Московский физико-технический институт
(государственный университет)

ЭКЗАМЕНАЦИОННАЯ РАБОТА

Дисциплина: **Уравнения математической физики** **Год: 1998/99**

Вариант: **1** **Курс:** **3** **Семестр:** **осенний**

Решить задачи:

1.
$$3y^2u_{xy} + u_{yy} + 9y^4u_x + \left(3y^2 - \frac{2}{y}\right)u_y = 0\ ,\ \ y<0\ ,\ \ x\in\mathbb{R}^1\ ;$$
$$u\Big|_{y=-1} = x+1, \qquad u_y\Big|_{y=-1} = 3(2-x), \quad x\in\mathbb{R}^1.$$

2.
$$u_{tt} = 9u_{xx} - 18e^x\sin 3t, \qquad t>0, \quad x>0\ ;$$
$$u\Big|_{t=0} = 2\sin x, \qquad u_t\Big|_{t=0} = 3e^x + 6\cos x, \quad x\geqslant 0;$$
$$u_x\Big|_{x=0} = 2\cos 3t + 3t, \quad t\geqslant 0.$$

3.
$$u_t = u_{xx} - \pi e^{-t}, \qquad t>0, \quad 0<x<\frac{\pi}{2}\ ;$$
$$u\Big|_{t=0} = 2x, \quad 0\leqslant x\leqslant\frac{\pi}{2};$$
$$u_x\Big|_{x=0} = 2e^{-t}, \qquad u\Big|_{x=\pi/2} = \pi e^{-t}, \quad t\geqslant 0.$$

4.
$$\Delta u = 12r^2\sin 2\varphi, \qquad r=\sqrt{x^2+y^2}, \quad 1<r<2\ ;$$
$$u_r\Big|_{r=1} = 4\sin 4\varphi + 24\cos 3\varphi + 4\sin 2\varphi,$$
$$u\Big|_{r=2} = 16\sin 4\varphi - \cos 3\varphi + 16\sin 2\varphi - 2.$$

5.
$$u_{tt} = \Delta u + \left(x^2 - \frac{5}{8}y^2 - \frac{3}{8}z^2\right)\operatorname{ch} t, \qquad t>0, \quad (x,y,z)\in\mathbb{R}^3\ ;$$
$$u\Big|_{t=0} = (x^2+y^2+z^2)\left(\operatorname{sh}\sqrt{x^2+y^2+z^2}\right)^3, \quad u_t\Big|_{t=0} = xe^z,\ (x,y,z)\in\mathbb{R}^3.$$

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Дисциплина: **Уравнения математической физики** **Год: 1998/99**

Вариант: **2** **Курс:** **3** **Семестр:** **осенний**

Решить задачи:

1. $$2xu_{xx} + 3x^3u_{xy} + (6x^3-4)u_x + 9x^5u_y = 0, \qquad x>0, \quad y\in\mathbb{R}^1\ ;$$
$$u\Big|_{x=1} = 2y-1, \qquad u_x\Big|_{x=1} = 6(1-y), \quad y\in\mathbb{R}^1.$$

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2. $$u_{tt} = 25u_{xx} - 50e^{5t}\cos x, \qquad t>0, \quad x>0\ ;$$
$$u\Big|_{t=0} = \cos x, \qquad u_t\Big|_{t=0} = 10\sin x - 5\cos x, \qquad x\geqslant 0;$$
$$u_x\Big|_{x=0} = 15t - \sin 5t, \quad t\geqslant 0.$$

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3. $$u_{tt} = u_{xx} + (\pi x - x^2 - 9)\sin 3t, \qquad t>0, \quad 0<x<\pi\ ;$$
$$u\Big|_{t=0} = 0, \qquad u_t\Big|_{t=0} = 3, \qquad 0\leqslant x\leqslant \pi;$$
$$u\Big|_{x=0} = \sin 3t, \qquad u\Big|_{x=\pi} = \sin 3t, \quad t\geqslant 0.$$

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4. $$\Delta u = 16r^3\sin 3\varphi, \qquad r=\sqrt{x^2+y^2}, \qquad 1<r<2\ ;$$
$$u_r\Big|_{r=1} = \sin 5\varphi - \cos 3\varphi + 5\sin 3\varphi,$$
$$u_r\Big|_{r=2} = 16\sin 5\varphi - \frac{1}{16}\cos 3\varphi + 80\sin 3\varphi.$$

