Московский физико-технический институт
(государственный университет)

ЭКЗАМЕНАЦИОННАЯ РАБОТА

Дисциплина: **Уравнения математической физики** **Год: 2000/2001**

Вариант: **1** **Курс:** **3** **Семестр:** **осенний**

Решить задачи:

1.
$$2u_{xx}+u_{xy}-u_{yy}+u_x+u_y=0\,,\quad (x,y)\in\mathbb{R}^2\,;$$
$$u|_{y=0}=1+\frac{x}{3}\,,\quad u_y|_{y=0}=\frac{2}{3}\,,\quad x\in\mathbb{R}^1\,.$$

2.
$$u_{tt}=4u_{xx}\,,\quad x>0\,,\quad t>0\,;$$
$$u|_{t=0}=x^2\,,\quad u_t|_{t=0}=x\,,\quad x\geqslant 0\,;$$
$$u_x|_{x=0}=t\sin t\,,\quad t\geqslant 0\,.$$

3.
$$u_{tt}-u_{xx}-5u=-\frac{10xt}{\pi}+\pi e^{-t}x\left(x-\frac{\pi}{2}\right)\,,\quad 0<x<\frac{\pi}{2}\,,\quad t>0\,;$$
$$u|_{t=0}=\sin 4x\,,\quad u_t|_{t=0}=\frac{2x}{\pi}\,,\quad 0\leqslant x\leqslant\frac{\pi}{2}\,;$$
$$u|_{x=0}=0\,,\quad u|_{x=\frac{\pi}{2}}=t\,,\quad t\geqslant 0\,.$$

4.
$$\Delta u=12x\,,\quad 1<r<2\,,\quad r=\sqrt{x^2+y^2}\,;$$
$$u|_{r=1}=2\cos^3\varphi+1-\sin\varphi\cos\varphi\,;$$
$$u|_{r=2}=16\cos^3\varphi-4\sin\varphi\cos\varphi\,.$$

5.
$$u_{tt}-\Delta u=\cos(x-2y-2z+3t)\,,\quad (x,y,z)\in\mathbb{R}^3\,,\quad t>0\,;$$
$$u|_{t=0}=x(x^2+y^2+z^2)\,,\quad u_t|_{t=0}=0\,,\quad (x,y,z)\in\mathbb{R}^3\,.$$

Московский физико-технический институт
(государственный университет)

ЭКЗАМЕНАЦИОННАЯ РАБОТА

Дисциплина: **Уравнения математической физики** **Год: 2000/2001**

Вариант: **2** **Курс:** **3** **Семестр:** **осенний**

Решить задачи:

1.
$$u_{xx} - 3u_{xy} + 2u_{yy} - 6u_x + 6u_y = 0\,, \quad (x,y) \in \mathbb{R}^2\,;$$
$$u|_{y=-x} = e^{4x}\,, \quad u|_{y=-2x} = e^{6x} - x\,, \quad x \in \mathbb{R}^1\,.$$

2.
$$u_{tt} = 4u_{xx}\,, \quad x > 0\,, \quad t > 0\,;$$
$$u|_{t=0} = \cos x\,, \quad u_t|_{t=0} = 2\sin x\,, \quad x \geqslant 0\,;$$
$$u_x|_{x=0} = te^t\,, \quad t \geqslant 0\,.$$

3.
$$u_{tt} - u_{xx} - 13u = x(2 - 13t^2) + 9e^{-2t}\left(x - \frac{\pi}{3}\right)^2\,, \quad 0 < x < \frac{\pi}{3}\,, \quad t > 0\,;$$
$$u|_{t=0} = -\frac{\pi^2}{27}\,, \quad u_t|_{t=0} = \frac{2\pi^2}{27}\,, \quad 0 \leqslant x \leqslant \frac{\pi}{3}\,;$$
$$u_x|_{x=0} = t^2\,, \quad u_x|_{x=\frac{\pi}{3}} = t^2\,, \quad t \geqslant 0\,.$$

4.
$$\Delta u = 24\,r^3 \sin\varphi\,, \quad r < 1\,, \quad r = \sqrt{x^2 + y^2}\,;$$
$$u_r|_{r=1} = \cos\varphi\,.$$

5.
$$u_{tt} - \Delta u = \operatorname{sh}(6x - 2y - 3z + 7t)\,, \quad (x,y,z) \in \mathbb{R}^3\,, \quad t > 0\,;$$
$$u|_{t=0} = xy^2 + yz^2 + zx^2\,, \quad u_t|_{t=0} = 0\,, \quad (x,y,z) \in \mathbb{R}^3\,.$$

