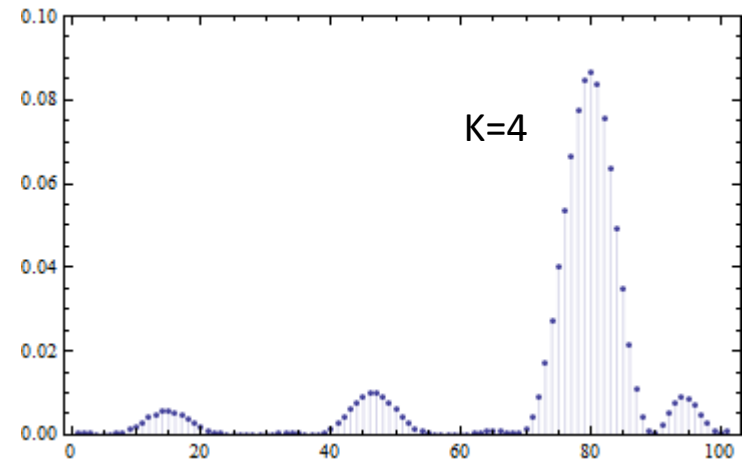
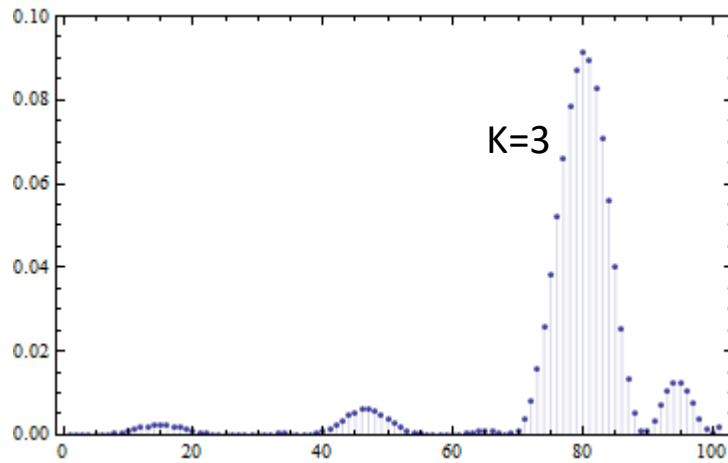
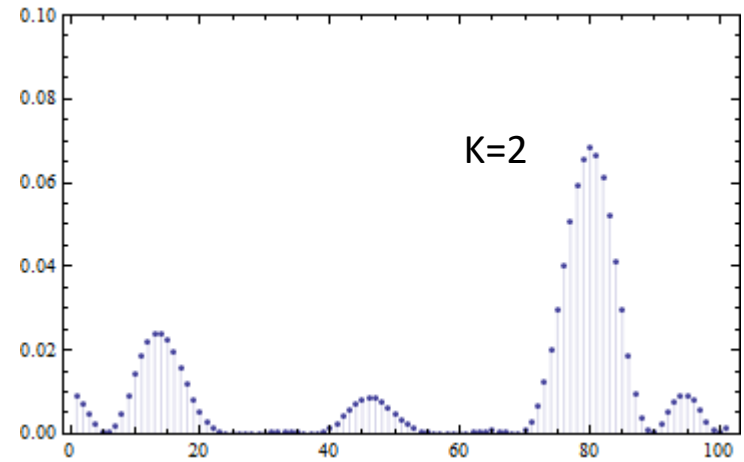
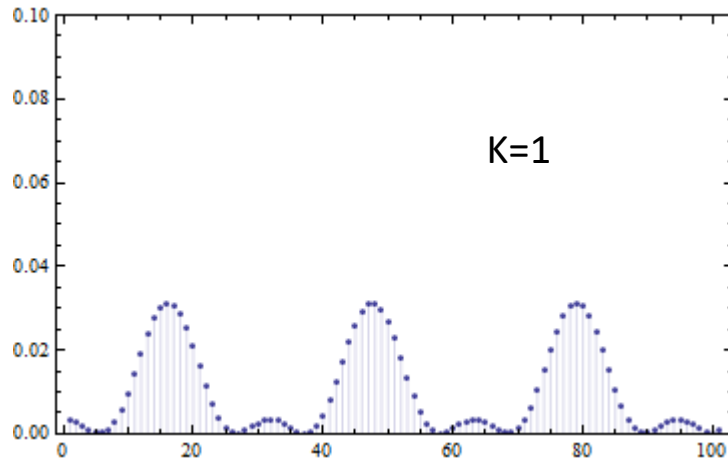
$$\langle \delta H(t) \rangle = 0, \quad \langle \delta H(t)\delta H(t') \rangle = \left(\frac{d\omega}{dH} \right)^{-2} \Gamma_{\varphi} \delta(t - t')$$

$$p_i^{(0)} = \frac{1}{3} \left[1 + 2 \cos \left(\omega(H)t - \frac{2\pi}{3}i \right) \right]^2, \\ i = 0, 1, 2$$

$$p_i \\ = \frac{1}{3} + \frac{2}{9} \cos \left(\omega(H)t - \frac{2\pi}{3}i \right) e^{-\frac{(\Gamma_{01} + \Gamma_{\varphi})}{2}t} \\ + \frac{2}{9} \cos \left(\omega(H)t - \frac{2\pi}{3}i \right) e^{-\frac{(\Gamma_{01} + \Gamma_{12} + \Gamma_{\varphi})}{2}t} \\ + \frac{2}{9} \cos \left(2\omega(H)t - \frac{4\pi}{3}i \right) e^{-\frac{(\Gamma_{12} + 4\Gamma_{\varphi})}{2}t}, \\ i = 0, 1, 2$$