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5. $$17u_t = \Delta u + te^{-3t}, \qquad t>0, \quad (x,y,z)\in\mathbb{R}^3\ ;$$
$$u\Big|_{t=0} = (15x+8y)e^{-\frac{(15x+8y)^2}{2}}\cos\left(\frac{\pi}{12} - z\right).$$

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Дисциплина: **Уравнения математической физики** **Год: 1998/99**

Вариант: **3** **Курс:** **3** **Семестр:** **осенний**

Решить задачи:

1. $$u_{xx} - 6x^2 u_{xy} - \left(6x^2 + \frac{2}{x}\right) u_x + 36x^4 u_y = 0, \qquad x > 0, \quad y \in \mathbb{R}^1 ;$$
$$u\Big|_{x=1} = y - 2, \qquad u_x\Big|_{x=1} = 6(y+2), \quad y \in \mathbb{R}^1.$$

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2. $$u_{tt} = 4u_{xx} - 8e^x \cos 2t, \qquad t > 0, \quad x > 0 ;$$
$$u\Big|_{t=0} = 2\cos x + e^x, \qquad u_t\Big|_{t=0} = -4\sin x, \quad x \geqslant 0;$$
$$u_x\Big|_{x=0} = \cos 2t - 4t, \quad t \geqslant 0.$$

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3. $$u_t = u_{xx}, \qquad t > 0, \quad 0 < x < \frac{\pi}{2} ;$$
$$u\Big|_{t=0} = 3x, \qquad 0 \leqslant x \leqslant \frac{\pi}{2};$$
$$u\Big|_{x=0} = 0, \qquad u_x\Big|_{x=\pi/2} = 3e^{-t}, \quad t \geqslant 0.$$

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4. $$\Delta u = 12r^2 \cos 2\varphi, \qquad r = \sqrt{x^2 + y^2}, \quad 1 < r < 2 ;$$
$$u_r\Big|_{r=1} = 4\cos 4\varphi - 24\sin 3\varphi + 4\cos 2\varphi,$$
$$u\Big|_{r=2} = 16\cos 4\varphi + \sin 3\varphi + 16\cos 2\varphi + 1.$$

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5. $$u_{tt} = \Delta u + \left(\frac{x^2}{2} - \frac{y^2}{4} - \frac{z^2}{4}\right)\cos t, \qquad t > 0, \quad (x,y,z) \in \mathbb{R}^3 ;$$
$$u\Big|_{t=0} = (x^2 + y^2 + z^2)\left(\sin\sqrt{x^2 + y^2 + z^2}\right)^3,$$
$$u_t\Big|_{t=0} = ze^y, \qquad (x,y,z) \in \mathbb{R}^3.$$

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Дисциплина: **Уравнения математической физики** **Год: 1998/99**

Вариант: **4** **Курс:** **3** **Семестр:** **осенний**

Решить задачи:

1. $$y^3 u_{xy} - y u_{yy} - 3y^5 u_x + (2+3y^3) u_y = 0, \qquad y>0, \quad x \in \mathbb{R}^1\ ;$$
$$u\Big|_{y=1} = 1+3x, \qquad u_y\Big|_{y=1} = 3(4+3x), \qquad x \in \mathbb{R}^1.$$

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2. $$u_{tt} = 16u_{xx} + 32e^{4t}\sin x, \qquad t>0, \quad x>0\ ;$$
$$u\Big|_{t=0} = -\sin x, \qquad u_t\Big|_{t=0} = 4\sin x + 8\cos x, \quad x \geqslant 0;$$
$$u_x\Big|_{x=0} = e^{4t} - \cos 4t - 1, \qquad t \geqslant 0.$$

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3. $$u_{tt} = u_{xx} - 2\cos 3t, \qquad t>0, \quad 0<x<\pi\ ;$$
$$u\Big|_{t=0} = x^2, \qquad u_t\Big|_{t=0} = 0, \qquad 0 \leqslant x \leqslant \pi;$$
$$u_x\Big|_{x=0} = 0, \qquad u_x\Big|_{x=\pi} = 2\pi\cos 3t, \quad t \geqslant 0.$$

