EXAMINATION WORK

Discipline:

Mathematical Analysis - Sequences and Series of Functions, Functions of Several Variables

Course: 2 **Semester:** 3 **2021–2022**

Student's name ______________________ **Group** ________

Total Score		Grade	
Checked by		Examiner	

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1. ④ Let $X \subset Y \subset \mathbb{R}^n$. Prove or disprove the following: every limit point of X is a limit point of Y.

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2. ⑦ Prove that the function $f(x, y)$ is differentiable at $(0, 0)$ if

$$f(x,y) = \begin{cases} \dfrac{y^2(e^{2x}-1)}{x} & , \quad x \neq 0 \\ y^3 + |y|^{5/3} & , \quad x = 0. \end{cases}$$

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3. ⑦ Evaluate the first and second differentials of

$$f(x,y) = x^2 + \sin\left(1 - \frac{1}{\sqrt{x+y}}\right)$$

at the point $M(1, 0)$, and represent $f(x, y)$ by the Taylor formula in a neighbourhood of M up to $o\left((x-1)^2 + y^2\right)$.

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4. ⑥ Find the volume (Jordan measure) of the solid of revolution generated when the subgraph of $y = \arcsin x$, $x \in [0, 1]$, is rotated around the Ox axis.

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5. ⑥ Expand the function $f(x) = \ln\left(x + \sqrt{3 + x^2}\right)$ into the Maclaurin series and find its radius of convergence.

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6. ④ Find the gradient of the function $z(x, y)$ at the point $(\pi/4, \pi/4, \pi/2)$ if the equation

$$(z - y)\tan(z - y) = x$$

defines z as a function of two independent variables x and y.

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7. ⑥ Find local extrema of the function $f(x, y) = 6x^2 - 2x^3 + 3y^2 + 6xy$.

«Students are prohibited to use any sources of information during the exam.»

Signature: ______________________ (Student's name)

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1. ④ Let $X \subset Y \subset \mathbb{R}^n$. Prove or disprove the following: every boundary point of X is a boundary point of Y.

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2. ⑦ Prove that the function $f(x, y)$ is differentiable at $(0, 0)$ if

$$f(x,y) = \begin{cases} \dfrac{x^2 \tan(3y)}{y} &, \quad y \neq 0 \\ x^2 + |x|^{3/2} &, \quad y = 0. \end{cases}$$

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3. ⑦ Evaluate the first and second differentials of

$$f(x, y) = y^2 + \arcsin\left(1 - \sqrt{x - y}\right)$$

at the point $M(0, -1)$, and represent $f(x, y)$ by the Taylor formula in a neighbourhood of M up to $o\left(x^2 + (y+1)^2\right)$.

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4. ⑥ Find the length of the curve $y = 1 - \ln x$, $\sqrt{3} \leq x \leq \sqrt{8}$.

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5. ⑥ Expand the function $f(x) = \arccos \dfrac{x}{\sqrt{4 + x^2}}$ into the Maclaurin series and find its radius of convergence.

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6. ④ Find the gradient of the function $z(x, y)$ at the point $(1, 1, 1)$, if the equation

$$y = z + z \ln(z/x)$$

defines z as a function of two independent variables x and y.

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7. ⑥ Find local extrema of the function $f(x, y) = 3y^2 - 2y^3 - 3x^2 + 6xy$.

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