

Program of the Course of Analytic Geometry

Vladimir Stukopin

1. Change-of-basis problem in vector space. Coordinate system for affine space. Coordinates of point. Change-of –coordinate system problem.
2. Algebraic Curves and Surfaces. Conic sections. Definitions of parabola, hyperbola, and ellipse. Their basic characteristic properties. Foci, directrices, and eccentricity.
3. Quadratic surfaces. Sphere, cylinder, and cone.
4. Quadratic surfaces. Ellipsoid, simple and double hyperboloids. Elliptic and hyperbolic paraboloids.
5. Systems of linear equations. Three possibilities for a number of solutions of a linear system. Equivalent linear systems. List of elementary operations on a system. Prove that elementary operations on a linear system produce the equivalent system. Gaussian elimination of unknowns.
6. Matrix and augmented matrix of a linear system. The rank of the matrix. Kronecker-Capelli theorem.
7. Matrices. Addition and scalar multiplication for matrices. Multiplication of matrices. Matrix transpose. Algebraic properties of matrices. Inverse of a matrix. Invertible (nonsingular) matrices. Solution of a linear system by matrix inversion. Solution of matrix equations. Powers of square matrices, matrix polynomials.
8. Vector space and subspace of vector space. Linear hull of vectors and space of solutions of linear system. Sum and intersection of linear subspaces. Theorem on relation between dimensions of sum and intersection of linear subspaces.
9. Determinant as polylinear and antisymmetric function. Properties and applications. Calculation of determinants.
10. Dot (Scalar) Product. Euclidean space. Orthogonalization.
11. Projection on hyperplane, orthogonal component. Distance problems. Distance between a point and a hyperplane.
12. Bilinear and quadratic forms, introduction. Diagonalization of quadratic form.
13. Affine transformations.
14. Quadratic curves in the plane. Classification of quadratic curves.
15. Quadratic surfaces. Classification of quadratic surfaces.