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4. $$\Delta u = 16r^3\cos 3\varphi, \qquad r=\sqrt{x^2+y^2}, \quad 1<r<2\ ;$$
$$u_r\Big|_{r=1} = \frac{5}{16}\cos 5\varphi + 5\cos 3\varphi - 48\sin 3\varphi,$$
$$u_r\Big|_{r=2} = 5\cos 5\varphi - 3\sin 3\varphi + 80\cos 3\varphi.$$

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5. $$13u_t = \Delta u + te^{2t}, \qquad t>0, \quad (x,y,z)\in\mathbb{R}^3\ ;$$
$$u\Big|_{t=0} = (12x+5y)e^{-\frac{(12x+5y)^2}{2}}\sin\left(\frac{\pi}{8} - z\right).$$

1.⑤ $\xi = x\ ,\ \eta = x - y^3\ . \qquad u_{\xi\eta} = u_\xi\ .$

$$u(x,y) = \varphi(x)e^{x-y^3} + \psi(x-y^3) = 3 + (x-2)e^{-1-y^3}\ .$$

2.⑤ $u(x,t) = e^x \sin 3t + 2\sin(x+3t) +$

$$+\begin{cases} 0, & \text{если } x \geqslant 3t \geqslant 0, \\ -\cos(x-3t) - \frac{1}{2}(x-3t)^2 + 1, & 0 \leqslant x \leqslant 3t. \end{cases}$$

3.⑦ $u = v + 2xe^{-t}\ .$

$$v_t - v_{xx} = (2x-\pi)e^{-t}\ , \quad v|_{t=0} = v_x|_{x=0} = v|_{x=\pi/2} = 0\ .$$

$$\lambda_n = (2n+1)^2\ ,\ X_n(x) = \cos(2n+1)x \quad (n = 0, 1, \ldots)\ .$$

$$2x - \pi = \sum_{n=0}^{\infty} \alpha_n \cos(2n+1)x\ ,\ \alpha_n = \frac{-8}{\pi(2n+1)^2}\ .$$

$$v = \sum_{n=0}^{\infty} T_n(t)\cos(2n+1)x\ .$$

$$T_n' + (2n+1)^2 T_n = \alpha_n \cdot e^{-t}\ ,\ T_n(0) = 0\ .\ \ T_0(t) = \alpha_0 t \cdot e^{-t}\ .$$

$u(x,t) =$

$$= 2xe^{-t} + \alpha_0 te^{-t}\cdot\cos x + \sum_{n=1}^{+\infty} \frac{\alpha_n}{4n(n+1)}\left(e^{-t} - e^{-(2n+1)^2 t}\right)\cos(2n+1)x\ .$$

4.④ $u = v + r^4 \sin 2\varphi\ .$

$$\Delta v = 0\ ,\ v_r|_{r=1} = 4\sin 4\varphi + 24\cos 3\varphi\ ,\ v|_{r=2} = 16\sin 4\varphi - \cos 3\varphi - 2\ .$$

$$u(r,\varphi) = r^4(\sin 2\varphi + \sin 4\varphi) - \frac{8}{r^3}\cos 3\varphi - 2\ .$$

5.⑥ $u(x,y,z,t) = \left(x^2 - \frac{5}{8}y^2 - \frac{3}{8}z^2\right)(\operatorname{ch} t - 1) + xe^z \operatorname{sh} t +$

$$+\frac{1}{2r}\left((r+t)^3 \operatorname{sh}^3(r+t) + (r-t)^3 \operatorname{sh}^3|r-t|\right)\ , \text{ где}$$

$$r = \sqrt{x^2+y^2+z^2}\ .$$

98/99 **МФТИ-92**

1.⑤ $\xi = 2y - x^3\ ,\ \ \eta = y\ ;\ \ u_{\xi\eta} = u_\eta\ ;$

$$u(x,y) = \varphi(y)e^{2y-x^3} + \psi(2y - x^3) = 1 + 2(y-1)e^{1-x^3}\ .$$

2.⑤ $u(x,t) = -e^{5t}\cos x + 2\cos(x - 5t) +$

$$+\begin{cases} 0, & \text{если } x \geqslant 5t \geqslant 0, \\ -3\cos(x-5t) - \frac{3}{2}(x-5t)^2 + 3, & 0 \leqslant x \leqslant 5t. \end{cases}$$