Московский физико-технический институт
(государственный университет)

ЭКЗАМЕНАЦИОННАЯ РАБОТА

Дисциплина: **Уравнения математической физики** **Год: 2000/2001**

Вариант: **3** **Курс:** **3** **Семестр:** **осенний**

Решить задачи:

1. $$u_{xx} - 2u_{xy} - 3u_{yy} - 2u_x - 2u_y = 0\,, \quad (x,y) \in \mathbb{R}^2\,;$$
$$u|_{y=0} = 1 - x\,, \quad u_y|_{y=0} = \frac{1}{3}\,, \quad x \in \mathbb{R}^1\,.$$

---

2. $$u_{tt} = u_{xx}\,, \quad x > 0\,, \quad t > 0\,;$$
$$u|_{t=0} = x\,, \quad u_t|_{t=0} = 4x\,, \quad x \geqslant 0\,;$$
$$u_x|_{x=0} = (t+1)\cos t\,, \quad t \geqslant 0\,.$$

---

3. $$u_{tt} - u_{xx} - 2u = 12x(1-t^2) + 3\pi e^{-t}\sin 2x\,, \quad 0 < x < \frac{\pi}{2}\,, \quad t > 0\,;$$
$$u|_{t=0} = -\frac{3}{5}\cos 3x\,, \quad u_t|_{t=0} = \frac{3}{5}\cos 3x\,, \quad 0 \leqslant x \leqslant \frac{\pi}{2}\,;$$
$$u_x|_{x=0} = 6t^2\,, \quad u|_{x=\frac{\pi}{2}} = 3\pi t^2\,, \quad t \geqslant 0\,.$$

---

4. $$\Delta u = 12y\,, \quad \frac{1}{2} < r < 1\,, \quad r = \sqrt{x^2+y^2}\,;$$
$$u|_{r=\frac{1}{2}} = \frac{1}{4}\sin^3\varphi + \frac{1}{2}\cos^2\varphi\,;$$
$$u|_{r=1} = 2\sin^3\varphi + \cos 2\varphi\,.$$

---

5. $$u_t - \Delta u = \mathrm{ch}\,(x - y + 2z + 6t)\,, \quad (x,y,z) \in \mathbb{R}^3\,, \quad t > 0\,;$$
$$u|_{t=0} = x^2y^2z^2\,, \quad (x,y,z) \in \mathbb{R}^3\,.$$

Московский физико-технический институт
(государственный университет)

ЭКЗАМЕНАЦИОННАЯ РАБОТА

Дисциплина: **Уравнения математической физики** **Год: 2000/2001**

Вариант: **4** **Курс:** **3** **Семестр:** **осенний**

Решить задачи:

1. $$u_{xx} - 4u_{xy} + 3u_{yy} - 7u_x + 7u_y = 0\,, \quad (x,y) \in \mathbb{R}^2\,;$$
$$u|_{y=-x} = e^{3x}\,, \quad u|_{y=-3x} = e^{7x} - 2x\,, \quad x \in \mathbb{R}^1\,.$$

---

2. $$u_{tt} = 9u_{xx}\,, \quad x > 0\,, \quad t > 0\,;$$
$$u|_{t=0} = \sin x\,, \quad u_t|_{t=0} = -3\cos x\,, \quad x \geqslant 0\,;$$
$$u_x|_{x=0} = (t+1)\,e^{t/2}\,, \quad t \geqslant 0\,.$$

---

3. $$u_{tt} - 2u_{xx} - 3u = (2x+1)(2-3t^2) + 6\pi e^{-t}\sin 2x\,, \quad 0 < x < \frac{\pi}{2}\,, \quad t > 0\,;$$
$$u|_{t=0} = \frac{3}{5}\sin 3x\,, \quad u_t|_{t=0} = -\frac{3}{5}\sin 3x\,, \quad 0 \leqslant x \leqslant \frac{\pi}{2}\,;$$
$$u|_{x=0} = t^2\,, \quad u_x|_{x=\frac{\pi}{2}} = 2t^2\,, \quad t \geqslant 0\,.$$

---

4. $$\Delta u = 12r^2\cos 2\varphi\,, \quad \frac{1}{2} < r < 1\,, \quad r = \sqrt{x^2+y^2}\,;$$
$$u_r|_{r=\frac{1}{2}} = 0\,, \quad u_r|_{r=1} = 0\,.$$

