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ABSTRACTS

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On Equivariant Boundary Value Problems and Some Applications

V. P. Burskii
Moscow Institute Physics and Technology, Moscow, Russia

Let Ω be an arbitrary bounded domain in the space $\mathbf{R}^n$ with the boundary $\partial\Omega$ and $\mathcal{L} = \sum_{|\alpha|\leqslant m} a_\alpha(x)D^\alpha, D^\alpha = (-i\partial)^{|\alpha|}/\partial x_1^{\alpha_1}...\partial x_n^{\alpha_n}$, $\alpha \in \mathbf{Z}_+^n, |\alpha| = \sum_k \alpha_k$ be some formally self-ajoint differential operation with smooth complex matrix coefficients $a_\alpha(x)$, i.e. their elements belong to $C^\infty(\overline{\Omega})$. Let L_0 and $L = (L_0)^*$ be the minimal and the maximal operators for $\mathcal{L}$ with domains $D(L_0)$ and $D(L)$, $C(L) = D(L)/D(L_0)$ be the boundary space, and $\Gamma : D(L) \to C(L)$ the factor-mapping. The boundary value problem $Lu = f, \Gamma u \in B \subset C(L)$ is called well-posed if the corresponding expansion $L_B = L|_{D(L_B)}$, $D(L_B) = \Gamma^{-1}B$ has a continuous two-sided inverse operator.

Let G be a Lie group acting smoothly in the closed domain $\overline{\Omega}$ and on the boundary $\partial\Omega$, and let this action preserve the volume of the domain. Let the differential operation $\mathcal{L}$ be invariant with respect to the group action, that is $g(\mathcal{L}u) = \mathcal{L}(gu)$. Then the spaces $D(L)$, $D(L_0), C(L)$ are invariant with respect to the action of the group G. The boundary value problem $Lu = f, \;\; \Gamma u \in B$, is called G-invariant if the space B is invariant with respect to the action of G. If the group G is compact, then, as is well known, the Hilbert representation space is decomposed into the direct sum of finite-dimensional invariant subspaces of irreducible representations. And if the group is also commutative, then such representations are one-dimensional. Let the representation space of the group G be the boundary space $C(L)$. If the group is compact, then we have the decompositions

$C(L) = \sum_{k=0}^\infty \oplus \tilde{C}^k, \; C(\ker L) = \sum_{k=0}^\infty \oplus C^k(\ker L), \; B = \sum_{k=0}^\infty \oplus B^k$.

If our G-invariant boundary value problem is well-posed, then the decomposition into the direct sum $C(L) = C(\ker L) \oplus B$ appears as decompositions into the direct sums $C^k := C^k(\ker L) \oplus B^k = \sum_l \tilde{C}^{k_l}$ with finite-dimensional projectors $\Pi^k : C^k \to C^k(\ker L)$ along B^k, and thus the check of **well-posedness of the G-invariant boundary boundary value problem** is reduced to **verification of the following two properties**:

$$1) \; C^k(\ker L) \cap B^k = 0; \quad 2) \; \exists \varkappa > 0, \; \forall k, \; \|\Pi^k\|_{C^k} < \varkappa.$$

About applications. We investigate the spectrum of the general well-posed SO-equivariant boundary value problem for the Poisson equation in a disk and in a ball, distinguishing violation of well-posedness of the same problem for the Helmholz equation as violation of exactly the first property. For the fulfilment of the second one is a consequence of well-posedness of the problem for the Poisson equation. One more application is related to the quantum mechanics. We consider the Schrödinger equation for a hydrogen-like atom with Coulomb potential and non-point ball nucleus. The eigenvalues and eigenfunctions of the operator given by an arbitrary rotation-invariant boundary value problem on the spherical boundary of the nucleus are found. The eigenvalues prove to be independent on the choice of any such boundary value problem, being the same as for the point nucleus, although the corresponding eigenfunctions are essentially different.

References

[1] Burskii V. P. Investigation methods of boundary value problems for general differ-