3.⑦ $u = v + \sin 3t\ .$

$$v_{tt} = v_{xx} + x(\pi - x)\sin 3t\ ,\ \ v|_{t=0} = v_t|_{t=0} = v|_{x=0} = v|_{x=\pi} = 0\ .$$

$$\lambda_n = n^2\ ,\ \ X_n(x) = \sin nx \quad (n \in \mathbb{N})\ .$$

$$x(\pi - x) = \sum_{n=1}^{+\infty} \alpha_n \cdot \sin nx\ ,\ \ \alpha_n = \frac{4}{\pi n^3}(1 - (-1)^n)\ .$$

$$v = \sum_{n=0}^{+\infty} T_n(t)\sin nx\ .$$

$$T_n'' + n^2 T_n = \alpha_n \sin 3t\ ,\ \ T_n(0) = 0\ ,\ \ T_n'(0) = 0\ .\ \ T_{n=2k}(t) \equiv 0\ .$$

$$T_3(t) = \frac{\alpha_3}{18}(\sin 3t - 3t\cos 3t)\ .$$

$$u(x,t) = \sin 3t + \frac{\alpha_3}{18}(\sin 3t - 3t\cos 3t)\sin 3x -$$

$$-\sum_{\substack{n=1, n\neq 3,\\ n=2k+1}}^{+\infty} \frac{\alpha_n}{n(n^2-9)}(3\sin nt - n\sin 3t)\sin nx\ .$$

4.④ $u = v + r^5 \sin 3\varphi\ .$

$$\Delta v = 0\ ,\ \ v_r|_{r=1} = \sin 5\varphi - \cos 3\varphi\ ,\ \ v_r|_{r=2} = 16\sin 5\varphi - \frac{1}{16}\cos 3\varphi\ .$$

$$u(r,\varphi) = r^5\left(\sin 3\varphi + \frac{1}{5}\sin 5\varphi\right) + \frac{1}{3r^3}\cos 3\varphi + C\ .$$

5.⑥ $u(x,y,z,t) =$

$$= \frac{1}{153} - \frac{e^{-3t}}{51}\left(t + \frac{1}{3}\right) + \frac{15x + 8y}{(1+34t)^{3/2}}\, e^{-\frac{(15x+8y)^2}{2+68t} - \frac{t}{17}} \cos\left(\frac{\pi}{12} - z\right)\ .$$

1.⑤ $\xi = y\ ,\ \ \eta = 2x^3 + y\ ;\ \ u_{\xi\eta} = u_\xi\ ;$

$$u(x,y) = \varphi(y)e^{y+2x^3} + \psi(y+2x^3) = (2+y)e^{2x^3-2} - 4\ .$$

2.⑤ $u(x,t) = e^x \cos 2t + 2\cos(x+2t) +$

$$+\begin{cases} 0, & \text{если } x \geqslant 2t \geqslant 0, \\ 2\cos(x-2t) + (x-2t)^2 - 2, & 0 \leqslant x \leqslant 2t. \end{cases}$$

3.⑦ $u = v + 3xe^{-t}\ .$

$$v_t = v_{xx} + 3x\cdot e^{-t}\ ,\ \ v|_{t=0} = v|_{x=0} = v_x|_{x=\pi/2} = 0\ .$$

$$\lambda_n = (2n+1)^2\ ,\ \ X_n(x) = \sin(2n+1)x \quad (n = 0,1,2\ldots)\ .$$

$$3x = \sum_{n=0}^{+\infty} \alpha_n \cdot \sin(2n+1)x\ ,\ \ \alpha_n = \frac{12(-1)^n}{\pi(2n+1)^2}\ .$$

$$v = \sum_{n=0}^{+\infty} T_n(t)\sin(2n+1)x\ .$$

$$T_n' + (2n+1)^2 T_n = \alpha_n e^{-t}\ ,\ \ T_n(0) = 0\ .\ \ T_0(t) = \alpha_0 t e^{-t}\ .$$

$$u(x,t) =$$

$$= 3xe^{-t} + \alpha_0 t e^{-t}\sin x + \sum_{n=1}^{+\infty} \frac{\alpha_n}{4n(n+1)}\left(e^{-t} - e^{-(2n+1)^2 t}\right)\sin(2n+1)x\ .$$