---

5. $$u_t - \Delta u = e^{-9t}\cos(2x - y + 2z)\,, \quad (x,y,z) \in \mathbb{R}^3\,, \quad t > 0\,;$$
$$u|_{t=0} = xy^2z^3 + \cos y\,\sin(x+y+z)\,, \quad (x,y,z) \in \mathbb{R}^3\,.$$

---

1. 4

$$\xi = x + 2y,\ \eta = x - y; \qquad 3u_{\xi\eta} + u_{\xi} = 0\,.$$

$$u(x,y) = f(x+2y)e^{\frac{y-x}{3}} + g(x-y) = e^{y} + \frac{x-y}{3}\,.$$

---

2. 4 $u(x,t) = \frac{5}{8}(x+2t)^2 +$

$$+\begin{cases} \frac{3}{8}(x-2t)^2, & x \geqslant 2t \geqslant 0,\\ \frac{5}{8}(x-2t)^2 - (x-2t)\cos\frac{x-2t}{2} + 2\sin\frac{x-2t}{2}, & 0 \leqslant x \leqslant 2t, \end{cases} =$$

$$=\begin{cases} x^2 + xt + 4t^2, & x \geqslant 2t \geqslant 0,\\ \frac{5}{4}x^2 + 5t^2 - (x-2t)\cos\frac{x-2t}{2} + 2\sin\frac{x-2t}{2}, & 0 \leqslant x \leqslant 2t. \end{cases}$$

---

3. 8 $u = v + \dfrac{2xt}{\pi}$. $\quad v_{tt} - v_{xx} - 5v = \pi e^{-t}x\left(x - \dfrac{\pi}{2}\right), \quad 0 < x < \dfrac{\pi}{2}, \quad t > 0;$

$$v|_{t=0} = \sin 4x, \quad v_t|_{t=0} = 0, \quad 0 \leqslant x \leqslant \frac{\pi}{2}; \quad v|_{x=0} = v|_{x=\frac{\pi}{2}} = 0, \quad t \geqslant 0\,.$$

$$\lambda_k = 4k^2, \quad X_k(x) = \sin\sqrt{\lambda_k}\,x, \quad (k \in \mathbb{N})\,.$$

$$x\left(x - \frac{\pi}{2}\right) = \sum_{k=1}^{\infty} a_k \sin 2kx, \quad a_{2m+1} = -\frac{2}{\pi(2m+1)^3}, \quad a_{2m+2} = 0\ (m = 0, 1, 2, \ldots).$$

$$v(x,t) = \sum_{k=1}^{\infty} T_k(t)\sin 2kx\,. \qquad T_1(t) = te^{-t} - \operatorname{sh} t\,, \quad T_2(t) = \cos\sqrt{11}\,t\,.$$

$$m \geqslant 1:\ T_{2m+2}(t) = 0,\ T_{2m+1}(t) = \frac{\pi a_{2m+1}}{16m(m+1)}\left(e^{-t} - \cos\omega_m t + \frac{\sin\omega_m t}{\omega_m}\right).$$

$$\omega_m = \sqrt{16m^2 + 16m - 1}\,.$$

$$u(x,t) = \frac{2xt}{\pi} + \left(te^{-t} - \operatorname{sh} t\right)\sin 2x + \cos\sqrt{11}t\ \sin 4x +$$

$$+\sum_{m=1}^{\infty} \frac{\pi a_{2m+1}}{16m(m+1)}\left(e^{-t} - \cos\omega_m t + \frac{\sin\omega_m t}{\omega_m t}\right)\sin 2(2m+1)x\,.$$

---

4. 4 $u = \dfrac{3}{2}r^3\cos\varphi + v$.

$$\Delta v = 0, \quad v|_{r=1} = \frac{1}{2}\cos 3\varphi + 1 - \frac{1}{2}\sin 2\varphi, \quad v|_{r=2} = 4\cos 3\varphi - 2\sin 2\varphi\,.$$

$$u(r,\varphi) = 1 - \frac{\ln r}{\ln 2} + \frac{r^3}{2}(\cos 3\varphi + 3\cos\varphi) - \frac{r^2}{2}\sin 2\varphi\,.$$

---

5. 6 $\widehat{u} = \dfrac{t}{6}\sin(x - 2y - 2z + 3t)$. $\quad u = \widehat{u} + v$.

