Program for the Exam on Geometry
Fall Semester 2020–2021

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1. Axioms of elementary Geometry.

2. Convex polygon. Types of triangles: isosceles, equilateral, and right. Prove congruence tests for triangles.

3. Types of angles: vertical, adjacent, complementary, supplementary). Interior and exterior angles for a triangle. Prove that an exterior angle of a triangle is greater than either of the nonadjacent interior angles. Prove that, in any triangle, the greater angle lies opposite the greater side.

4. Prove the triangle inequality. Prove that a line segment is shorter than any broken line with the same endpoints. Prove that perimeter of a convex broken line is less than that of any broken line with the same endpoints which surrounds it.

5. Perpendicular and oblique line segments. Distance from the point to the line. Prove that the locus of points equidistant from two given points is the perpendicular bisector of the segment joining these two points.

6. Congruence tests for right triangles. Prove the main property of the bisector of an angle.

7. Prove that in a triangle, the median are concurrent, the altitudes are concurrent, and the angle bisectors are concurrent.

8. Formulate Fundamental Theorem on cutting transversals by parallel lines (Thales' theorem) and prove it for the case of equal segments. Prove theorem on division of a side of a triangle by the bisector of the opposite angle.

9. Definition of similar triangles. Prove similarity tests for triangles.

10. Pythagorean theorem, sine and cosine theorems. Prove Pythagorean theorem and some other geometric relations in a right triangle.

11. Prove theorems on areas of triangles, parallelograms, trapezoids. Formulas for finding areas of a triangle.

12. Definition of a vector. Equal vectors, zero vector. Definition and properties of addition and scalar multiplication. Introduction of coordinates, position vectors, components of vectors.

13. Represent the position vector of a point lying on a segment in terms of position vectors of its endpoints. Represent the position vector of the center of mass of a triangle in terms of position vectors of vertices. Prove that the medians of a triangle are concurrent using vectors.

14. Lines in space. Direction vector. Vector equation and parametric equations of a line. Method of determining whether two lines intersect or not.

15. Systems of linear equations. Three possibilities for a number of solutions of a linear system. Illustrate these options using systems of two equations in two variables.Consistent and inconsistent systems.

16. Equivalent linear systems. List of elementary operations on a system. Prove that elementary operations on a linear system produce the equivalent system.

17. Solving linear systems using matrices. Augmented matrix, coefficient matrix, and constant matrix. Elementary operations on rows the augmented matrix. Gaussian elimination method (unknown elimination method).

18. Determinants of matrices of order 2 and 3. Cofactor expansion of determinants along rows and columns. Solving linear systems by Cramer's rule.

19. Matrix Operations: matrix addition and scalar multiplication. Properties of these operations. Definition of a vector space. Set of matrices as a vector space.

20. Multiplication of matrices.

21. Definition of a vector. Equal vectors, zero vector. Definition and properties of addition and scalar multiplication. Axioms of vector space. Abstract and arithmetic vector spaces.

22. Linear combinations of vectors. Linearly dependent and independent set of vectors. The concept of a basis. Coordinates of vectors relative to a basis.

23. Norm of a vector, dot product of vectors. Distance between two points in n-space. Angles between vectors. Cauchy–Bunyakovsky–Schwarz Inequality. Orthogonal (perpendicular) vectors.

24. Cross product of vectors. Properties of cross product.

25. Relationships involving cross product and dot product.

26. Geometric interpretation of cross product. Areas of parallelogram and triangle which vertices are given.

27. Projection theorem. Triple product of vectors. Properties of triple product.

28. Vector and parametric equations of lines. Lines through two points. Symmetric equations of a line.

29. Vector and parametric equations of planes. General equations of lines and planes. Normal vectors. Planes through tree points. Plane throw point and parallel two given non coplanar vector.

30. Relative position of lines and planes. Angles between lines and planes.

31. Distance problems. Distance between a point and a plane. Distance between a point and a line. Distance between skew lines.

32. Changing coordinate of vector when changing the basis.

33. Changing coordinate of point when changing the reference system.