

# Program for the Exam on Introduction to Calculus

Fall Semester 2020–2021

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1. The equivalence of two forms of completeness axiom (the existence of the least upper bound).
2. The principle of nested intervals.
3. The set of rational numbers is countable, the set of points of an interval is uncountable.
4. Convergent sequence is bounded.
5. Uniqueness of a limit of a sequence.
6. Comparison theorem, two policemen theorem (for sequences).
7. Limit theorems related to arithmetic operations (for sequences).
8. Monotone convergence theorem.
9. The number  $e$ .
10. The Bolzano-Weierstrass theorem.
11. Theorems about Cauchy sequences.
12. Theorems about limit superior and limit inferior.
13. Sequential characterization of limits (the equivalence of Cauchy and Heine definitions).
14. Cauchy's criterion (for functions).
15. The existence of one-sided limits of monotone functions.
16. Extreme value theorem (Weierstrass theorem on a compact).
17. Intermediate value theorem (Bolzano-Cauchy theorem).
18. Continuity of the inverse function.
19. Necessary and sufficient conditions for uniform continuity.
20. Uniform continuity on compacts.
21. Differentiation and algebraic operations.
22. The chain rule.
23. The derivative of an inverse function.
24. The Leibnitz formula.
25. Fermat's rule.
26. Rolle's theorem.
27. Lagrange's mean value theorem.
28. Cauchy's mean value theorem.
29. L'Hospital's rule (case  $\left(\frac{0}{0}\right)$ ).
30. L'Hospital's rule (case  $\left(\frac{\infty}{\infty}\right)$ ).