$$v_{tt} - \Delta v = 0, \quad (x,y,z) \in \mathbb{R}^3, \quad t > 0;$$

$$v|_{t=0} = x(x^2+y^2+z^2), \quad v_t|_{t=0} = -\frac{1}{6}\sin(x - 2y - 2z), \quad (x,y,z) \in \mathbb{R}^3\,.$$

$$v = x(x^2+y^2+z^2) + 5t^2x - \frac{1}{18}\sin 3t\sin(x - 2y - 2z)\,.$$

$$u(x,y,z,t) = \frac{t}{6}\sin(x-2y-2z+3t) + x(x^2+y^2+z^2) + 5t^2x - \frac{1}{18}\sin 3t\sin(x-2y-2z)\,.$$

1. 4 $\xi = 2x + y$, $\eta = x + y$; $u_{\xi\eta} + 6u_\xi = 0$.

$$u(x,y) = f(2x+y)e^{-6x-6y} + g(x+y) = e^{2x-2y} + x + y\,.$$

2. 4 $$u(x,t) = \begin{cases} \cos(x-2t), & x \geqslant 2t \geqslant 0, \\ (x-2t+2)\,e^{t-x/2} - 1, & 0 \leqslant x \leqslant 2t. \end{cases}$$

3. 8 $u = v + xt^2$. $v_{tt} - v_{xx} - 13v = 9e^{-2t}\left(x - \frac{\pi}{3}\right)^2$, $0 < x < \frac{\pi}{3}$, $t > 0$;

$$v|_{t=0} = -\frac{\pi^2}{27}, \quad v_t|_{t=0} = \frac{2\pi^2}{27}, \quad 0 \leqslant x \leqslant \frac{\pi}{3}; \quad v_x|_{x=0} = v_x|_{x=\frac{\pi}{3}} = 0, \quad t \geqslant 0\,.$$

$$\lambda_k = 9k^2, \quad X_k(x) = \cos\sqrt{\lambda_k}\,x, \quad (k = 0, 1, 2, \dots)\,.$$

$$\left(x - \frac{\pi}{3}\right)^2 = \sum_{k=0}^{\infty} a_k \cos 3kx, \quad a_k = \frac{4}{9k^2} \ (k \neq 0), \quad a_0 = \frac{\pi^2}{27}.$$

$$v(x,t) = \sum_{k=0}^{\infty} T_k(t)\cos 3kx\,. \quad T_0(t) = -\frac{\pi^2}{27}e^{-2t}, \qquad T_1(t) = \frac{1}{2}\,\mathrm{sh}\,2t - te^{-2t}\,.$$

$$k \neq 0, 1: \quad T_k(t) = \frac{a_k}{k^2-1}\left(e^{-2t} - \cos\omega_k t + \frac{2\sin\omega_k t}{\omega_k}\right), \quad \omega_k = \sqrt{9k^2 - 13}\,.$$

$$u(x,t) = xt^2 - \frac{\pi^2}{27}e^{-2t} + \left(\frac{1}{2}\,\mathrm{sh}\,2t - te^{-2t}\right)\cos 3x +$$

$$+ \sum_{k=2}^{\infty} \frac{a_k}{k^2-1}\left(e^{-2t} - \cos\omega_k t + \frac{2\sin\omega_k t}{\omega_k}\right)\cos 3kx\,.$$

4. 4

$$u = r^5 \sin\varphi + v\,.$$

$$\Delta v = 0, \qquad v_r|_{r=1} = \cos\varphi - 5\sin\varphi\,.$$

$$u(r,\varphi) = (r^5 - 5r)\sin\varphi + r\cos\varphi + C\,.$$

5. 6

$$\widehat{u} = \frac{t}{14}\,\mathrm{ch}(6x - 2y - 3z + 7t)\,.$$

$$u = \widehat{u} + v\,.$$

$$v_{tt} - \Delta v = 0, \quad (x,y,z) \in \mathbb{R}^3, \quad t > 0;$$

$$v|_{t=0} = xy^2 + yz^2 + zx^2, \quad v_t|_{t=0} = -\frac{1}{14}\,\mathrm{ch}(6x - 2y - 3z), \quad (x,y,z) \in \mathbb{R}^3\,.$$

$$v = xy^2 + yz^2 + zx^2 + t^2(x+y+z) - \frac{1}{98}\,\mathrm{sh}\,7t\,\mathrm{ch}(6x - 2y - 3z)\,.$$

$$u(x,y,z,t) = \frac{t}{14}\,\mathrm{ch}(6x - 2y - 3z + 7t) + xy^2 + yz^2 + zx^2 + t^2(x+y+z) -$$