4.④ $u = v + r^4\cos 2\varphi\ . \qquad \Delta v = 0\ ,$

$$v_r|_{r=1} = 4\cos 4\varphi - 24\sin 3\varphi\ ,\ \ v|_{r=2} = \sin 3\varphi + 16\cos 4\varphi + 1\ .$$

$$u(r,\varphi) = r^4(\cos 2\varphi + \cos 4\varphi) + \frac{8}{r^3}\sin 3\varphi + 1\ .$$

5.⑥ $u(x,y,z,t) = \left(\dfrac{x^2}{2} - \dfrac{y^2}{4} - \dfrac{z^2}{4}\right)(1-\cos t) + ze^y \operatorname{sh} t +$

$$+\frac{1}{2r}\left((r+t)^3\sin^3(r+t) + (r-t)^3\sin^3|r-t|\right)\ ,\ \text{где}$$

$$r = \sqrt{x^2+y^2+z^2}\ .$$

98/99 **МФТИ-94**

1.⑤ $\xi = x\ ,\ \eta = 3x + y^3\ ;\ u_{\xi\eta} = u_\xi\ ;$

$$u(x,y) = \varphi(x)e^{3x+y^3} + \psi(3x+y^3) = (4+3x)e^{y^3-1} - 3\ .$$

2.⑤ $u(x,t) = e^{4t}\sin x - 2\sin(x-4t) +$

$$+\begin{cases} 0, & \text{если } x \geqslant 4t \geqslant 0, \\ \sin(x-4t) - (x-4t), & 0 \leqslant x \leqslant 4t. \end{cases}$$

3.⑦ $u = v + x^2\cos 3t\ .$

$$v_{tt} = v_{xx} + 9x^2\cos 3t\ ,\ v|_{t=0} = v_t|_{t=0} = v_x|_{x=0} = v_x|_{x=\pi} = 0\ .$$

$$\lambda_n = n^2\ ,\ X_n(x) = \cos nx \quad (n = 0,1,2\ldots)\ .$$

$$9x^2 = \sum_{n=0}^{+\infty} \alpha_n \cos nx\ ,\ \alpha_n = (-1)^n\frac{36}{n^2} \quad (n \in \mathbb{N})\ ,\ \alpha_0 = 3\pi^2\ .$$

$$v = \sum_{n=0}^{+\infty} T_n(t)\cos nx\ .$$

$$T_n'' + n^2T_n = \alpha_n\cos 3t\ ,\ T_n(0) = 0\ ,\ T_n'(0) = 0\ .$$

$$T_0(t) = \frac{\alpha_0}{9}(1-\cos 3t)\ ,\ T_3(t) = \frac{\alpha_3}{6}t\sin 3t\ .$$

$$u(x,t) = x^2\cos 3t + \frac{\alpha_0}{9}(1-\cos 3t) + \frac{\alpha_3}{6}t\sin 3t\cos 3x +$$

$$+\sum_{n=1, n\neq 3}^{+\infty} \frac{\alpha_n}{n^2-9}(\cos 3t - \cos nt)\cos nx\ .$$

4.④ $u = v + r^5\cos 3\varphi\ . \quad \Delta v = 0\ ,\ v_r|_{r=1} = \frac{5}{16}\cos 5\varphi - 48\sin 3\varphi\ ,$

$$v_r|_{r=2} = 5\cos 5\varphi - 3\sin 3\varphi\ .$$

$$u(r,\varphi) = r^5\left(\cos 3\varphi + \frac{1}{16}\cos 5\varphi\right) + \frac{16}{r^3}\sin 3\varphi + C\ .$$

5.⑥ $u(x,y,z,t) =$

$$= \frac{1}{52} - \frac{1}{26}\left(t + \frac{1}{2}\right)e^{-2t} + \frac{12x+5y}{(1+26t)^{3/2}}e^{-\frac{(12x+5y)^2}{2+52t} - \frac{t}{13}}\sin\left(\frac{\pi}{8} - z\right)\ .$$