

# Program for the Exam on Linear Algebra

Fall Semester 2020–2021

Olga Orel

1. Systems of linear equations. Elementary Row Operations. Gaussian Algorithm.
2. Inverse Matrix. Row Operations by Matrix Multiplication. Matrix Inverse Using Row Operations.
3. Determinant as a Sum of Elementary Signed Products.
4. Properties of the Determinant.
5. Determinant by Cofactor Expansion. Cramer's Rule.
6. Determinant of a Product. Matrix Inverse Using Cofactors.
7. Vandermonde Determinant.
8. Real Vector Spaces. Vector Space Axioms. Subspaces. Definitions and Examples.
9. Linear Dependence and Basis. Fundamental Theorem. Steinitz Exchange Lemma. Invariance Theorem.
10. Rank of a Matrix. Equality of Dimensions of Column Space and Row Space.
11. Change of Basis, Transition Matrix. Null Space of a Matrix. Kronecker–Capelli Consistency Theorem.
12. Dimension Theorem for Matrices. Fredholm Alternative Theorem.
13. Theorem Concerned with Bases and Dimensions of Vector Spaces and Their Subspaces.
14. Sum and Intersection of Subspaces. Grassman Formula for Vector Space Dimensions. Direct Sum of Subspaces.
15. Linear Transformations. Examples and Elementary Properties. Matrix Transformations.
16. Kernel and Image of a Linear Transformation. Definition and Properties of One-to-One and Onto Transformations.
17. The Dimension Theorem. Isomorphisms.
18. Change of Basis and Linear Transformations. Similarity of Matrices. Eigenvalues and Eigenvectors.
19. Similarity Invariants. Diagonalization of Matrices.
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21. Theorems about Diagonalizable Matrices.
22. Symmetric Matrices. Dot Product, Length, and Distance. Cauchy–Bunyakovsky-Schwarz Inequality. Triangle Inequality.
23. Orthogonal Sets and the Expansion Theorem. Pythagoras' Theorem. Orthogonal Lemma. Gram-Schmidt Orthogonalization Algorithm.