$$- \frac{1}{98}\,\mathrm{sh}\,7t\,\mathrm{ch}(6x - 2y - 3z)\,.$$

1. 4

$$\xi = 3x + y\,,\ \eta = x - y\,; \qquad 2u_{\xi\eta} - u_\xi = 0\,.$$

$$u = f(3x+y)e^{\frac{x-y}{2}} + g(x-y) = e^{-\frac{2}{3}y} - x + y\,.$$

---

2. 4 $u(x,t) = \dfrac{x+t}{2} + (x+t)^2 +$

$$+\begin{cases} \frac{x-t}{2} - (x-t)^2, & x \geqslant t \geqslant 0 \\ \frac{t-x}{2} + (x-t)^2 + (1+t-x)\sin(x-t) - \cos(x-t) + 1, & 0 \leqslant x \leqslant t \end{cases} =$$

$$= \begin{cases} 4xt + x, & x \geqslant t \geqslant 0, \\ 2x^2 + 2t^2 + t + (1+t-x)\sin(x-t) - \cos(x-t) + 1, & 0 \leqslant x \leqslant t. \end{cases}$$

---

3. 8 $u = v + 6xt^2\,.\ v_{tt} - v_{xx} - 2v = 3\pi e^{-t}\sin 2x\,,\ 0 < x < \dfrac{\pi}{2}\,,\ t > 0\,;$

$$v|_{t=0} = -\frac{3}{5}\cos 3x,\ v_t|_{t=0} = \frac{3}{5}\cos 3x,\ 0 \leqslant x \leqslant \frac{\pi}{2};\ v_x|_{x=0} = v|_{x=\frac{\pi}{2}} = 0,\ t \geqslant 0\,.$$

$$\lambda_k = (2k+1)^2, \qquad X_k(x) = \cos\sqrt{\lambda_k}\,x, \qquad (k = 0,1,2,\ldots)\,.$$

$$\sin 2x = \sum_{k=0}^{\infty} a_k \cos(2k+1)x, \qquad a_k = \frac{8}{\pi(3+2k)(1-2k)}\,.$$

$$v(x,t) = \sum_{k=0}^{\infty} T_k(t)\cos(2k+1)x\,. \quad T_0(t) = 4\left(\operatorname{sh} t - te^{-t}\right), \quad T_1(t) = -\frac{3}{5}e^{-t}\,.$$

$$k \neq 0,1: \quad T_k(t) = \frac{3\pi a_k}{4k(k+1)}\left(e^{-t} - \cos\omega_k t + \frac{\sin\omega_k t}{\omega_k}\right), \quad \omega_k = \sqrt{4k^2+4k-1}\,.$$

$$u(x,t) = 6xt^2 + 4\left(\operatorname{sh} t - te^{-t}\right)\cos x - \frac{3}{5}e^{-t}\cos 3x +$$

$$+ \sum_{k=2}^{\infty} \frac{3\pi a_k}{4k(k+1)}\left(e^{-t} - \cos\omega_k t + \frac{\sin\omega_k t}{\omega_k}\right)\cos(2k+1)x\,.$$

---

4. 4

$$u = \frac{3}{2}r^3\sin\varphi + v\,.$$

$$\Delta v = 0, \qquad v|_{r=\frac{1}{2}} = -\frac{1}{16}\sin 3\varphi + \frac{1}{4}\cos 2\varphi + \frac{1}{4}, \qquad v|_{r=1} = -\frac{1}{2}\sin 3\varphi + \cos 2\varphi$$

$$u(r,\varphi) = -\frac{\ln r}{4\ln 2} + \frac{r^3}{2}(3\sin\varphi - \sin 3\varphi) + r^2\cos 2\varphi\,.$$

---

5. 6 $\widehat{u} = \dfrac{t}{2}\operatorname{ch}(x-y+2z+6t) + \left(\dfrac{t}{2} + \dfrac{1}{12}\right)\operatorname{sh}(x-y+2z+6t)\,. \quad u = \widehat{u} + v\,.$

$$v_t - \Delta v = 0\,,\ (x,y,z) \in \mathbb{R}^3\,,\ t > 0\,;$$

$$v|_{t=0} = x^2y^2z^2 - \frac{1}{12}\operatorname{sh}(x-y+2z)\,,\ (x,y,z) \in \mathbb{R}^3\,.$$

$$v = (x^2+2t)(y^2+2t)(z^2+2t) - \frac{1}{12}e^{6t}\operatorname{sh}(x-y+2z)\,.$$

$$u(x,y,z,t) = \frac{t}{2}\operatorname{ch}(x-y+2z+6t) + \left(\frac{t}{2} + \frac{1}{12}\right)\operatorname{sh}(x-y+2z+6t) +$$

$$+ (x^2+2t)(y^2+2t)(z^2+2t) - \frac{1}{12}e^{6t}\operatorname{sh}(x-y+2z)\,.$$

1. 4

$$\xi = 3x + y\,,\ \eta = x + y\,; \qquad 2u_{\xi\eta} + 7u_{\xi} = 0\,.$$

$$u(x,y) = f(3x+y)e^{-\frac{7}{2}x-\frac{7}{2}y} + g(x+y) = e^{x-2y} + x + y\,.$$