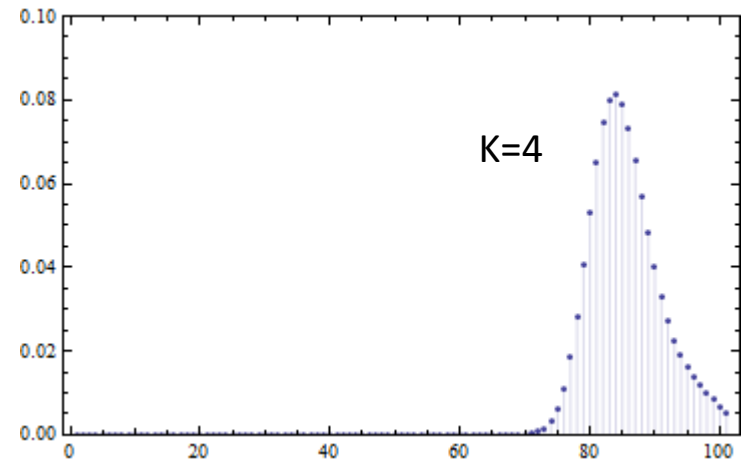
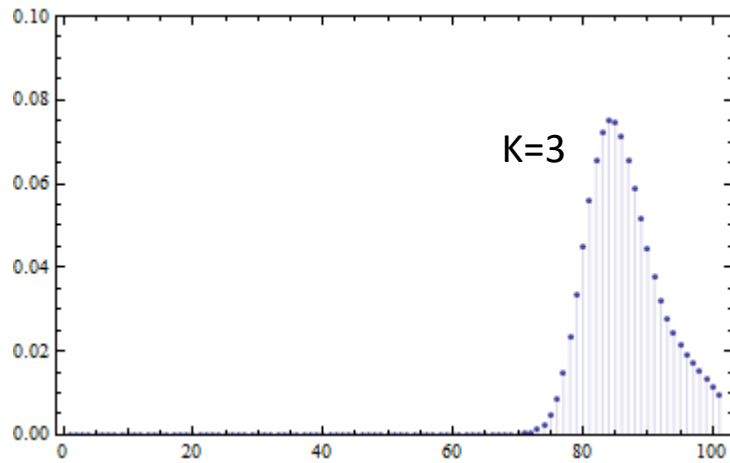
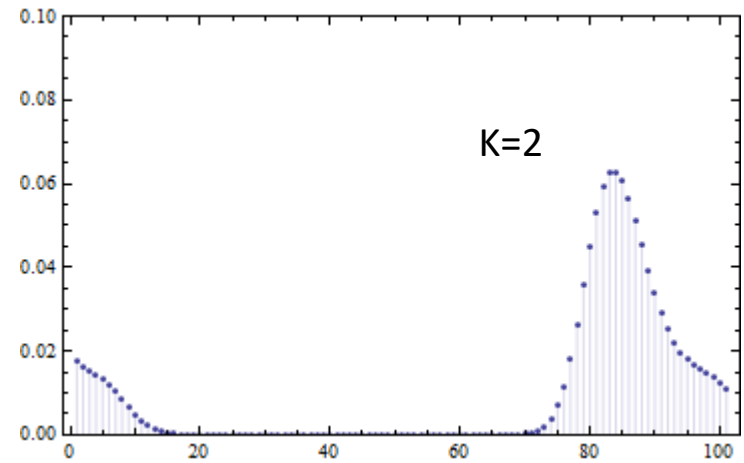
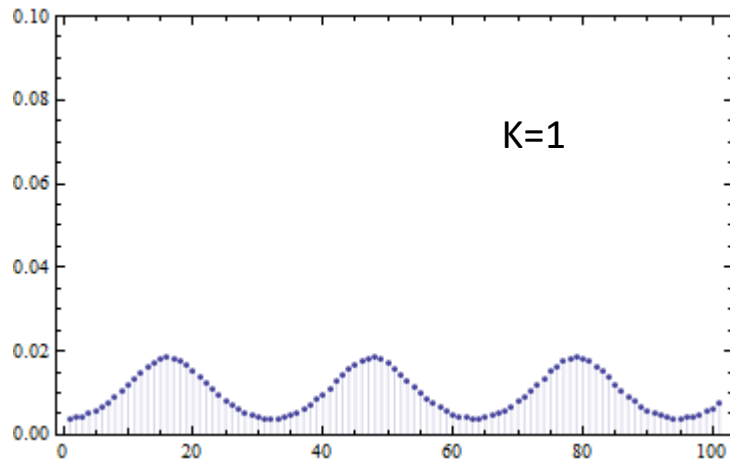


Случай слабой декогеренции: $t \ll T_2$



$$P(\omega(H) - \tilde{\omega}|\vec{t}) = \frac{1}{2\pi} \frac{\sin^2[3^K (\omega(H) - \tilde{\omega})t_K/2]}{3^K \sin[(\omega(H) - \tilde{\omega})t_K/2]}$$

Случай сильной декогеренции: $t \approx T_2$



$N=10$

- $H(P^{(i)}(\omega)) = -\sum_{m=0}^M P^{(i)}(\omega_m) \ln P^{(i)}(\omega_m)$

- $H(P^{(i)}(\omega)) \rightarrow \min \Rightarrow t_1^*$