---

2. 4

$$u(x,t) = \begin{cases} \sin(x-3t), & x \geqslant 3t \geqslant 0, \\ 2(x-3t+3)\,e^{\frac{1}{3}t-\frac{1}{6}x} - 6, & 0 \leqslant x \leqslant 3t. \end{cases}$$

---

3. 8

$$u = v + (2x+1)t^2\,.$$

$$v_{tt} - 2v_{xx} - 3v = 6\pi e^{-t}\sin 2x, \qquad 0 < x < \frac{\pi}{2}, \qquad t > 0;$$

$$v|_{t=0} = \frac{3}{5}\sin 3x, \qquad v_t|_{t=0} = -\frac{3}{5}\sin 3x, \qquad 0 \leqslant x \leqslant \frac{\pi}{2};$$

$$v|_{x=0} = v_x|_{x=\frac{\pi}{2}} = 0, \qquad t \geqslant 0\,.$$

$$\lambda_k = (2k+1)^2, \qquad X_k(x) = \sin\sqrt{\lambda_k}\,x, \qquad (k = 0,1,2,\ldots)\,.$$

$$\sin 2x = \sum_{k=0}^{\infty} a_k \sin(2k+1)x, \qquad a_k = \frac{8(-1)^k}{\pi(3+2k)(1-2k)}\,.$$

$$v(x,t) = \sum_{k=0}^{\infty} T_k(t)\sin(2k+1)x\,. \quad T_0(t) = 8\left(\operatorname{sh} t - te^{-t}\right), \quad T_1(t) = \frac{3}{5}e^{-t}\,.$$

$$k \neq 0,1: \quad T_k(t) = \frac{3\pi a_k}{4k(k+1)}\left(e^{-t} - \cos\omega_k t + \frac{\sin\omega_k t}{\omega_k}\right), \qquad \omega_k = \sqrt{8k^2+8k-1}\,.$$

$$u(x,t) = (2x+1)t^2 + 8\left(\operatorname{sh} t - te^{-t}\right)\sin x + \frac{3}{5}e^{-t}\sin 3x +$$

$$+ \sum_{k=2}^{\infty} \frac{3\pi a_k}{4k(k+1)}\left(e^{-t} - \cos\omega_k t + \frac{\sin\omega_k t}{\omega_k}\right)\sin(2k+1)x\,.$$

---

4. 4

$$u = r^4\cos 2\varphi + v\,.$$

$$\Delta v = 0, \qquad v_r|_{r=\frac{1}{2}} = -\frac{1}{2}\cos 2\varphi, \qquad v_r|_{r=1} = -4\cos 2\varphi\,.$$

$$u(r,\varphi) = \left(r^4 - \frac{21}{10}r^2 - \frac{1}{10\,r^2}\right)\cos 2\varphi + C\,.$$

---

5. 6

$$\widehat{u} = te^{-9t}\cos(2x - y + 2z)\,. \quad u = \widehat{u} + v\,.$$

$$v_t - \Delta v = 0, \qquad (x,y,z) \in \mathbb{R}^3, \qquad t > 0;$$

$$v|_{t=0} = xy^2z^3 + \cos y\,\sin(x+y+z) = xy^2z^3 + \frac{1}{2}[\sin(x+2y+z) + \sin(x+z)],$$

$$(x,y,z) \in \mathbb{R}^3\,.$$

$$v = x(y^2+2t)(z^3+6tz) + \frac{1}{2}e^{-6t}\sin(x+2y+z) + \frac{1}{2}e^{-2t}\sin(x+z)\,.$$

$$u(x,y,z,t) = te^{-9t}\cos(2x-y+2z) + x(y^2+2t)(z^3+6tz) +$$

$$+ \frac{1}{2}e^{-6t}\sin(x+2y+z) + \frac{1}{2}e^{-2t}\sin(x+z)\,.$$